Thinking Like Mathematicians: Challenging All Grade 3 Students

An Instructional Guide to Developing Rigorous, Differentiated Mathematics Lessons

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2022

Table of Contents

Introduction ········ 1
Proposing the Differentiating Mathematics by Design (DMbD) Model ········ 5
    Trusting the Design Process ········ 7
    Previewing the Instructional Guide ········ 8
    Stage 1 (Identify Desired Results): A Closer Look ········ 8
Anchoring the Lesson on Standards ········ 8
    Content Standards ········ 11
    General Process Standards ········ 14
Integrating Standards Into Big Ideas ········ 15
    Clarity of Content and Process ········ 15
    Connection to Overarching Concepts ········ 16
    Description of Authentic Applications ········ 17
Writing Student Learning Objectives ········ 18
    Stage 2 (Determine Acceptable Evidence): A Closer Look ········ 19
    Application to TLM Lesson: Strolling in Space: Preparing for a Space Walk ········ 21
    Stage 3 (Plan Learning Experiences and Instruction): A Closer Look ········ 22
    Remodeling Textbook Lessons ········ 22
    Designing New Lessons ········ 28
Conclusion ········ 39
References ········ 41

List of Tables

Table 1: Differentiation Strategies and Guiding Questions–Thinking Like Mathematicians: Challenging all Grade 3 Students 3

Table 2: Problem-Solving Storyboard Example (Barrell, 2010) 10

Table 3: Aligning Third and Fourth Grade Common Core State Standards 13

Table 4: Standards for Mathematical Practice (MP) 14

Table 5 Student Learning Objectives (SLOs): Know, Do, Understand 19

Table 6: 21st Century Skills (4Cs: Creativity, Critical Thinking, Collaboration, Communication: Partnership for 21st Century Learning, 2019) 28

Table 7: Talk Moves and Sample Prompts 34

Table 8: Sample Talk Moves Conversation 36

List of Figures

Figure 1: Differentiation of Instruction 2
Figure 2: Differentiating Mathematics by Design (DMbD) 6
Figure 3: Stage 1 (Identify Desired Results) 9
Figure 4: Overarching Concepts 16
Figure 5: Designing Big Ideas & Objectives 18
Figure 6: Stage 2 (Determine Acceptable Evidence) 20
Figure 7: Stage 3 (Plan Learning Experiences and Instruction) 23
Figure 8: Scaffold Examples 27
Figure 9: Launch: Thinking Like Mathematicians: Centering the Mathematical Practice—Sample Section 30
Figure 10: Explore: Communicating Information From Number Lines 32
Figure 11: Highlight Students' Mathematical Thinking 35
Figure 12: Tiered Lesson Water Gauges 37
Figure 13: Debrief Content and Mathematical Practices 38
Figure 14: Assess: What Students Learned 39

List of Appendices

Appendix A: Strolling in Space: Preparing for A Space Walk ··43

Thinking Like Mathematicians: Challenging All Grade 3 Students
An Instructional Guide to Developing Rigorous, Differentiated Mathematics Lessons[1]

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Introduction

Thinking Like Mathematicians (TLM): Challenging All Grade 3 Students is a federally funded project to develop, pilot, and evaluate a series of differentiated curriculum lessons for third grade math classrooms. When the TLM research team began the curriculum design process, we selected several curriculum models and existing theoretical frameworks to provide inspiration and underlying principles to guide curriculum lesson design; however, we initially struggled to operationalize these principles into a systematic design process. For example, we knew we could not design challenging math tasks by simply increasing the size of the numbers within the task; however, *consistently* designing differentiated tasks that truly facilitated deeper mathematical thinking proved to be our challenge. We knew differentiation models recommended adjusting the process, product, content, and learning environment but operationally, we needed to develop specific steps to reliably transform a generic lesson into a differentiated lesson.

Just as some students arrive at correct mathematical answers without showing their work, some curriculum designers develop differentiated lessons without articulating their process. Ideas seem to appear to these curriculum designers just as some characters appear fully formed to literary authors. Some TLM designers seemed to be able to develop lessons in this fashion; however, for others, it seemed random whether an idea would simply appear and worse, whether that idea would yield a rigorous, differentiated lesson. Further, at times it was a challenge for the team to provide concrete, step-by-step guidance to pre- and in-service teachers as they developed their own differentiated lessons.

Through these experiences, we recognized the need for a generalizable system to demystify differentiation, such that any curriculum writer and/or teacher could systematically develop differentiated mathematics' lessons and units. This instructional guide is the result of years of refining and reflecting upon the differentiation process. The purpose of this instructional guide is to delineate the process of designing differentiated math lessons that will support all students' growth, accounting for students' initial readiness levels. This guide provides a series of steps for differentiation as well as a glimpse into the minds of the TLM curriculum authors, including the rationale behind their design decisions. Importantly, just as differentiation is a dynamic process that requires deliberate reflection guiding future instructional adaptations, our curriculum writing model is also dynamic. We hope you will be able to take what we have learned and adapt it to meet your own needs.

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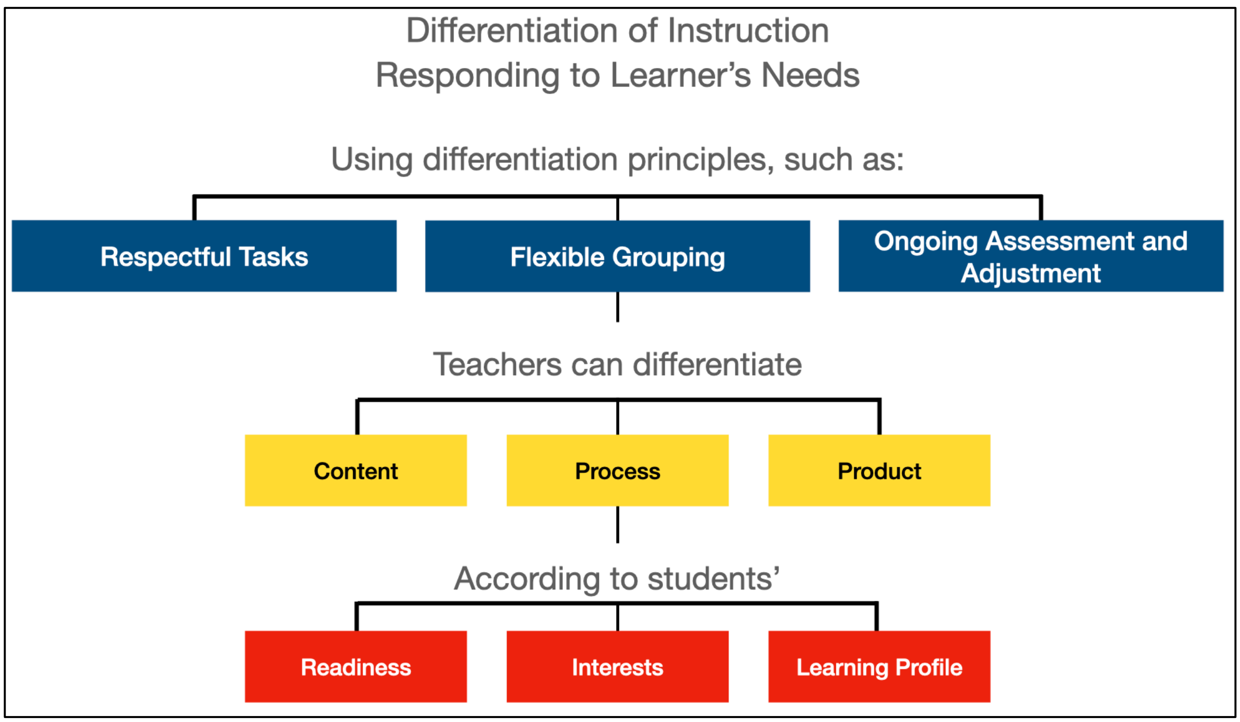
[1] This work was supported by Jacob K. Javits Gifted and Talented Students Education Program, United States Department of Education PR/Award # S206A170023, Thinking Like Mathematicians: Challenging All Grade 3 Students.

As you begin your journey, consider what you already know about differentiation and the specific strategies you already implement. Many educators are familiar with Tomlinson's (1999) model that details how teachers could respond to learners' needs (see Figure 1). Differentiation of instruction is guided by three general principles: respectful tasks, flexible grouping, and ongoing assessment and adjustment. The approach to differentiation is proactive by reflecting on content, process, and product. Key reflective questions include:

- Are students ready for the content based on their current knowledge, skills, and understanding?
- Do students have the necessary critical and/or creative thinking skills to process the content?
- How will interest-based content promote student achievement and engagement in the related activities and tasks?
- How do students like to learn: flexible grouping strategies: small group, whole group, or individually; visual, auditory, or figural scaffolds; technology options?

Figure 1

Differentiation of Instruction



(Adapted from Figure 2.1, Tomlinson, 1999, p. 15)

In addition to those broad questions, Gubbins (2021) created a detailed list of differentiation strategies with targeted questions to inspire the modification of content, process, product, and learning environment to address students' needs in academically diverse classrooms (see Table 1). This list can be used as a further check on what strategies can be selected and practiced now or in the future. Collectively, your thoughtful reflections will provide a foundation on which you can experiment with new techniques from this manual to enhance your pedagogical practice and promote student learning.

Table 1

Differentiation Strategies and Guiding Questions–Thinking Like Mathematicians: Challenging all Grade 3 Students

Content ***Guiding Questions***	
	• learning objectives *What do you want students to know, understand, and be able to do?*
	• prior knowledge or learner readiness *What evidence do you have about students' current knowledge and skills?*
	• tiered activities *How will you design tiered activities on the same mathematical concept with varied levels of difficulty?*
	• formative assessment *What techniques will you use to assess students' prior knowledge and skills?*
	• varied levels of challenge *How will you vary the level of difficulty for each tiered activity?*
	• "teaching up" (aim high, provide scaffolding) *How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?*
	• know (information, facts, vocabulary), understand (concepts, big ideas, connections), apply (skills, processes) *How will you ensure students have a deep understanding of mathematical concepts and skills?*
	• real-world application *What real-world connections will you make explicit about mathematical concepts and skills?*
Process **Guiding Questions**	
	• questioning strategies *How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?*
	• 4Cs (21st Century Skills) • critical thinking *How will you promote a learning environment in which students question data and view issues or problems from multiple perspectives?*
	• 4Cs (21st Century Skills) • creative thinking *How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?*

	• 4Cs (21st Century Skills) • collaboration *How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?*
	• 4Cs (21st Century Skills) • communication *How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?*
	• hands-on activities/manipulatives *How will you incorporate activities promoting the use of manipulatives to clarify or illustrate mathematical concepts?*
	• connections *How will you use "big ideas" to emphasize connections between and among mathematical concepts and skills and their connections to real-world situations?*
Product ***Guiding Questions***	
	• oral, visual, and written opportunities *How will you encourage students to represent their thinking and problem solving using different communication strategies?*
	• multiple ways to demonstrate knowledge, understanding, and skills *How will you encourage students to share their understanding of mathematical concepts and skills?*
	• multiple models and representations *What techniques of lesson design will you include to support students' deep understanding and the ability to apply mathematical concepts and skills?*
	• summative assessment *How will you assess student learning upon completion of the lesson?*
Learning Environment ***Guiding Questions***	
	• flexible grouping *How will you use your tiered lesson to support flexible grouping?*
	• whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?*
	• growth mindset *How will you promote the perspective that it is important to view abilities as malleable?*
	• learning community *How will you support a positive learning community as students are encouraged to think, work, and communicate like mathematicians?*

(Gubbins, 2021)

Proposing the Differentiating Mathematics by Design (DMbD) Model

While Tomlinson (1999) and Gubbins (2021) provided inspiration for reflection, question-asking, and lesson development, we, as curriculum designers needed a system, a step-by-step approach to integrate these fundamental components of differentiation into our curriculum units. The Differentiating Mathematics by Design (DMbD) Model was our team's solution to operationalize and systematize differentiating mathematics lessons. Our model is anchored on the broader Understanding by Design model (UbD; Wiggins & McTighe, 2005), which proposes using three stages (i.e., identifying the goals/objectives, developing assessments, and then, creating the learning experiences) to facilitate curricular alignment and provide a systematic approach to designing curriculum. UbD was developed to guide full curriculum units across domains; however, designing a single differentiated plan could make use of the same steps. While UbD provides a solid operational structure, it does not deliberately incorporate strategies for differentiation, depth, complexity, or enrichment. To begin to address this deficit, Tomlinson and McTighe (2006) co-authored a book that conceptualized how students' needs should be considered and addressed within each stage of the UbD framework. As our curriculum team explored these connections, we still did not have a systematic process to consistently promote instructional depth and complexity, especially within math curriculum.

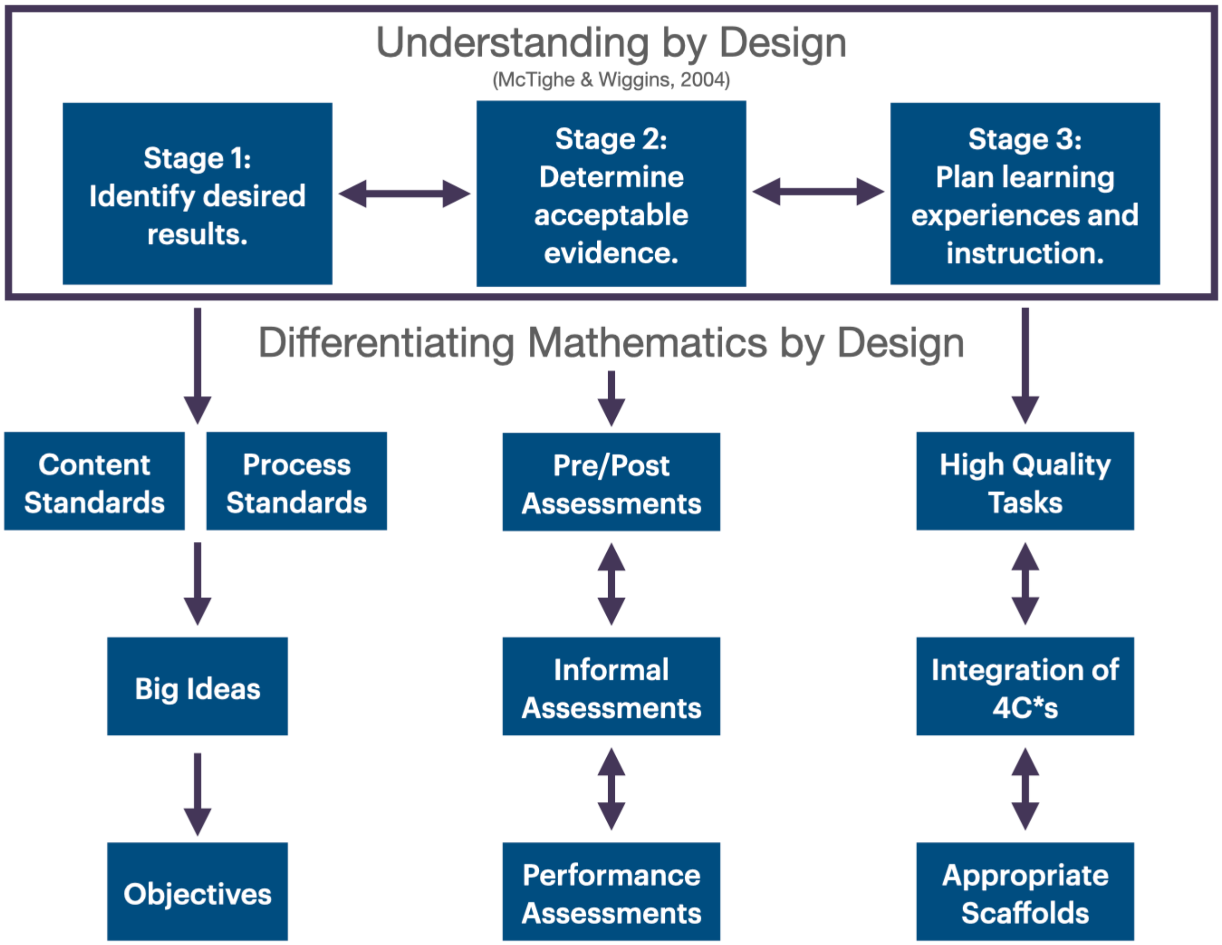
Mathematics presents unique challenges to curriculum differentiation. Historically, students have experienced minimal differentiated content, suggesting that teachers' own educational experiences contained few models of high-quality differentiation (Westberg et al., 1993). Further, elementary teachers often demonstrate significantly high levels of math anxiety stemming from their time as students (e.g., Gresham, 2018). High levels of math anxiety negatively affect teachers' self-efficacy for teaching both math and science (Bursal & Paznokas, 2006) and their ability to promote students' math achievement (Beilock et al., 2010; Schaeffer et al., 2020). This confluence of a lack of modelling, low mathematical self-efficacy, and high math anxiety may complicate an already challenging task of differentiating math curriculum.

To address this need, our team developed DMbD (see Figure 2) by expanding upon UbD in several concrete and fundamental ways. In **Stage 1 (Identify Desired Results)**, we dissect the standards into specific content understandings, discrete processes, and generalizable processes. Standards (e.g., Common Core State Standards [CCSS], National Governors Association Center/Council of Chief State School Officers, 2010) provide this information, but curriculum designers must take the time to explore the nuances before advancing to the next planning stages. In addition to these specific distinctions, the TLM team also translates these standards into big ideas, which we define as an initial summary that synthesizes the content and process standards with transferable concepts and authentic applications. *(Big ideas will be explored in more depth in subsequent sections.)*

These initial steps are as much for the lesson designer as they are for the students. This process helps designers internalize the goals and identify what can be adjusted and in what ways. During this stage, the instructional and pedagogical techniques may not be obvious yet, but it is important to trust the process and keep going.

Figure 2

Differentiating Mathematics by Design (DMbD)



*To prepare students within the 21st century, teachers are encouraged to incorporate the four Cs (i.e., creativity, critical thinking, collaboration, and communication) into their lesson plans. Each of the 4Cs will be discussed more thoroughly in the Stage 3: Plan Learning Experiences and Instruction section.

Stage 2 (Determine Acceptable Evidence) is guided by the big ideas, curriculum designers conceptualize multiple types of assessments, including pre/post, informal, and performance assessments. In general, these assessments can be created using generic assessment guidelines, such as providing clear expectations and multiple ways of demonstrating knowledge (Mizala Salcés et al., 2015). However, these assessments also need to uncover specific student understandings/misconceptions to enable teachers to design readiness groups and to determine levels of necessary scaffolding. These assessments are designed with differentiation in mind.

Stage 3 (Plan Learning Experiences and Instruction) requires additional considerations to differentiate math lessons. Using the previously identified goals and assessments, designers must carefully consider how to craft appropriate learning experiences to meet the varied needs of students. When designing the curriculum unit and lesson collection, we often started with high-quality mathematics tasks that can be scaffolded for struggling students and/or extended for advanced students. The selection of a high-quality math task is essential for meaningful differentiation. Some math tasks are too simple to provide opportunities for differentiation, leaving teachers with no options other than to simply incorporate bigger numbers into the problem to challenge advanced student mathematicians. Through the application of these stages, the TLM team ultimately designed the curriculum unit entitled *TLM If Aliens Taught Algebra: Multiplication and Division Would be out of This World!* (Cole et al., 2019a). They also developed challenging pre-differentiated and enriched math lessons with teachers to create the *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022).

Trusting the Design Process

In general, the compilation of these steps provides a reassuring process that when followed, yields a lesson that should address varied students' academic needs. As curriculum designers, we occasionally get lost, but we often return to Pixar's initial mantra: "Trust the process." This encourages designers that ideas require time to develop, that they should not panic if the product is not immediately perfect. "Just keep swimming," as Dory encourages Nemo. Within curriculum design, the process moves from Stage 1 (Identify Desired Results) to Stage 2 (Determine Acceptable Evidence) to Stage 3 (Plan Learning Experiences and Instruction).

While it may be tempting to skip stages, trusting the structured process will ensure that the lesson has the appropriate foundation. Stage 1 (Identify Desired Results) provides the essential foundation to ensure a lesson meets its objectives and provides high-quality differentiated learning experiences. This does not prohibit curriculum designers from *revisiting and refining* Stage 1 (Identify Desired Results) components after drafting an assessment plan or a learning experience. However, every curriculum designer must spend initial time dissecting the objectives, making connections, and identifying authentic processes. Without that deep thinking, the learning experiences may be fun, but the lesson will be more challenging to differentiate. Trust us, we know. We tried to skip steps. We thought we knew the standards well enough, and/or we had a great learning experience in mind. However, every time, we found ourselves needing to revisit Stage 1 (Identify Desired Results) because we got stuck with providing a differentiated learning experience. When we did not identify the mathematical process, we were unable to add appropriate scaffolds. When we did not identify the key concepts, we were unable to increase the conceptual challenge of activities. When we spent time dissecting and rewriting standards, crafting big ideas, and building student objectives, then, we were able to access the foundational knowledge that was necessary for all of the other stages. This stage cannot be rushed nor skipped, yet it can always be refined after subsequent stages are developed.

In addition to following the process, we also spent considerable time detailing each stage within our lesson plans. Teacher candidates (i.e., students training to become teachers) may question the process of writing detailed lesson plans, suggesting they will never do this when they are in their own classrooms. They grumble that the assignment is impractical. They may never be assigned to teach the grade or content of the lesson they just spent weeks developing. To some extent, their claims are accurate. However, designing detailed lessons is not about the actual 10-page product, but rather, it is about developing a transferable process, a way of thinking about lesson planning. Writing these detailed lessons gives teacher candidates an opportunity to consider what makes a good lesson, how to develop and generalize a pattern of thinking that can be applied in any classroom situation. They may choose to adopt the practices they find the most useful, or these actions may become automatic such that the long script is no longer necessary.

This process prepares teachers to differentiate math. Identifying the big ideas provides the anchor for the lesson, and then, the tasks can be extended or scaffolded, depending upon students' needs. Writing is thinking, so as the plan expands, so does the teachers' thinking. This process is important not only for developing original lessons, but perhaps more importantly for evaluating and supplementing existing math curriculum. Currently, the United States remains somewhat ambiguous in terms of curriculum, so teachers must be prepared to implement and adapt any curriculum they are provided within their specific context. Many curricula provide great learning experiences, but often, they are missing opportunities to truly challenge advanced learners. Therefore, DMbD can also be used to address gaps by both adapting or supplementing existing curriculum.

Previewing the Instructional Guide

Throughout this instructional guide, each stage of the design process will be explained in greater depth with sample questions and examples to support curriculum designers as they create their own differentiated lessons. Then, after each stage's description, the TLM team members will present an example of how these stages were applied to develop a third-grade lesson plan on fractions (i.e., Strolling in Space: Preparing for a Space Walk [Appendix A], which is one of the lessons from the *TLM Mission to Mars Lesson Collection* [Gubbins et al., 2022]). This lesson plan will be dissected throughout this guide to illuminate how the TLM team progressed through the design process, wrestled with design decisions, and ultimately produced a differentiated learning opportunity. Although the lesson will be presented in individual sections for the purpose of this instructional guide, we encourage curriculum designers to view the entire lesson as it appears in Appendix A. We often present the final lesson plans without explaining our own process; however, this mistakenly communicates that these lessons simply appeared to us. In reality, each lesson may have taken weeks to write and rewrite, even before piloting in the classroom and then revising again. We hope this "behind-the-scenes" tour of our process demystifies the differentiation process, providing a systematic, concrete approach to developing mathematics lessons that will challenge all learners.

Stage 1 (Identify Desired Results): A Closer Look

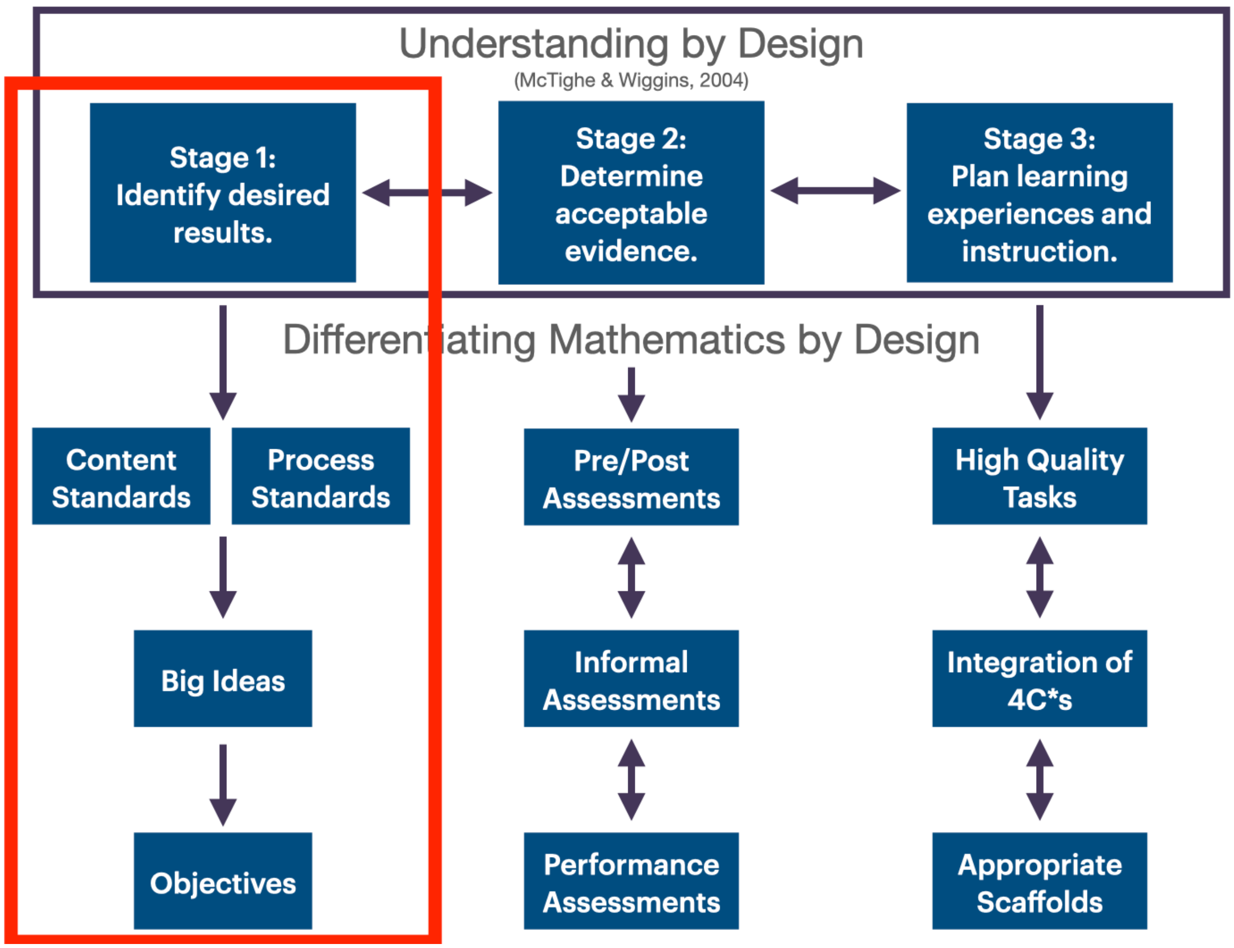
Stage 1 (Identify Desired Results) is a critical component of backwards design, including the construction of (a) transparent and defensible student outcomes that reflect content and process standards, (b) big ideas that reveal the purpose of the content and skills, and (c) the objectives that clearly link standards and big ideas (see Figure 3).

Anchoring the Lesson on Standards

We have witnessed teacher candidates write lesson plans, only asking for help to identify appropriate standards to match their lesson *after* the lesson had already been developed. These teacher candidates may erroneously believe the standards do not deserve (or require) initial consideration, that standards are merely a bureaucratic mandate to be tolerated. However, standards are often written to be helpful, as they offer conceptual insights and guidance for differentiating instruction when they are carefully considered and dissected. The TLM curriculum unit and lesson collection emphasize two key types of standards: mathematical content (i.e., what students should know) and mathematical processes (i.e., what students should be able to do). Before beginning to develop the lesson, the lesson designer must dissect, simplify, and be able to communicate both content and process standards separately.

Figure 3

Stage 1 (Identify Desired Results)



The CCSS anchor the TLM lessons because they provide the broadest guidance, and they often map onto other state standards. CCSS describe both (a) discrete processes, such as multiplying two-digit numbers or measuring using a ruler, and (b) general processes, such as constructing viable mathematical arguments. CCSS provide developmental progressions to delineate when certain content and discrete processes are introduced and mastered. Additionally, CCSS also provide a series of eight mathematical practices, which describes the general processes that all students need across all grade levels and domains. During this first stage of lesson design, we dissect these aspects separately, using clues from each to develop the differentiated options.

Let's consider CCSS.Math.Practice.MP.1: "Make sense of problems and persevere in solving them." The MP.1's description includes this guidance: [Students should] "analyze givens, constraints, relationships, and goals." That is a process that can be taught. Teachers could develop graphic organizers to support students in identifying givens, constraints, relationships, and goals. Now that we understand the process standard, we can integrate existing scaffolds and strategies to support student growth. For example, the Problem-Solving Storyboard Example (Barrell, 2010, see Table 2) provides guiding questions, like "What do you know?" or "What do you want to find out?" Importantly, all these scaffolds were inspired by first understanding which process standard the lesson would address.

Table 2

Problem-Solving Storyboard Example (Barrell, 2010)

1. What do you already know?	2. What do you need to find out?
3. How will you solve the problem?	4. What have you learned?
5. How can we apply this to other problems?	6. What new questions do you have?

Shifting to content standards, CCSS.Math.Content.3.NF.3.d states that students should ". . . recognize that comparisons are only valid when the two fractions refer to the same whole." While we may have many fraction comparison lessons already, when we go back to the original standard, we recognized a key phrase: "same whole." This could inspire scenarios in which the whole was not given or the wholes were different. For example, as students are working with the denominator 5 (comparing 1/5 to 4/5, 2/5 to 3/5), the teacher might introduce a fraction that does not have the denominator 5 (asking students to compare 3/5 to 6/8, 4/5 to 4/6). As we conceptually wrestle with this standard, we may become aware of how to develop students' understanding of this standard beyond numerical operations.

Both examples demonstrate how standards can be leveraged for inspiration, for building solid differentiated lessons. While both content and process standards must be deliberately assessed and taught, students may not need the same level of support in each. For example, one student may not be able to define "fraction" (i.e., content standard), but may be an excellent problem solver (i.e., process standard), whereas another student may be able to determine which fraction is larger but struggle with the ambiguity of open-ended questions. Thus, being able to delineate both the content and process standards provides the foundation for developing a tiered, differentiated lesson, designed to meet the needs of a variety of students. In the subsequent sections, we will explore both in more depth.

Content Standards

Many of the content standards (National Governors Association Center/Council of Chief State School Officers, 2010) are concise yet complicated. To unpack these standards, curriculum designers may find a series of steps helpful: (a) translate the standard into clear language for yourself, (b) identify the fundamental concept and purpose of the standard, and (c) situate the standard within the broader domain progression. The outcomes from each step serve as building blocks for constructing the lesson's big idea and developing differentiated learning experiences. Below is an example of how to dissect content standards for the sample lesson.

Translate the Standard

Some teachers and curriculum designers may find it helpful to translate the standards into language that is more accessible to them. Most math standards are both precise and compact; however, this precision may make it more challenging to make sense of them. Consider the following example:

> *CCSS.Math.Content.3.NF.2.a:* Represent a **fraction** 1/*b* on a **number line** diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* **equal parts**. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. (National Governors Association Center/Council of Chief State School Officers, 2010, p. 24)

This standard contains cumbersome notations and language, which needs to be simplified to capture the essence of the standard. So, as curriculum designers, we started by dissecting the standard to clarify the variables (see italics above). The **basic** components require students to define the core features of a **fraction**, use a **number line** to display a fraction, and describe the importance of **equal parts** for fractions. While this may seem basic, this process will position designers to be successful in the next stages.

Identify the Purpose

The next step that curriculum designers must take is to identify the fundamental concept and purpose of the standard. Once the standard's language has been simplified and, hopefully, internalized, the designer should consider the standard's fundamental, transferrable concept and its purpose. Why is this standard important? This question can be repeated to uncover additional nuances of the standard. Again, using the above standard as the example, note how the questions below continue to prompt deeper understandings for the lesson designer:

- *Why do fractions matter?* Without fractions, we would not be able to conceptualize parts of a whole.
- *Why do we need to understand parts of a whole?* We need to be able to provide precise responses when a quantity is between two whole numbers.
- *Why are equal parts important to fractions?* Fractions provide a consistent frame of reference to be able to communicate parts of a whole.
- *Why is it important to understand fractions on number lines?* Number lines provide a visual of how much of a whole is present. Fractions can be used to read a tape measurer to determine if appliances will fit within the kitchen. Fractions can be used to determine how much gas is left in the tank.

These questions explore each of the basic components of the standard identified in the previous step. As the questions become more refined, the reasoning also becomes more refined. This is an iterative process, so it is important to keep digging until the big concept and authentic applications become clear.

To encapsulate the foundational concepts, we rewrote the standard as follows:

> Fractions communicate a specific location on a number line (in this standard between 0 and 1). For **consistent communication**, the distance between 0 and 1 must be **equally** divided into parts. The total number of parts is the denominator, and each part is one of the total number of parts, which can be represented in a fraction form.

In this summary, we eliminated most notations (e.g., 1/b) and rephrased in language. We considered the *purpose* of the content through a series of questions; the identified purpose is to be able to locate/communicate points along a number line. Then, we summarized what is fundamental for understanding the standard. In this case, the concept of **equally dividing** the whole was foundational. Thus, we knew our lesson will explore equality in some capacity. Dissecting this standard shifted our focus from labeling fractions on a number line to emphasizing the core concept of equality. This conceptual understanding provides the foundation for more advanced learning and will provide important guidance for differentiation. Again, we are collecting building blocks for the differentiated lesson. We will use this standard translation to build the big idea in the next section.

Situate the Standard

Finally, curriculum designers should consider where the standard fits within the domain progression. The current standard can be situated within the developmental scope to determine what students should already know, what they should learn now, and what they will be expected to learn next year. This can also serve to guide differentiation of content. Students who are struggling may need scaffolds and reminders from the previous years' standards, whereas students who demonstrate mastery of grade-level standards may want to explore the future grade's expectations (see Table 3).

The sample third-grade standard is the first within the fraction progression, which suggests many third-grade students may not have background understandings of fractions. Thus, in the differentiated lesson, we may want to integrate significant visuals and manipulatives to support these new learners. Next, we examine the fourth-grade aligned standards to provide inspiration for higher levels of challenge, and these standards explore equivalent fractions, unit fractions, and decimals as fractions. This suggests we may want to incorporate some of these concepts in the more advanced tiers. At this moment, we can note these ideas as part of the process and return to them as needed.

Table 3

Aligning Third and Fourth Grade Common Core State Standards

<table>
<tr><th>Third Grade Standards</th><th>Fourth Grade Standards</th></tr>
<tr><td>CCSS.MATH.CONTENT.3.NF.3
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
CCSS.MATH.CONTENT.3.NF.3.A
Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
CCSS.MATH.CONTENT.3.NF.3.B
Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
CCSS.MATH.CONTENT.3.NF.3.C
Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.
CCSS.MATH.CONTENT.3.NF.3.D
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.</td><td>CCSS.MATH.CONTENT.4.NF.1
Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
CCSS.MATH.CONTENT.4.NF.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.</td></tr>
<tr><td colspan="2">Translating the Standards:

CCSS.MATH.CONTENT.3.NF.3.D

Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

My students will need to compare two fractions that have the same numerator (top number of fraction) or the same denominator (bottom number). They will need to practice reasoning about their size. My students should know that they will only compare fractions that refer to the same whole, such as comparing 1/3 and 2/3 rather than comparing 2/3 and 5/2. They will need to demonstrate their thinking visually and use the following symbols: >, =, or <, to do so.</td></tr>
</table>

(National Governors Association Center/Council of Chief State School Officers, 2010, pp. 24, 30)

General Process Standards

In addition to content standards, CCSS provides a series of eight mathematical practices (MPs) to anchor mathematical process objectives across grade levels and mathematical contexts see Table 4). These general process standards are both transferrable and foundational for mathematical problem-solving. Thus, when designing any differentiated lesson plan, designers should become familiar with all the mathematical practices before selecting the best one for the current lesson.

Table 4

Standards for Mathematical Practice (MP)

MP1	Make sense of problems and preserve in solving them.
MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.
MP4	Model with mathematics.
MP5	Use appropriate tools strategically.
MP6	Attend to precision.
MP7	Look for and make use of structure.
MP8	Look for and express regularity in repeated reasoning.

(National Governors Association Center/Council of Chief State School Officers, 2010, p. 8)

For our sample lesson, we selected *CCSS.Math.Practice.MP6* (i.e., attend to precision) because it dovetailed with the content standard delineated above (i.e., *CCSS.Math.Content.3.NF.2.a)*, which emphasized using equal parts to communicate parts of a whole consistently and precisely. At their core, fractions provide opportunities for precision between whole numbers.

While the brief standard itself captures the heart of the practice, it does not provide enough specific guidance on how to teach and assess the mathematical process. Thus, these broad standards must be dissected to provide clear guidance for instructional design. For example, *CCSS.Math.Practice.MP6* states "Attend to precision." While it might be tempting to simply copy this language as the lesson's process standard and move onto the next stage, the broadness does not contain the clarity necessary for high quality differentiation. Specifically, process standards should be dissected to understand (a) how this process would be **taught** and (b) how the process could be **assessed** to ensure students' growth. Thus, the initial standard does not specifically describe *how to* attend to precision. How do students demonstrate they are able to attend to precision?

Fortunately, under the initial practice, CCSS provides a paragraph narrative with significant additional information:

> *CCSS.Math.Practice.MP6:* Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical

> answers with a degree of precision appropriate for the problem context. In elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. (National Governors Association Center/Council of Chief State School Officers, 2010, p. 7)

Once again, however, curriculum designers need to spend time analyzing the components. In this mathematical process standard, several key components could anchor a third-grade lesson on using number lines to understand fractions; however, it would be challenging to capture all of them in a single lesson (or even unit), so we identified the most relevant components aligned with the content. These are the components we want to deliberately teach and evaluate students' thinking to ensure they are developing mathematical reasoning skills. Students will:

- Use precise communication to specify how many equal parts are present.
- Use tools and visual representations (or other strategies) to precisely demonstrate equal parts within a fraction.
- Determine the degree of precision appropriate for specific contexts.
- Use clear definitions of fractions, denominators, and numerators in discussion with others and within their own reasoning.

Integrating Standards Into Big Ideas

The insights gathered from dissecting the content and process standards can be folded into a Big Idea for the lesson. Big Ideas provide a general overview of the lesson's purpose and importance, including both the content and process standards but also the bigger picture. Developing a lesson's Big Idea establishes the lesson foundation and the constants that *every* student should explore within the lesson. High-quality Big Ideas meet several key criteria. They provide: (a) clarity of content and process, (b) connection to an overarching concept, and (c) description of an authentic application. These Big Ideas demonstrate how the TLM team has been influenced by other curriculum models. For example, Big Ideas are a fundamental component described in Kaplan's (2018) Depth and Complexity Model. Further, the Big Idea's authentic applications include opportunities for students to assume the role of a practicing professional, which is a fundamental practice within the Schoolwide Enrichment Model (Renzulli & Reis, 2014).

Clarity of Content and Process

Returning to the lesson design sample, we began crafting the Big Idea with the content and process standards' summaries. This ensures alignment between the required content standards, mathematical practices, and the lesson. Every stage of the design process is built upon this foundation. Here is the initial section of the lesson's big idea:

> Numbers provide a consistent method to communicate a **precise** quantity, and specifically, fractions are numbers that **precisely** describe a situation where a whole has been broken up into equal parts. The "denominator" communicates how many equal parts there are in the whole, and it is written on the bottom of the fraction. The "numerator" communicates how many of those equal parts are present, and it is recorded at the top of the fraction. In this lesson, we will be examining number lines that span from 0 to 1. In this case, [0 to 1] is the whole that will be split into equal parts. Fractions can be used to

communicate the **precise** distance between 0 and 1. **Precision** is important to establish a common understanding of distance that most closely communicates the true or desired value.

This Big Idea demonstrates how both the content and the bolded process standards can be woven together. The MP words are bolded to demonstrate how this practice supports content understanding and how the content can be leveraged to develop a generalizable process.

Connection to Overarching Concepts

In addition to incorporating content and process standards, Big Ideas also include an overarching concept. Overarching concepts are concepts that are abstract, span/link disciplines, and can be explored at a variety of levels of depth. Occasionally, when overarching concepts are not obvious, curriculum designers may wish to examine a pre-existing list (e.g., VanTassel-Baska, 2003, 2017; Wiggins & McTighe, 2005). Some overarching concepts that may be particularly helpful for math lessons include change, communication, patterns, relationships, scale, structure, and systems (see Figure 4).

Figure 4

Overarching Concepts

Overarching Concepts

Change	Good/Evil	Relationships
Communication	Knowledge	Scale
Conflict	Life/Death	Signs
Constancy	Models	Structure
Evolution	Order	Systems
Exploration	Origins	Time
Family	Patterns	Truth
Force	Power	Wisdom

(Rubenstein, 2022a; Schmidt, 2005; VanTassel-Baska, 2003, 2017; Wiggins & McTighe, 2005)

Alternatively, these overarching concepts may become evident when dissecting the standards. Within the sample, two overarching concepts became essential to fulfilling the standards: communication and equality. We decided to emphasize "communication" because it unified the content purpose and the mathematical practice. Further, "communication" demonstrates all the characteristics of an overarching concept, as it is abstract (e.g., you cannot physically hold communication), it can be applied across disciplines (e.g., authors communicate using precise words), and it can be explored throughout students' development at different levels (e.g., kindergartens may explore how to best communicate with friends when playing, whereas middle schoolers may learn how to use communication techniques for persuasion). In our drafted Big Idea above, we integrated the overarching concept, "communication" into several spaces. For example, "Numbers provide a consistent method to **communicate** a precise quantity . . . [in this lesson] fractions can be used to **communicate** the precise distance between 0 and 1."

Description of Authentic Applications

Authentic applications provide a context for why this lesson is important or how this knowledge will provide unique insight. Within *TLM If Aliens Taught Algebra: Multiplication and Division Would be out of This World!* (Cole et al., 2019a), we often had a consistent setting for the entire unit: outer space. This was helpful because it established a throughline for the unit. While it is sometimes helpful to have students work towards a larger project, it is not essential for differentiation. Differentiation can be successfully applied in individual lessons that may have their own unique contexts.

Within the sample lesson Strolling in Space: Preparing for a Space Walk from *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022), we wanted to situate the content, but the authentic application was not immediately apparent. Initially, we brainstormed generic connections between communication and fractions. We thought about students walking to a friend's home. We imagined explaining to a student: "Let's suppose you want to tell your friend how far you are from their house. A fraction could communicate the relative distance from your starting point to their house, like 'I am halfway there.' Your friend will be better able to estimate your arrival time when you are more precise. Generally, understanding fractions on number lines will support the measurement of distance and comparing relative lengths."

Writing this lesson helped us see several authentic connections; we may or may not actually use this initial brainstorm, but it inspired the next phase. Now we could brainstorm more engaging or interesting situations that would place the students in the role of practicing professionals. We considered when astronauts might need to communicate a location on a number line, including communicating to Mission Control their distance from Earth or how much fuel remains. Each of these applications were either too complicated or too limited. Then, we thought about all the gauges that might be on a space suit. This is a number line from 0 to 1, but there are multiple gauges that could offer more complexity without becoming overwhelming. This led us to insert "reading gauges" to the Big Idea to foreshadow this specific lesson. The final Big Idea follows:

> Numbers provide a consistent method to communicate a precise quantity, and specifically, fractions are numbers that precisely describe a situation where a whole has been broken up into equal parts. The "denominator" communicates how many equal parts there are in the whole, and it is written on the bottom of the fraction. The "numerator" communicates how many of those equal parts are present, and it is recorded at the top of the fraction. In this lesson, we will be examining number lines that span from 0 to 1. In this case, [0 to 1] is the whole that will be split into equal parts. Fractions can be used to communicate the precise distance between 0 and 1. Precision is important to establish a common understanding of distance that most closely communicates the true or desired value. Generally, understanding fractions on number lines will support the measurement of distance, reading gauges, and comparing relative lengths. Number lines are helpful to visualize and compare distances and amounts.

Curriculum designers can add other applications or details to help them understand the big idea, lesson objectives, purpose, and applications (see Figure 5). These details can be used to frame the assessments and learning experience tasks.

Figure 5

Designing Big Ideas & Objectives

(Rubenstein, 2022b)

Writing Student Learning Objectives

Student Learning Objectives (SLOs) serve as the final component of Stage 1 (Identify Desired Results), which is the foundation for differentiated lessons. Without the previous, initial groundwork, writing objectives may not provide the depth necessary for differentiation. SLOs translate the content/process standards and big ideas into simple, measurable student objectives. The SLO components have already been identified in the previous sections. The hard work is already completed. These objectives organize what you want students to know (content standards), be able to do (process standards), and understand (Big Ideas) into a list that can be used to guide the design of the assessments and learning experiences.

Revisiting the sample lesson, we dissected the Big Idea and analyzed what it stated students should know, do, and understand. We then translated these concepts into the following SLOs (see Table 5).

Table 5

Student Learning Objectives (SLOs): Know, Do, Understand

Know Students will explain that fractions communicate a part of a whole unit. (In this lesson, students will recognize the whole unit to be the distance between 0 and 1 on a number line.) Students will explain that a fraction is composed of equal parts.
Do Students will partition the distance between 0 and 1 on a number line into equal parts. Students will communicate with precision by incorporating definitions of terms and explaining key problem-solving decisions.
Understand Mathematicians decide how much precision is necessary to appropriately communicate a mathematical situation.

Stage 2 (Determine Acceptable Evidence): A Closer Look

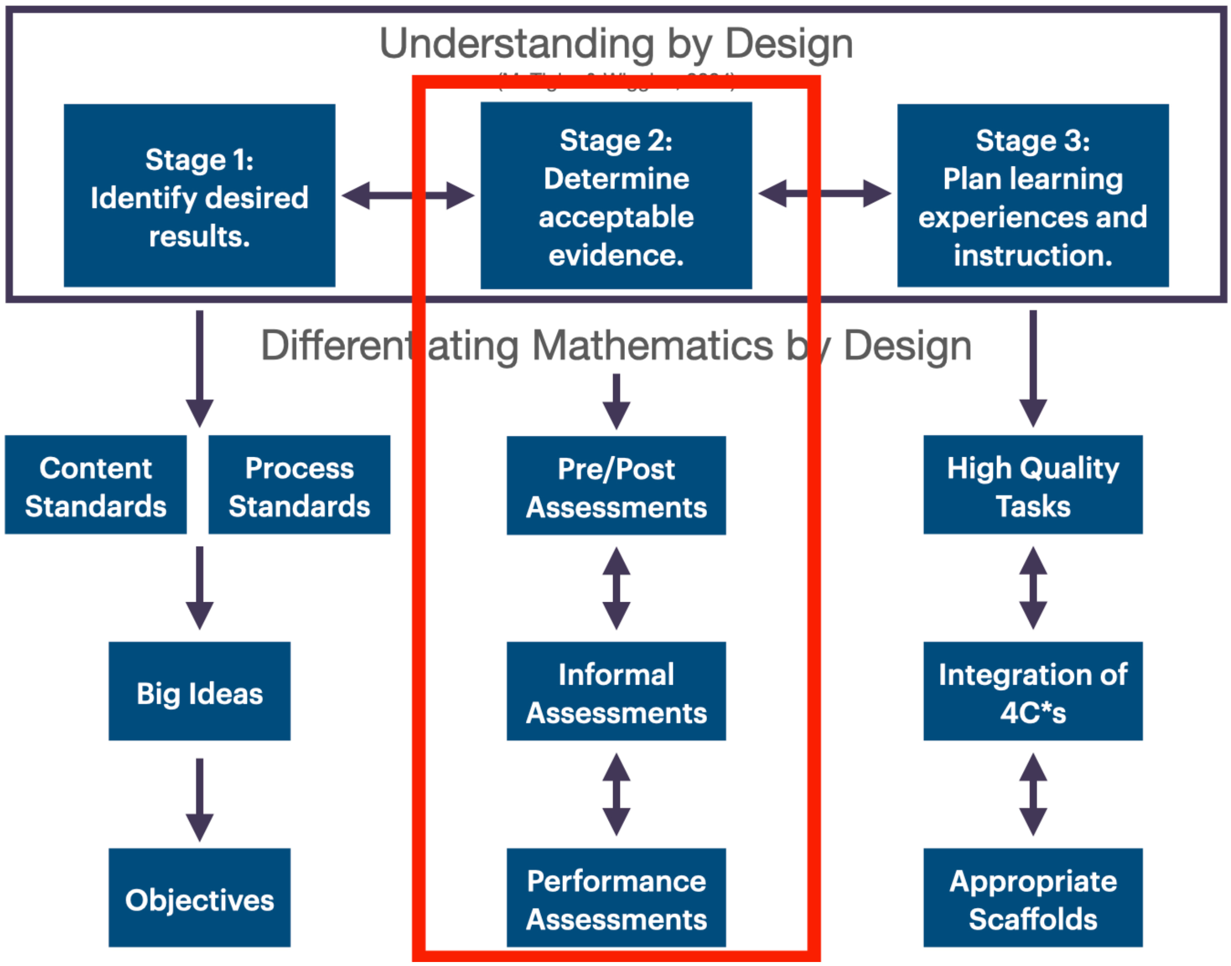
Assessments provide the essential connection between lesson objectives and learning experiences. Without appropriate assessments, teachers are operating blindly, uncertain of whether students have achieved the objectives and unable to make informed decisions about differentiation options (see Figure 6). Within the *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022), we anchored our work on several key assessment principles:

- Students should be provided multiple opportunities and methods to demonstrate their growth in the targeted objectives, including pre- and post-assessments, informal assessments/tasks, exit cards, practice tasks, and performance assessments.
- Assessments must be directly aligned with the objectives, the lesson tasks, and other assessments.
- The grading criteria are clearly delineated for both content and process skills.
- Assessments must measure every learning objective. If they are unable to do so, there are either too many objectives or the assessment needs to be revised.
- The assessment outcomes are used to determine levels of scaffolding necessary for specific groups of students. Frequent assessments provide opportunities for teachers to flexibly adjust grouping formations.

In general, an assessment plan for an entire unit should take a scrapbook approach. We all have those photographs we want to bury (for eternity). Any single photo is subject to a bad hair day, a bad camera angle, or an unfortunate moment when raspberry seeds are stuck in our teeth. None of these photos accurately depict us as we usually appear. However, if you take enough photos to fill a scrapbook, you are more likely to get a realistic picture of the person. This is the same for students. If we only have one piece of data, we may fail to understand students' true present level of performance. Using different assessments to collect diverse data over time will reveal a more accurate and in-depth understanding of students' comprehension and potential.

Figure 6

Stage 2 (Determine Acceptable Evidence)



In the *TLM If Aliens Taught Algebra: Multiplication and Division Would be out of This World!* (Cole et al., 2019a), the unit begin with a pre-assessment to help teachers gather initial information about their students' knowledge and skills. Each question on the pre-assessment is aligned with a specific lesson. Therefore, students could express mastery on one item but struggle with the next item. Using this pre-assessment, the teacher would be able to place a student in the advanced group for one lesson, while also being placed in the basic group for a different lesson.

While this worked for our *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022), we could also start to see the benefits of collecting single snapshots more frequently. A common approach involved using entrance tickets is to determine students' readiness for each learning activity. For example, within the single lesson, Strolling in Space: Preparing for a Space Walk, we had an opening task that could be used to place students within groups. Then, embedded within the lesson are informal assessments in which teachers can continue to gather insights into students' levels of understanding. At the end of the lesson, teachers may use additional practice tasks, like exit cards and homework assignments, to guide future learning experiences.

Application to TLM Lesson: Strolling in Space: Preparing for a Space Walk

Within this lesson, we identified the objectives in Stage 1 (Identify Desired Results) of the Understanding by Design process and adopted the selected task context of a Mission to Mars. Then, we started to develop a task to guide both the assessments and the learning experience Stage 2 (Determine Acceptable Evidence). Across all TLM lessons, we tended to use similar tasks in the pre-assessment as we were planning to use within the learning experience. This provided the best information to determine how well students would do with the specific learning tasks.

Therefore, we needed a task that would allow students to demonstrate competencies across all objectives, including understanding fractions on a number line and constructing arguments. At the simplest level, we could give students an unlabeled number line and ask them to show 1/4 and then add a follow-up question, "Sasha does not believe you. How could you explain your reasoning?"

By the end of the lesson, we would hope students would correctly identify 1/4 on the number line, but more importantly, we would want to see that there was an effort to break the number line down into 4 equal parts. Then, they should precisely communicate with Sasha, using appropriate mathematical vocabulary: the denominator "4" describes how many total equal parts of the whole, and the numerator "1" communicated the current level present. Additionally, the effort to divide the number line into equal parts could be more precise with specific tools.

Now, we have a task and a basic answer guide, we can consider how to group students based on their responses to this opening task. Importantly, *all* groups are given mathematician names. This is a foundational belief within the *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022). All students practice "thinking like mathematicians," are treated like mathematicians, and are called mathematicians. All students are given opportunities to experience different groups throughout the lesson collection depending on their needs, and all group tasks respectful of students' abilities. Further, the curriculum provides options for students who finish early beyond mere busy work or free time that otherwise may be provided. In these lessons, if students finish their targeted task, they can either work on the next level's task or receive a challenge card to further extend their mathematical thinking.

- **Tier 1: Peggy Whitson:** Students who do not demonstrate a conceptual understanding of fractions on number lines should be placed in Tier 1. *[Note: This should inform the design of the Tier 1 learning experience. These students need to understand what fractions are and why equal parts are essential for fractions.]*
- **Tier 2: Guion Bluford:** If students demonstrate a vague sense of fractions (i.e., they try to establish equal parts) but do not demonstrate a specific strategy or fail to arrive at the correct answer, they should be placed in Tier 2. *[Note: This should inform the design of the Tier 2 learning experience. These students may understand equality, but we could probe this further by exploring inequality. We could create a task that will require the use of strategies to increase precision.]*

- **Tier 3: Ellen Ochoa:** If students can communicate the fraction is 1/4 (or an equivalent fraction) and they used a specific strategy, they should be placed in Tier 3. *[Note: This should inform the design of the Tier 3 learning experience. These students will need to explore more challenging concepts within fractions. When numerators are 1 (such as 1/3, 1/5), the fractions with smaller denominators represent larger quantities (1/3 is larger than 1/5). Fractions with larger denominators have been separated into more parts.]*

In the next stage, we may improve upon our assessment task by integrating the Mission to Mars context; however, we know what types of scaffolds and challenges students may need based on their pre-assessment performance.

Stage 3 (Plan Learning Experiences and Instruction): A Closer Look

Through the first two stages of lesson design, we have determined the lesson's big ideas, objectives, and assessments, and in doing so, we have also identified what can be adjusted without compromising the goals of the lesson. This is like remodeling a house when the construction team identifies load-bearing walls before beginning the process. Those load-bearing walls cannot be removed without endangering the structural integrity of the house; however, other walls could be eliminated or adjusted to provide a more open floor plan. Once the initial work of identifying those walls is complete, the fun can begin, as the team eliminates extra walls, installs new windows, and freshly paints the ceilings. These details are what makes a house a home; however, they do not matter if the house is not structurally sound.

When designing differentiated lessons, we often imagine ourselves starting with nothing and developing new curriculum. However, most school districts have adopted curriculum that could be remodeled. Within DMbD Stage 3 (Plan Learning Experiences and Instruction), curriculum designers can evaluate learning experiences and decide whether they are worth remodeling. Within this section, we will first discuss how to remodel an existing lesson and then, we will also examine how to build your own (see Figure 7).

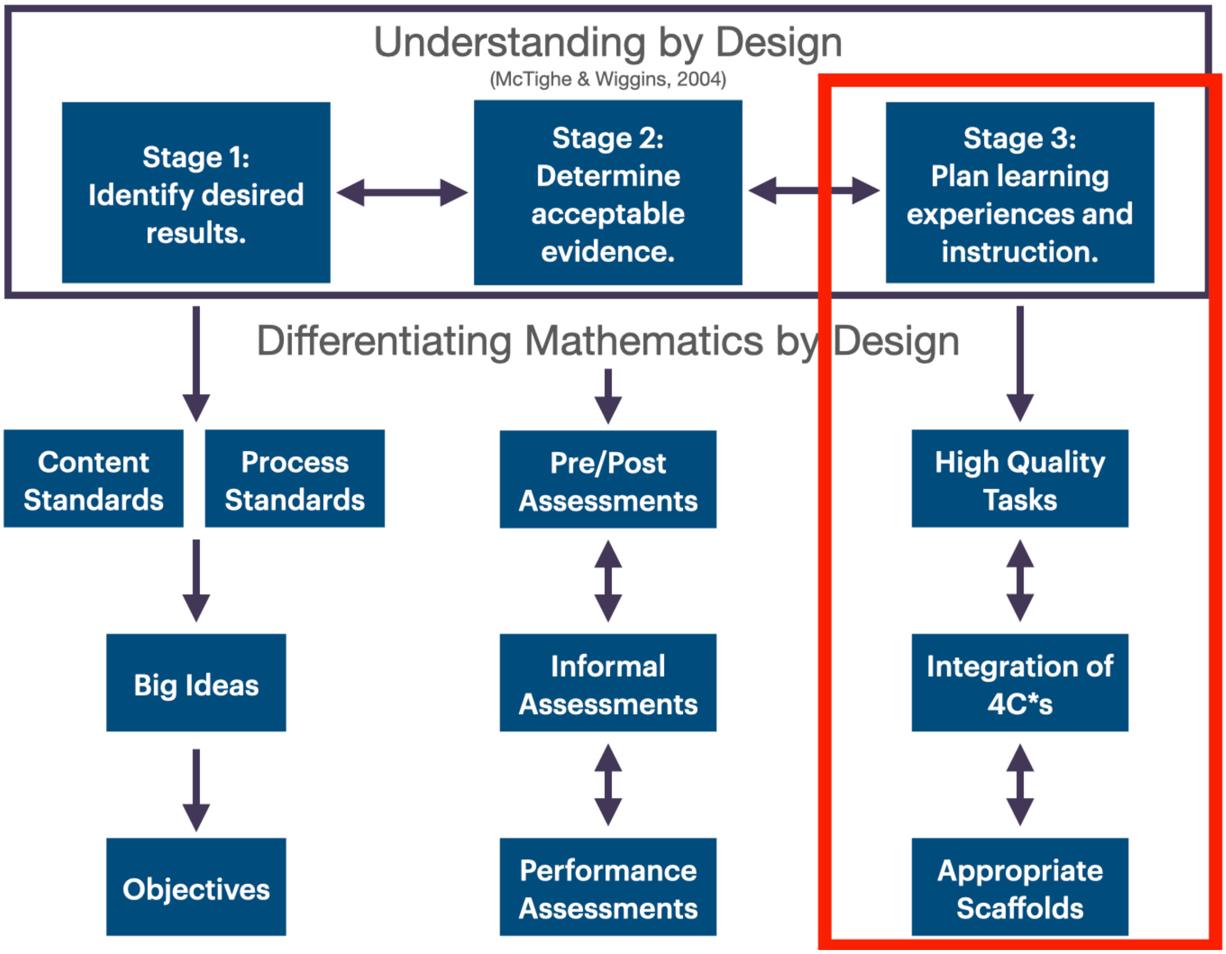
Remodeling Textbook Lessons

The key to differentiated math lessons is starting with a high-quality task. Not all tasks can be differentiated efficiently or effectively, so it is important to start with a task that meets certain criteria. Specifically, high-quality, differentiated tasks must:

- Have multiple pathways to arrive at a solution or have multiple solutions.
- Encourage productive struggle.
- Allow students to plan their approach.
- Promote productive conversations.
- Have possible scaffolds and extension options.

Figure 7

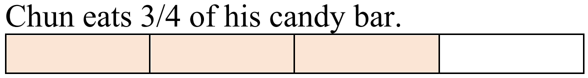
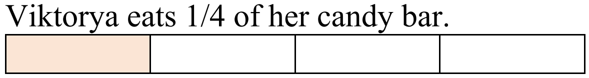
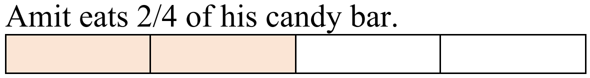
Stage 3 (Plan Learning Experiences and Instruction)



Myriad curricula present tasks with multiple pathways; however, some textbook tasks simply cannot be salvaged for a differentiated exploration, such as "What fraction of this circle is shaded?"

This is not necessarily a bad question. It is helpful to practice identifying and using fractions, yet the question is not rich enough to anchor a high-quality, differentiated lesson. As the task is currently written, would students be able to debate the answer or process? How could the challenge level be adjusted? Using this type of task to anchor differentiated lessons sets teachers and curriculum designers up for failure. The task is only easily adjusted by using bigger numbers, which does not actually increase the depth or complexity of the task.

Other textbook tasks are too well-defined with too much scaffolding; however, they can be salvaged by deconstructing and adapting certain key task components. For example, let's explore how we can remodel one problem to provide differentiated learning opportunities:

> Chun, Viktorya, and Amit each have an equal sized candy bar.
> Each candy bar is divided into four equal parts.
>
> Chun eats 3/4 of his candy bar.
>
> 
>
> Viktorya eats 1/4 of her candy bar.
>
> 
>
> Amit eats 2/4 of his candy bar.
>
> 
>
> Who ate the most? Who ate the least?

As is, this problem is not easy to differentiate, but it has potential. As written, this task is too helpful and does not embed a specific mathematical practice. First, the abundance of information does not provide an opportunity for students to struggle, to take different approaches, or to discuss their process. They do not need to understand much about fractions to use the provided visual to determine that Chun ate the most and Viktorya ate the least.

After recognizing this flaw, the first step is to revisit the lesson's content and process objectives (i.e., determine the load-bearing walls). Most content and process standards will provide a foundation and inspiration for differentiation. The content standard for this task reads:

> *CCSS.Math.Content.3.NF.3.d.* Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (National Governors Association Center/Council of Chief State School Officers, 2010, pp. 24, 24)

This content standard provides insight on how to remodel this task. First, "comparisons are valid only when the two fractions refer to the same whole" suggests students should understand the "same whole," yet in this task, they are already given the same whole. They do not have the opportunity to consider the importance of the whole. Further, the standard states students must "justify the conclusions, e.g., by using a visual fraction model." In this task, the visual model is already provided. Because the task is too helpful, the students are not wrestling with the key content objectives, which makes it difficult to introduce differentiated scaffolds and challenges.

In addition to the content standards, we also want to consider the process standards. In the example above, a specific MP is not included; however, selecting and aligning an MP will provide the necessary inspiration to remodel this task. We could apply multiple MPs to this task. However, because the existing task and content standard is about "comparing," MP3 would be especially applicable (i.e., "Construct viable arguments and critique the reasoning of others"). Now, we can consider how to revise the task, such that it provides students the opportunity to construct or critique an argument.

The task has several components that could be adjusted to incorporate the MP. Task components include the task premise, key questions, and process supports. The task can be dissected as follows:

- *Task Premise* is "Chun, Viktorya, and Amit each have an equal sized candy bar. Each candy bar is divided into four equal parts. Chun eats 3/4 of his candy bar. Viktorya eats 1/4 of her candy bar. Amit eats 2/4 of his candy bar."
- *Key Questions* are "Who ate the most? The least?"
- *Process Support* is the visual of shaded area fractions.

Evaluating each of these components alongside the lesson objectives provides a systematic approach for developing differentiated tasks. Below, each component is examined and applied to the sample remodeled task.

Evaluate the Task Premise

To revise the task premise, curriculum designers might ask a series of questions:

- How might the task premise ensure students are developing understanding of both content and process standards?
- Does the task premise provide too much support?
- Does it allow for the discussion of conceptual understandings and applications of mathematical processes?
- How would the task change if you eliminated the numbers? Can you remove the numbers and generalize the process?

In this task remodel, the premise prevents students from discussing the core concept of equal parts and wholes: "Chun, Viktorya, and Amit each have an equal sized candy bar." Then, "each candy bar is divided into four *equal* parts." This premise prevents a powerful conceptual discussion by giving too much information. As designers, we want students to have the opportunity to experience this conceptual foundation (i.e., load-bearing wall). How can students wrestle with unequal wholes and/or unequal parts? How can that concept be combined with MP3 of constructing and evaluating an argument? Some students may not be ready for a completely conceptual discussion, but their discussions could be additionally scaffolded using more concrete visuals. Now, the tiers and/or levels of support start to take shape.

We could adjust the premise: "Both Huzzah and Wowzers candy bars are pure chocolate. Viktorya ate 4 pieces of a Huzzah candy bar, and Chun ate 6 pieces of a Wowzers candy bar. Strangely, Viktorya claims she consumed more chocolate than Chun." *[Note: this premise no longer specifies equal wholes or equal parts, forcing students to wrestle with how that will affect the comparisons.]*

Broaden the Key Questions

Next, we could remodel the task's key questions. The question in the initial task is "Who ate the most? The least?" This simply requires a name in response. Similarly, other math problems may simply require a number. When the question can be answered with a name or a number, we cannot expect students to give more than what is asked. We can, however, rephase the question such that more is required. This is a great way to intentionally integrate the MP. TLM designers created new questions:

- How might the question be expanded, prompt a debate, or encourage deep thinking?
- How might the question prompt students to make a generalization that could apply to a variety of situations?
- When would their approach not work? Could someone argue differently?

The remodeled premise is primed for an argument (MP3), so the key question must force that argument. Thus, a key question might be: How could Viktorya prove she is right? [Follow-up question: What is another way Viktorya can prove that she is correct?] These are the broadest questions to the broad task premise. This demonstrates how the TLM team tends to develop the most challenging level/tier first, and then, we consider how to add more information or scaffolds to make the task more concrete. Teachers may find this approach easier than beginning with a lower-level tier, as they are asked to start with the most challenging concepts and simplify them, rather than being asked to make basic concepts more complex.

Eliminate Process Scaffolds

In the initial problem, the textbook may provide an unfortunate visual of the task premise. Students could simply look at the visual and not read anything more or understand anything about fractions and still arrive at the correct answer. In general, we could evaluate the scaffolds through these questions:

- What supports provide too much information or guidance?
- How might the scaffolds prevent students from making greater leaps between steps?
- What can be eliminated? How could students make their own scaffolds?

Thus, in the task, the visuals must be eliminated to give students the opportunity to master the objectives. After eliminating the supports, then, teachers can control which supports can be added back when needed. This approach is evident in many of our TLM tiers. Supports may include specific manipulatives, tools, graphic organizers, tables, drawings, or even a series of hint questions to guide students through the steps. All these scaffolds are only provided when students need them.

High-quality tasks lend themselves to a range of scaffolds. These scaffolds could be provided in the beginning to some students who demonstrated they need them based on a pre-assessment or the scaffolds could be released in real time using hint cards. Rather than giving too much support too quickly, teachers can encourage students to engage in productive struggle and provide planned scaffolds when students need them (see Figure 8).

Figure 8

Scaffold Examples

Fraction bars:

<table>
<tr><td colspan="4">1/4</td><td colspan="4">1/4</td><td colspan="4">1/4</td><td colspan="4">1/4</td></tr>
<tr><td colspan="2">1/8</td><td colspan="2">1/8</td><td colspan="2">1/8</td><td colspan="2">1/8</td><td colspan="2">1/8</td><td colspan="2">1/8</td><td colspan="2">1/8</td><td colspan="2">1/8</td></tr>
<tr><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td><td>1/16</td></tr>
</table>

Number line:

0/4 1/4 2/4 3/4 4/4
0/8 1/8 2/8 3/8 4/8 5/8 6/8 7/8 8/8
0/16 1/16 2/16 3/16 4/16 5/16 6/16 7/16 8/16 9/16 10/16 11/16 12/16 13/16 14/16 15/16 16/16

Hint Cards:

Hint Card #1

What would happen if the chocolate bars were divided into a different number of parts?

The scaffolds can include both content and process support. In the task remodel, we could create tiers or hint cards at varied levels:

- Small Scaffold: Could you add pictures to make your argument more convincing?
- Medium Scaffold (more concrete): What if Viktorya's candy bar had 8 total pieces and Chun's had 16?
- Large Scaffold (most concrete): Look at Viktorya's and Chun's candy bars. (The picture demonstrates 4/8 of one candy bar and the other is 6/16.) Now, why does Viktorya think she ate more than Chun? (Even with this scaffold, this is still a better task than the original task because the question requires students to construct an argument, and they must recognize the effect of unequal denominators.)

We could adjust the task further, such that in the most scaffolded tier, the denominators would be the same and Viktorya is wrong. We could also increase the challenge level by creating a broad extension question, like "What information do you need to know if you want to determine who ate the most chocolate in any situation?" In general, evaluating and remodeling existing textbook tasks is an important skill for teachers to develop. Examining the task premise, key questions, and process scaffolds provides a systematic, structured approach to differentiation. Other strategies to increase the level of challenge include:

- solving more complex and/or more abstract problems;
- addressing higher grade-level standards;
- working through real-world, open-ended, and/or multi-faceted problems;
- communicating understanding to others;
- generalizing learning to new situations; and
- emphasizing more challenging applications of 21st century skills (see Table 6).

Table 6

21st Century Skills (4Cs: Creativity, Critical Thinking, Collaboration, Communication: Partnership for 21st Century Learning, 2019)

CREATIVITY
Use a wide range of idea creation techniques (such as brainstorming)
Create new and worthwhile ideas
Elaborate, refine, analyze, and evaluate their own ideas in order to improve and maximize creative efforts
Demonstrate originality and inventiveness in work and understand the real world limits to adopting new ideas
CRITICAL THINKING
Use various types of reasoning as appropriate to the situation
Analyze how parts of a whole interact with each other to produce overall outcomes in complex systems
Analyze and evaluate alternative points of view
Synthesize and make connections between information and/or arguments
Interpret information and draw conclusions based on the best analysis
Reflect critically on learning experiences and processes
Solve different kinds of non-familiar problems in both conventional and innovative ways
COLLABORATION
Demonstrate ability to work effectively and respectfully with diverse teams
Exercise flexibility and willingness to be helpful in making necessary compromises to accomplish a common goal
Assume shared responsibility for collaborative work
Value the individual contributions made by each team member
COMMUNICATION
Articulate thoughts and ideas effectively using oral, written, and nonverbal communication skills
Listen effectively to decipher meaning, including knowledge, values, attitudes, and intentions
Use communication for a variety of purposes (e.g., to inform, instruct, motivate, and persuade)

Designing New Lessons

While some lessons can be remodeled, some lessons simply need to be demolished and rebuilt. The TLM team took this approach when building the *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022) by following the DMbD model, using a new learning experience (i.e., lesson plan) template and designing tasks to replace the existing curriculum.

Within this section of the instructional guide, we examine this new learning experience template and discuss how each section was crafted within the *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022). In general, the TLM lesson template has five key lesson components in which the teacher assumes different roles:

- Launch: Introduce objectives, context, and initial instruction.
- Explore: Provide high-quality, differentiated tasks.
- Examine & Elaborate: Guide a rich discussion of how each group approached the task.
- Debrief & Look Ahead: Summarize key learnings and preview next lesson.
- Assess: Identify misconceptions, track student progress, and design future instruction.

Launch

First, the Launch section delineates how the lesson will be introduced to students, which serves multiple purposes when done effectively: outlining the lesson's big idea, building student excitement, highlighting why the content/process is important, and/or connecting the lesson to a practicing professional's work. The TLM team often emphasizes the mathematical practice in this first section because these are the transferrable processes that mathematicians use that serve as the cornerstone for the entire lesson collection. Further, because the launch is a whole-class activity, the introduction needs to have an inclusive entry point such that all students can engage in the initial discussion. This section may also provide students with any necessary background knowledge to be successful with their tasks.

Application to TLM lesson: Strolling in Space: Preparing for a Space Walk. Within this lesson, we planned to address the mathematical content (CCSS.Math.Content.3.NF.2.a: fractions on number lines) and processes (MP6: attend to precision) within the context of a space mission. When designing the Launch section, we wanted to start with the important MP6 (attending to precision) as this is a key transferrable skill, necessary for all successful mathematicians, across contexts. We started with a set of prompting questions:

- When would third graders care about precision? When does precision matter?
- How can we connect precision to third graders' existing knowledge?
- When would a lack of precision be frustrating?

These questions helped us see how frustrating it would be if you did not get a precise response in a variety of situations. For example, how frustrating would it be if you did not know precisely how much money you won, how much something costs, or how much gas you have left. The winning money scenario seemed to be the most fun, so we framed the initial Launch discussion around winning a contest but only knowing the range of the award. Your response would be very different if you won one million dollars compared to $1.00. A non-precise range can be very frustrating. This is also a good Launch discussion because all students regardless of their mathematical acumen can imagine the difference between winning $1.00 and one million dollars.

After this initial example, we wanted students to have an opportunity to consider other areas where precision would be important. We added a creative thinking opportunity through the question: What are three other examples of when you need to be precise? What about three examples when you do not need to be precise? These questions represent a creative thinking opportunity because they require fluency of thought: students must generate multiple, different responses.

Next, we connected MP6 to the lesson's content (i.e., fractions on a number line) and context (i.e., space). The purpose of fractions is to provide precision, and precision in space is essential for survival. In the Launch section, we wanted to make the connection real, so we found a clip of astronauts completing a spacewalk and using precision to communicate back to Mission Control (see Figure 9). This sets the stage for the Explore section, during which students will have the opportunity to assume the role of an astronaut or a Mission Control specialist.

Figure 9

Launch: Thinking Like Mathematicians: Centering the Mathematical Practice—Sample Section

Launch

1. Thinking like Mathematicians: Centering the Mathematical Practice

CCSS.Math.Practice.MP6: Attend to Precision

Within this lesson, students will be developing the following mathematical practices:

- Precise communication by specifying how many equal parts are present.
- Precise use of tools and visual representations (or other strategy) of equal parts.
- Determining the degree of precision appropriate for specific contexts.
- Use of clear definitions of fractions, denominators, and numerators in discussion with others and in their own reasoning.

Explain: Mathematicians often need to be precise in their answers. Imagine you received a phone call, informing you that you won a major contest! The person said, "You won our third-grade student of the year award that comes with a cash prize that is somewhere between $1 and $1 million!

Would you be excited? Why or why not? [Gather student responses.]

Explain: Honestly, I am not sure how I would feel. There is a big difference between $1 and 1 million dollars, so I know I would need the person to be a little more precise in their description. Similarly, mathematicians also need to be precise to communicate clearly.

(See Appendix A, p. 49.)

Explore

Following the Launch section, the *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022) provides students with tiered, differentiated tasks to promote mathematical thinking for students across readiness levels. This section is often the most difficult to construct because it requires synthesizing across the background work (e.g., standards dissection, student learning objectives) within a specific context and task. We can start, however, with the same components and questions used to remodel a textbook task. For example, we could reframe the design questions as follows:

- *Task Premise*: How might the task premise promote discussion of conceptual understandings and applications of mathematical processes?
- *Key Questions*: How might the question prompt a debate, encourage deep thinking, or prompt students to make a generalization that could apply to a variety of situations?
- *Process Support*: What scaffolds could be added back if the students are struggling? How could students make their own supports?

After identifying the key pieces of the task, we can work backwards to ensure all students have appropriate levels of challenge. The most advanced learners may not need any process supports and they may not receive any numbers in the task premise, whereas some learners may need additional manipulatives or more concrete numbers within the task.

Application to TLM lesson: Strolling in Space: Preparing for a Space Walk. To develop this task, we revisited the carefully constructed Big Idea from Stage 1 (Identify Desired Results):

> Numbers provide a consistent method to communicate a precise quantity, and specifically, fractions are numbers that precisely describe a situation where a whole has been broken up into **equal parts**. The "denominator" communicates how many equal parts there are in the whole, and it is written on the bottom of the fraction. The "numerator" communicates how many of those equal parts are present, and it is recorded at the top of the fraction. In this lesson, we will be examining number lines that span from 0 to 1. In this case, [0 to 1] is the whole that will be split into equal parts. Fractions can be **used to communicate the precise distance between 0 and 1**. Precision is important to establish a common understanding of distance that most closely communicates the true or desired value.

The highlighted portions provide the inspiration for the task. We developed an anchoring task (see Figure 10) that provides an opportunity for students to explore equality, precision, and number lines. Then, we dissected the task to determine multiple ways to increase challenge or provide additional scaffolds. Here is the thought process behind the construction of the differentiated tiers:

- **Task Premise:** What is the least helpful we could be? We could ask students to read a number line without numbers. Then, we could add in the context, so ask students to read a gauge on an astronaut's suit without numbers. (This requires precision and communication.)
- **Key Question:** The basic question is the amount of oxygen in the tank. However, we do not want to simply ask: "How much oxygen is in the tank?" We add small disagreements that require explanations and precise communication. For example, one astronaut wants to use this approach while the other astronaut suggests an alternative. This requires students to think about both approaches and then construct an argument using evidence.

- **Process Support:** What additional scaffolds could we provide? If we have no numbers, how might we add something concrete to support students? In this case, we added paperclips for students to use when dividing the number line, and then, we added a few hint cards to provide additional support, like encouraging students to fold their paper or use graph paper to help divide the number line into equal parts.

Figure 10

Explore: Communicating Information From Number Lines

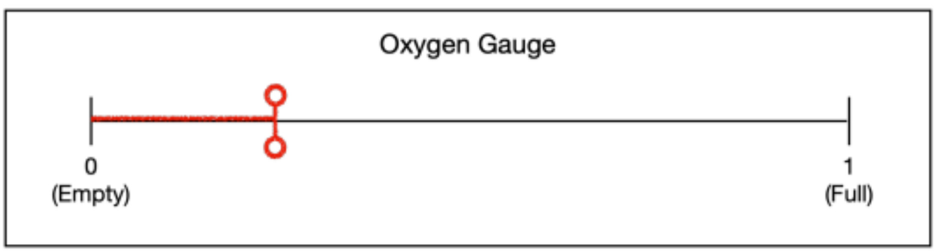
Explore

2. Communicating Information From Number Lines

Use this introductory task to place students in appropriate differentiated groups.

Say: When astronauts prepare for their spacewalk, they check their equipment. They examine their oxygen tank gauge. This gauge tells them how much oxygen they have in their suit.

Let's imagine you are preparing for a spacewalk. Strangely, the numbers had worn off your gauge. How would you communicate to Mission Control how much oxygen is in your tank?



Encourage students to write down ideas in their journals and share with their neighbors. After students have this entry discussion, start to probe their thinking by asking for evidence.

(See Appendix A, pp. 52–53.)

Examine and Elaborate

After students have an opportunity to work with their groups on a differentiated task, the lesson moves into the "Examine and Elaborate" section. While this section could include a variety of experiences, like a mini-lecture or a follow-up task, a hallmark of the *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022) is the whole-class discussion after students explore their differentiated tasks. All students are developing the same overarching understandings, simply at varied levels of content and processes; therefore, after the tiered groups explore their tasks, they all gather back together to share their unique experiences and approaches. Effective differentiation ensures all students can contribute something insightful to the whole class discussion.

To guide these discussions, the TLM team developed a *generalizable discussion skeleton* to unify students from different tiers, even though they may have different tasks and levels. The discussion often starts with students in Tier 1: Peggy Whitson because they tend to have access to unique scaffolds and manipulatives that none of the other students experienced. If they communicate the scaffolds clearly, all the other students can analyze and double check their own approaches using these scaffolds or processes. Questions may include:

- **Teacher to Students in Tier 1: Peggy Whitson:** You had a unique mathematical tool [or scaffold]. Tell us how you used this tool.
- **Teacher Follow-Up to Students in Tier 2: Guion Bluford and Tier3: Ellen Ochoa:** You didn't have that same tool. Could you have used that tool to help you on your tasks? How would that have worked? Would that be a more efficient strategy?

The discussion then proceeds to Tier 2: Guion Bluford tasks/scaffolds. The teacher asks students to describe what was unique about their task and how they approached it. All students should reflect on how that task or process supports or contradicts their experiences. Questions may include:

- **Teacher to Students in Tier 2: Guion Bluford:** Now, let's turn our attention to the Bluford group's problem. What was different between your task and the task the Whitson group described? How did it change the way you think about [the mathematical practice or content]?
- **Teacher Follow-Up to Students in Tier 2: Guion Bluford and Tier 3: Ellen Ochoa:** Can you also see how you applied [the mathematical practice/content] to understand your task? Do your new understandings align with each other? How so? What might need to be adjusted?

Finally, the teacher addresses several questions to students in Tier 3: Ellen Ochoa. This group often has a slightly different task, so the teacher asks these students to explain their task, but not to explain their process or answer, yet. The teacher gives all students an opportunity to think about the advanced task. Finally, the students in the Tier 3: Ellen Ochoa explain how they approached the task. At the end of the discussion, the teacher can return to the overarching objectives to evaluate how the different experiences contribute to a more robust understanding of the big idea. Questions may include:

- **Teacher to Students in Tier 3: Ellen Ochoa:** Now, let's turn our attention to the Ochoa group's task. Can you describe your task without telling us your process or answer?
- **Teacher Follow-Up to Students in Tier 1: Peggy Whitson and Tier 2: Guion Bluford:** This is an interesting task, why doesn't everyone think how they might solve it. How do you think the Ochoa approached the task knowing what you know?
- **Teacher to Students in Tier 3: Ellen Ochoa:** Is that the process you used? Can you explain why or why not?

These discussions do not always need to progress through all these stages, but all groups should have the opportunity to contribute value to the discussion. Further, within this discussion skeleton, we also integrate *Talk Moves*, a series of discussion prompts that encourages deeper thinking and more student involvement (Chapin et al, 2009). Both students and teachers can learn to use Talk Moves to facilitate mathematical discourse (Firmender et al., 2017). *TLM If Aliens Taught Algebra: Multiplication and Division Would be Out of This World!* (Cole et al, 2019a) and the *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022) incorporate the following Talk Moves and sample prompts (see Table 7):

Table 7

Talk Moves and Sample Prompts

Re-voicing: I heard you saying this . . . is that correct?
Repeat/rephrase: Could you (or someone else) repeat/rephrase what was just said?
Reasoning: Does anyone agree or disagree with this response? Why?
Adding On: Could someone add onto this response? Could someone give an example or non-example?
Wait Time: We have plenty of time to consider this problem. Let's all take a minute to think and then, we will chat.

Collectively, the discussion skeleton combined with the Talk Moves provides a strong foundation for meaningful mathematical discourse.

Application Within TLM Lesson: Strolling in Space: Preparing for a Space Walk. Within this sample lesson, we included broad questions that teachers could use throughout the discussion as well as a sample conversation to illustrate the flow for a full class discussion. The TLM team has myriad teaching experiences across grade levels, from elementary to graduate students, and within all environments, we have found it important to plan the big questions. If left to chance, those powerful questions may not be asked because in the moment, we may resort to surface-level questions that only require a one-word answer or questions that do not address the heart of the lesson, which prevents students from engaging with the lesson in greater depth. In this lesson, we revisited the dissection of the MPs and then, connected those components to the current lesson and context. We also considered questions that would encourage students to think more critically and creatively about the content.

Talk Moves provide multiple opportunities for teachers to conduct a formative assessment of students' knowledge and understanding of content and skills and promote their ability to incorporate math vocabulary accurately. As a check on student thinking and understanding, students are encouraged to think like mathematicians who have to think about solutions to challenging problems in multiple ways. Figure 11 highlights students' mathematical thinking by integrating sample questions and specific mathematical practices.

In addition to designing questions, we also determine which key ideas should emerge from the conversation:

> In this discussion, teachers should stress that mathematicians decide the level of precision necessary based on the situation and materials. In this case, astronauts need to be very precise regarding how much oxygen or water they have [basic survival], but they may not need to be as precise with how much shampoo they have left [level of comfort]. During this discussion, teachers should also remind students that these gauges are number lines, 0 to 1 is the "whole" that can be broken into equal parts, and finally, fractions and fraction notation can be used to describe an amount, emphasizing fractions only describe when a whole is divided into EQUAL parts.

Figure 11

Highlight Students' Mathematical Thinking

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**
Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might even be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss
After the differentiated groups have an opportunity to explore communicating fractions on number lines, bring the class back together for a full group discussion.

Guiding MP questions for all groups should include:

- What tools could you use to make your conclusion? (MP5: Encourages appropriate use of tools)
- How precise are your tools? (MP6: Attend to precision)
- What is another way you could determine the level of oxygen that would be more precise? Less precise? (MP6: Attend to precision and promotes fluency of thought, which is a component of creativity)

(See Appendix A, pp. 54–55.)

Finally, we wrote a sample conversation to demonstrate how a teacher might build understandings using each tier's learning process, honoring each of their experiences and integrating Talk Moves to prompt additional responses (see Table 8 and Figure 12).

Debrief and Look Ahead

After students have an opportunity to share their responses and mathematical thinking, the "Debrief and Look Ahead" section provides teachers with the opportunity to synthesize students' experiences and connect to the next lesson. This may take many different formats, but in general, both the content and process standards should be represented in the lesson summary. Teachers may want to use the exit card task as the guide for debriefing responses and practices. The "Debrief and Look Ahead" section provides teachers with the opportunity to synthesize students' experiences and connect to the next lesson.

Table 8

Sample Talk Moves Conversation

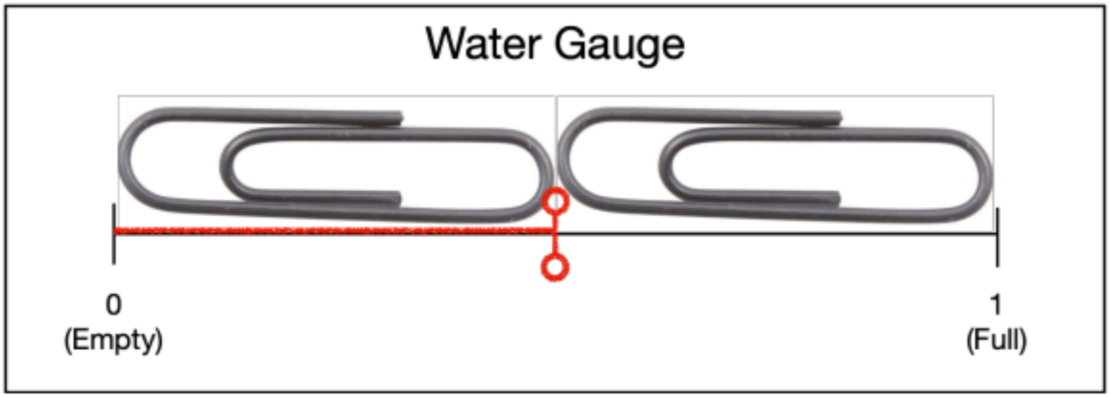
Teacher:	Let's start with Peggy Whitson's group. You had a water gauge that had 2 paper clips. How did you use those paper clips to determine how full the water tank was?
Monroe:	We used our paper clips. The water line covered one out of two paper clips.
Mahmood:	Right, so we said it was 1/2 full.
Teacher:	Interesting. I'm curious how this related to Guion Bluford group's first gauge. Can you describe what is different between your gauge and Peggy Whitson group's gauge?
Avalyn:	Yes! We had more paper clips.
Teacher:	That is true! Can someone add on to Avalyn's observation? (*Talk Move: Adding on.)*
Gerardo:	I noticed that Peggy Whitson group's paper clips are the same size, but our paper clips were different sizes.
Natalie:	It made it harder to measure properly.
Teacher:	Wow, great observation! So, why is that important? What about someone from Ellen Ochoa group?
Katie:	If the parts are not equal, we cannot use fractions.
Teacher:	And why is using fractions important?
Monroe:	It helps us to communicate to Mission Control! When we just said 2 paper clips, Mission Control may not understand the size of the paper clips, but if we said 1/2, Mission Control understands how full our tank is.
Teacher:	I hear you saying that using fractions helps us to communicate more precisely than paper clips. Is that correct? (*Talk Move: Revoicing.)*
Monroe:	Yes!

(Gubbins et al., 2022, Reprinted with permission)

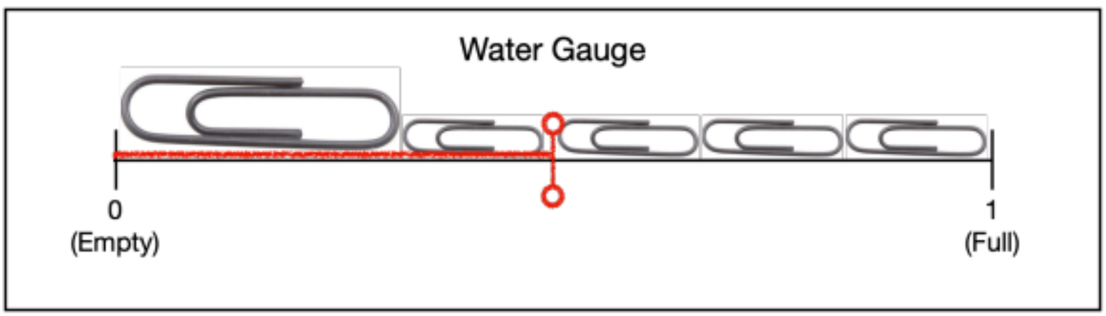
Figure 12

Tiered Lesson Water Gauges

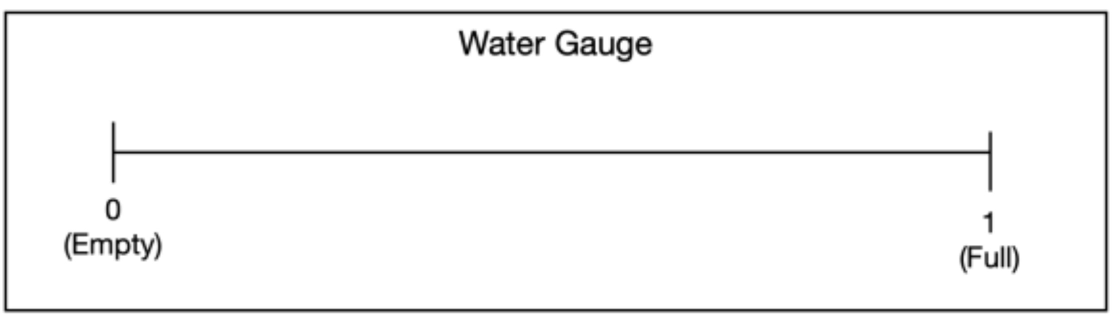
Tier 1: Peggy Whitson



Tier 2: Guion Bluford



Tier 3: Ellen Ochoa



(See Appendix A, pp. 60, 64, 67.)

Application Within TLM Lesson: Strolling in Space: Preparing for a Space Walk. This lesson section is guided by the identified questions and expected responses from the “Examine and Elaborate” section. In this debriefing session, teachers should stress that mathematicians decide the level of precision necessary based on the situation and materials. During this discussion, teachers should also remind students that these gauges are number lines, 0 to 1 is the “whole” that can be broken into equal parts, and finally, fractions and fraction notation can be used to describe an amount, emphasizing fractions only describe when a whole is divided into EQUAL parts. Within this sample debrief, the key content and processes are simply identified again, the mathematical vocabulary is integrated, and any misconceptions may be addressed (see Figure 13).

Figure 13

Debrief Content and Mathematical Practices

Debrief and Look Ahead

4. Debrief Content and Mathematical Practices
Remind students that the mathematical practice for this lesson focused on how mathematicians use precision to communicate amounts. Astronauts reading gauges is one example of when precision is important. In this case, students communicate the denominator is how many total EQUAL pieces are in the whole unit, and the numerator indicates how many of those pieces are present. They precisely label the number line with equal pieces using a strategy or tool. Students should be able to communicate their strategy for establishing equal parts.

(See Appendix A, p. 56.)

Assess

In general, within a single lesson, we designed the exit card as the final summative assessment (see Figure 14). Tiered student pages can also be assessed to determine students’ progress toward learning objectives. These assessments can also be formative, however, because teachers may use the results to further adjust future instructions to address student misconceptions. In some cases, students may receive extra practice (e.g., homework) depending upon their performance on the exit card.

Application Within TLM Lesson: Strolling in Space: Preparing for a Space Walk. Within this lesson, we used an exit card task to evaluate students’ content knowledge and mathematical processes. We did not create different homework options; however, the hint and challenge cards may provide insight into how to develop additional practices. For example, for the students who demonstrated proficiency on the exit card, they may be given a single question for homework: “Are larger or smaller denominators more precise? How do you know?” Whereas students who need additional scaffolds may be asked to read a variety of gauges or draw a variety of gauges from astronauts' descriptions.

Figure 14

Assess: What Students Learned

Assess

5. **What Students Learned**
Use the following exit card to assess what students learned from this lesson.

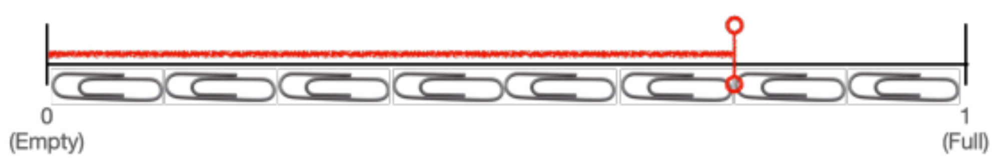
Differentiate Further as Needed
Please see the Hint and Challenge cards at the end of the lesson. The hint cards remind students to incorporate precise language and how fractions require equal parts. The challenge cards begin to probe students thinking on equivalent fractions on number lines.

Exit Card

The astronauts need to communicate how much fuel they have.

1. Use the paper clips to **label** the fuel gauge using fractions.

Fuel Gauge



2. How could you **precisely communicate** how much fuel is in the tank? (Use the definition of fractions in your answer.)

(See Appendix A, pp. 56–57.)

Conclusion

Throughout this project, the TLM team developed multiple, differentiated curriculum lessons, which were implemented in classrooms across the United States. We have seen students so excited by their math tasks that they asked to take their *TLM If Aliens Taught Algebra: Multiplication and Division Would be out of This World! Student Mathematician Notebooks* (Cole et al., 2019b) home to share with their parents. We have witnessed teachers implementing new discussion questions and experimenting with new grouping practices. Overall, we have found this work to be extraordinarily rewarding. However, we can still recall the earlier stages of curriculum development, when we were struggling to ensure lessons were adequately differentiated without sacrificing conceptual rigor. Those struggles led us to realize the gap within the curriculum design literature, the need for systematic approach to designing differentiated math lessons. In response, we developed DMbD and this instructional guide to describe how we used the model to design our *TLM Mission to Mars Lesson Collection* (Gubbins et al., 2022). This instructional guide provides specific details for those who do not consider themselves natural curriculum writers. With time and experience, the instructional guide should help develop a design process that can be generalized to different situations. We find comfort in the system, a system that will ask a series of questions to ensure we capture the conceptual foundations of math through meaningful and student-centered learning experiences. We hope that this instructional guide will provide all curriculum designers the opportunity to consistently differentiate mathematics lessons as you trust the process, and just keep swimming.

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Appendix A

Strolling in Space: Preparing for A Space Walk

Lesson Designer: Lisa DaVia Rubenstein

Lesson Designer: Lisa DaVia Rubenstein

LESSON 1

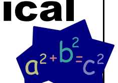
Defining/Labeling Number Lines—

Preparing for a Space Walk: Gauging Oxygen, Water, and Fuel

Big Ideas

Numbers provide a consistent method to communicate a precise quantity, and specifically, fractions are numbers that precisely describe a situation where a whole has been broken up into **equal** parts. The “denominator” communicates how many equal parts there are in the whole, and it is written on the bottom of the fraction. The “numerator” communicates how many of those equal parts are present, and it is recorded at the top of the fraction. In this lesson series, we will be examining number lines that span from 0 to 1. In this case, [0 to 1] is the whole that will be split into equal parts. Fractions can be used to communicate the precise distance between 0 and 1. Precision is important to establish a common understanding of distance that most closely communicates the true or desired value.

For example, let’s suppose you want to tell your friend how far you are to their house. A fraction could communicate the **relative distance** from your starting point to their house, like “I am halfway there.” The more precise you are; your friend will be better able to estimate when to expect you. When completing a fund raiser, a number line could be used to show how close a group is to meeting their goal. Generally, understanding fractions on number lines will support the measurement of distance, reading gauges, and comparing relative lengths. Number lines are helpful to visualize and compare distances and amounts.

Lesson Objectives 	• Students will recognize and describe that a fraction communicates when a whole unit is divided into **equal parts**. In this lesson, students will conceptualize the whole unit as the distance between 0 and 1 on a number line. • Students will break down the distance between 0 and 1 into equal parts. • Students will use mathematical language to communicate precisely with others. Students provide precise explanations and definitions in their communication. **Mathematical Content Standard** Develop understanding of fractions as numbers. <u>CCSS.MATH.CONTENT.3.NF.A.2.A</u> Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
Materials 	Students should have access to graph paper, tiles, fraction rods, paper clips, paper strips, scissors, scrap paper, and any other available materials. Students may not choose to build their responses using these tools, but in general, mathematicians have various tools at their disposal that they can use to test their hypotheses.
Mathematical Terms 	• **Denominator:** bottom number in a fraction that identifies the number of pieces the whole has been divided into equal parts • **Equal:** the same portion, piece, or segment • **Fraction:** a number that represents part of a whole • **Number Line:** a line with numbers placed in their correct position • **Numerator:** top number in a fraction that identifies the number of equal pieces considered as part of the whole • **Precise:** describes responses that are exact, accurate, careful about details

Selected Mathematical Practices	• MP1: Make sense of problems and persevere in solving them. *I never give up on a problem and I do my best to get it right.* • MP2: Reason abstractly and quantitatively. *I can solve problems in more than one way.* • MP3: Construct viable arguments and critique the reasoning of others. *I can explain my math thinking and talk about it with others.* • MP5: Use appropriate tools strategically. *I know how to choose and use the right tools to solve a math problem.* • MP6: Attend to precision. *I can work carefully and check my work.*
Differentiation	**Content** ***Guiding Questions*** • prior knowledge or learner readiness *What evidence do you have about students' current knowledge and skills?* • tiered activities *How will you design tiered activities on the same mathematical concept with varied levels of difficulty?* • formative assessment *What techniques will you use to assess students' prior knowledge and skills?* • varied levels of challenge *How will you vary the level of difficulty for each tiered activity?* • "teaching up" (aim high, provide scaffolding) *How will you increase the depth, breadth, complexity, and abstractness of lessons to challenge and support student learning?* **Process** ***Guiding Questions*** • questioning strategies *How will you pose and how will you encourage students to pose open-ended, closed-ended, lower-level, and higher-level questions to promote mathematical discourse?*

	4Cs (21st Century Skills) - creative thinking *How will you encourage students to "think outside the box" and synthesize information in new, different, and useful ways?* 4Cs (21st Century Skills) - collaboration *How will you encourage students to work with other students and appreciate their contributions to solving problems or making connections to prior work?* 4Cs (21st Century Skills) - communication *How will you promote students' opportunities to communicate face-to-face, in large and small groups, in online environments, and with print and non-print resources using their oral, written, and non-verbal skills?* - hands-on activities/manipulatives *How will you incorporate activities promoting the use of manipulatives to clarify or illustrate mathematical concepts?* **Product** ***Guiding Questions*** - oral, visual, and written opportunities *How will you encourage students to represent their thinking and problem solving using different communication strategies?* - multiple ways to demonstrate knowledge, understanding, and skills *How will you encourage students to share their understanding of mathematical concepts and skills?* - multiple models and representations *What techniques of lesson design will you include to support students' deep understanding and the ability to apply mathematical concepts and skills?* - summative assessment *How will you assess student learning upon completion of the lesson?*

	Learning Environment ***Guiding Questions*** • flexible grouping *How will you use your tiered lesson to support flexible grouping?* • whole group/small group/individual instruction *How will you incorporate different grouping plans to address students' learning needs?*

Lesson Preview

The content goal of this lesson is to establish fractions require **equal** parts, and specifically, a number line must be broken into **equal** parts to precisely communicate distance between 0 and 1. As students are preparing for a spacewalk, they must be able to solve unanticipated problems, and in this lesson, they are provided a scenario in which their gauge readers fail, and they must be able to communicate how much oxygen, water, and fuel are present in their space suit to Mission Control. The mathematical practice emphasis is communicating precisely to others.

Launch

1. **Thinking like Mathematicians: Centering the Mathematical Practice**

CCSS.Math.Practice.MP6: Attend to Precision

Within this lesson, students will be developing the following mathematical practices:

- Precise communication by specifying how many equal parts are present.
- Precise use of tools and visual representations (or other strategy) of equal parts.
- Determining the degree of precision appropriate for specific contexts.
- Use of clear definitions of fractions, denominators, and numerators in discussion with others and in their own reasoning.

Explain: Mathematicians often need to be precise in their answers. Imagine you received a phone call, informing you that you won a major contest! The person said, "You won our third-grade student of the year award that comes with a cash prize that is somewhere between $1 and $1 million!

Would you be excited? Why or why not? [Gather student responses.]

Explain: Honestly, I am not sure how I would feel. There is a big difference between $1 and 1 million dollars, so I know I would need the person to be a little more precise in their description. Similarly, mathematicians also need to be precise to communicate clearly.

Let's imagine now the person states you won between $1 and $7.

While you may be disappointed, is that a more precise description of your winnings? [Yes.]

Is it precise enough? [It depends.] Knowing you won between $1 and $7 helps you decide not to buy a private jet. So, it is precise enough to make that decision.

However, if you were trying to decide on a fast-food meal, you might need ever more precision. With $7 you could get a full meal with a drink, fries, and sandwich, and with $1 you would not be able to purchase anything with tax.

Let's brainstorm- in general, **when** would you need to be more precise? When could you be less precise? (If students struggle, you might want to be very precise when building a house so all the walls are straight, but you may want to be less precise in building with blocks.

Ask: How might mathematicians be more precise in their work?

If students struggle to generate ideas, consider mathematicians can be more precise by communicating the context, using clear definitions and units to explain their reasoning, and expressing answers with a degree of precision appropriate for the problem context. They may also use appropriate tools and language to provide more precise responses.

Being precise can be very important for mathematicians as well as for astronauts, which we will see today.

Situating the Lesson Context: Strolling in Space

Explain: In this 3-Lesson Series, we will be practicing and planning for a spacewalk. In the past, several astronauts have had to troubleshoot equipment and spacesuits that were malfunctioning. They must be extremely precise to ensure the success of their missions.

Watch: Let's watch a clip of actual astronauts completing a spacewalk. This is a condensed 15-minute clip of an actual spacewalk outside of the International Space Station that took close to 8 hours!

https://youtu.be/qStW1FysHLY

While you are watching, look for examples of **precise communication**. (EVA is an acronym that stands for **E**xtra **V**ehicular **A**ctivity, so EV1 is the first person out of the vehicle/station.

Ask: How were the astronauts precise in their work?
Potential examples:

- As they were leaving the station, they reported information on all their gauges, for example their suit gauges (e.g., 4.4 for EV2).
- There were specific numbers along the side of the station. As they moved down the station, there were markers spaced equidistant from each other. These numbers helped to give them information on where they were on the station. They stated they were going to the “P6 Truss site.” They label all the sites.
- While they were working, they were given specific instructions on which bolt needed tightened and how many rotations were necessary (e.g., 17.9 turns on the Nader bolt).
- In general, both Mission Control and the astronauts were precise on all their instructions.

Explain: These astronauts demonstrated extreme precision with their communication. When might astronauts not be so precise? (Sample responses may include the amount of toothpaste they use or the number of minutes they sleep. Astronauts may round to the nearest whole number in those cases. Often, however, astronauts are extraordinarily precise, even in their daily routines.)

Why is precision important for astronauts? While everything went well for these astronauts, that is not always the case. For example, when Italian astronaut Luca Parmitano was on a spacewalk, water started to fill his spacesuit helmet. He had to navigate his way back to the hatch while water was flooding in. After that event, NASA started to run considerable tests on the helmets to fix the issue. They needed to replicate the problem. Below you can see the empty spacesuit helmet in an Aug. 27, 2013 test of the faulty spacewalking gear. This water leak confirmation helped NASA engineers devise repair methods for the spacesuit. If one component is slightly off, it may be a matter of survival.

References
Spacesuit Malfunction
https://www.sciencetimes.com/articles/31751/20210616/spacesuit-problems-preempt-2-astronauts-completing-new-solar-panel-installation.htm

All-Woman Spacewalk (all 7.5 hours-October 2019)
https://www.youtube.com/watch?v=Iji5hTQ3CUo

Current Task
Explain: For today's Mission to Mars task, we are going to practice **precisely** communicating and understanding data from our space suit gauges. oxygen and water gauges. This will ensure our spacewalks will be successful.

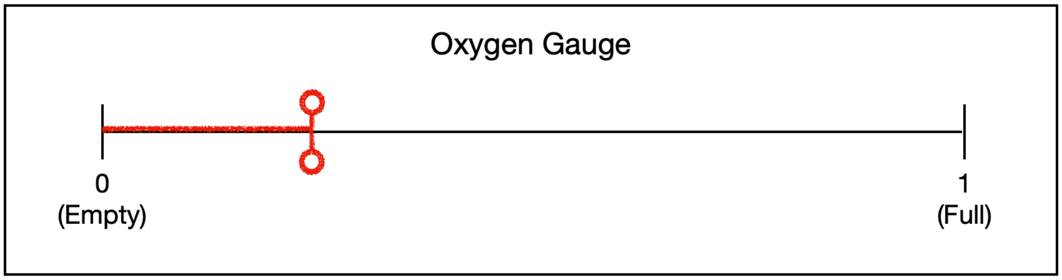
Explore

2. Communicating Information From Number Lines

Use this introductory task to place students in appropriate differentiated groups.

Say: When astronauts prepare for their spacewalk, they check their equipment. They examine their oxygen tank gauge. This gauge tells them how much oxygen they have in their suit.

Let's imagine you are preparing for a spacewalk. Strangely, the numbers had worn off your gauge. How would you communicate to Mission Control how much oxygen is in your tank?



Encourage students to write down ideas in their journals and share with their neighbors. After students have this entry discussion, start to probe their thinking by asking for evidence.

Differentiated Examination of Additional Options
During this introductory task discussion, teachers should look for developmental levels of two key concepts to demonstrate students' readiness levels:

- **Fraction Concept:** The correct answer is 1/4 (or an equivalent fraction, like 2/8), but it is more important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine and communicate the fraction. They also need to use the total parts as the denominator, and the level of oxygen present as the numerator.
- **Mathematical Practice 6:** Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts. They should be able to use to precise mathematical terms: fraction, denominator, and numerator appropriately by the end of this lesson.

In this investigation, students will be working on one of the Student Pages in their differentiated groups based on readiness levels. The groups are based on teacher's observations of students' conceptual understanding and mathematical practice acumen as described above.

- **Tier 1:** Students who do not demonstrate a conceptual understanding of fractions on number lines should be placed in Tier 1.
- **Tier 2:** If students demonstrate a vague sense of fractions (i.e., they try to establish equal parts) but do not demonstrate a specific strategy or fail to arrive at the correct answer, they should be in Tier 2.
- **Tier 3:** If students can communicate the fraction is 1/4 (or an equivalent fraction) and they used a specific strategy, they should be placed in Tier 3.

Explain to students that you are excited to see so many interesting approaches to determine and communicate the level of oxygen. Now, they are going to consider the level of water the astronauts have in their space suits, which is also key for astronaut survival on a spacewalk.

Groups Formed by Student Readiness		
Lab Group 1	**Lab Group 2**	**Lab Group 3**
Student Names	**Student Names**	**Student Names**

Collaborate and Communicate

Have students record their ideas for on their individual worksheets or one for the small group. Help them clarify their ideas by asking questions like, "What do you mean here?" and "How might you share that idea with the rest of the class?" Point out that mathematicians use definitions, examples/non-examples, and various representations to help support their conclusions. Below are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes.

A. [Possible response]	**B.** [Possible response]	**C.** [Possible response]
This group . . .	*This group . . .*	*This group . . .*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might even be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss

After the differentiated groups have an opportunity to explore communicating fractions on number lines, bring the class back together for a full group discussion.

Guiding MP questions for all groups should include:

- What tools could you use to make your conclusion? (MP5: Encourages appropriate use of tools)
- How precise are your tools? (MP6: Attend to precision)
- What is another way you could determine the level of oxygen that would be more precise? Less precise? (MP6: Attend to precision and promotes fluency of thought, which is a component of creativity)

In this discussion, it is important to stress that mathematicians decide to be either more or less precise in their responses based on the situation and materials. In this case, astronauts need to be very precise regarding how much oxygen or water they have, but they may not need to be as precise with how much shampoo they have left.

During this discussion, continue to discuss how these gauges are number lines, how 0 to 1 is the whole that can be broken into equal parts, and then, fractions and fraction notation can be used to describe an amount. Fractions only describe when a whole is divided into EQUAL parts. As they share, connect back to fraction notation, demonstrating how the number line can be divided into equal parts.

Here is a sample conversation on how to bring all the Tiers together to develop mathematical understanding.

Teacher: Let's start with the Whitson group. You had a water gauge that had 2 paper clips. How did you use those paper clips to determine how full the water tank was?

Monroe: The water line covered one out of two paper clips, so we said it was 1/2 full.

Teacher: Interesting. I'm curious how this related to the Bluford Group's first gauge. Can you describe what is different between your gauge and the Whitson gauge?

Avalyn: Yes! We had more paper clips.

Teacher: That is true! Can someone add on to Avalyn's observation? *(Adding On talk move)*

Gerardo: I noticed that the Whitson group's paper clips are the same size, but our paper clips were different sizes.

Teacher: Woah. So why is that important? What about someone from the Ochoa group?
Katie: If the parts are not equal, we cannot use fractions.
Teacher: And why is using fractions important?
Monroe: It helps us to communicate to Mission Control! When we just said 2 paper clips, Mission Control may not understand the size of the paper clips, but if we said 1/2, Mission Control understands how full our tank is.
Teacher: I hear you saying that using fractions helps us to communicate more precisely than paper clips. Is that correct? *(Revoicing talk move)*
Monroe: Yes!

Differentiate Further as Needed
Please see the Hint and Challenge cards at the end of the lesson. The hint cards remind students to incorporate precise language and how fractions require equal parts. The challenge cards begin to probe students thinking on equivalent fractions on number lines.

Debrief and Look Ahead

4.

Debrief Content and Mathematical Practices

Remind students that the mathematical practice for this lesson focused on how mathematicians use precision to communicate amounts. Astronauts reading gauges is one example of when precision is important. In this case, students communicate the denominator is how many total EQUAL pieces are in the whole unit, and the numerator indicates how many of those pieces are present. They precisely label the number line with equal pieces using a strategy or tool. Students should be able to communicate their strategy for establishing equal parts.

Assess

5.

What Students Learned

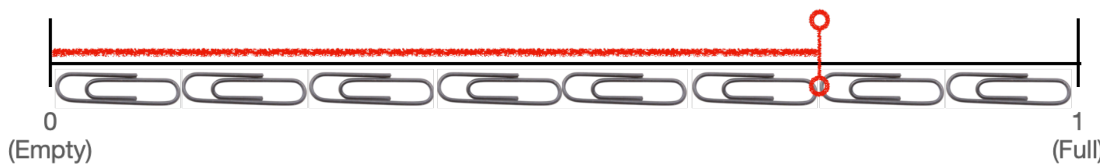
Use the following exit card to assess what students learned from this lesson.

Exit Card

The astronauts need to communicate how much fuel they have.

1. Use the paper clips to **label** the fuel gauge using fractions.

Fuel Gauge

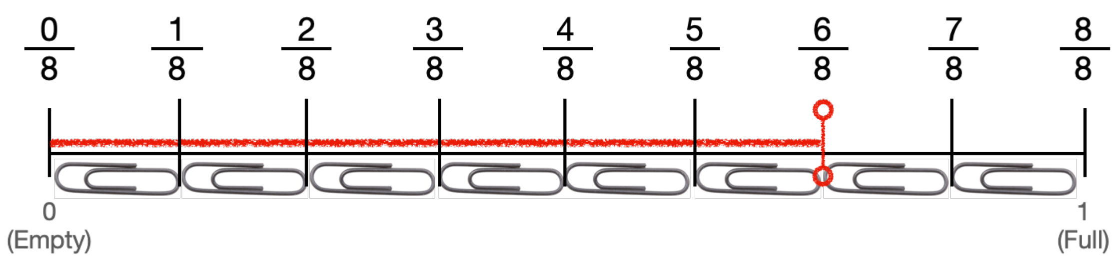


2. How could you **precisely communicate** how much fuel is in the tank? (Use the definition of fractions in your answer.)

Exit Card Answer Key

1. *Students will label the number line like the picture below. (They may use equivalent fractions in some cases.)*

Fuel Gauge



2. *Students will communicate the amount of fuel by explaining the line has been broken down into 8 equal parts, and 6 of them are filled. Therefore, the tank is 6/8 full. Students may also describe there are 6 paperclips worth of fuel, but they need to be able to communicate the fraction form. The fraction is more precise because Mission Control would not need to know the size of the paper or the gauge to determine how much fuel is available if they understand fractions. Using fractions to communicate is more efficient and precise than paper clips. (Although paper clips can help break the whole into equal pieces IF they are the same size.)*

Space Walker ________________________________ Date __________________

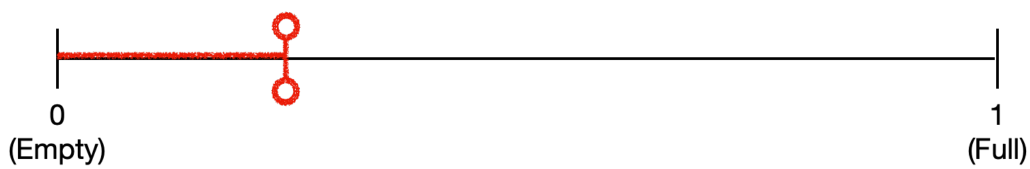
Opening Task

You are preparing for your spacewalk and checking your equipment.

Mission Control: Good Morning Team! Please give us an update on your oxygen tank levels.

You examine your oxygen tank gauge to report how much oxygen you have. Strangely, all the markings had worn off! How would you communicate to Mission Control how much oxygen is in your tank?

Oxygen Gauge



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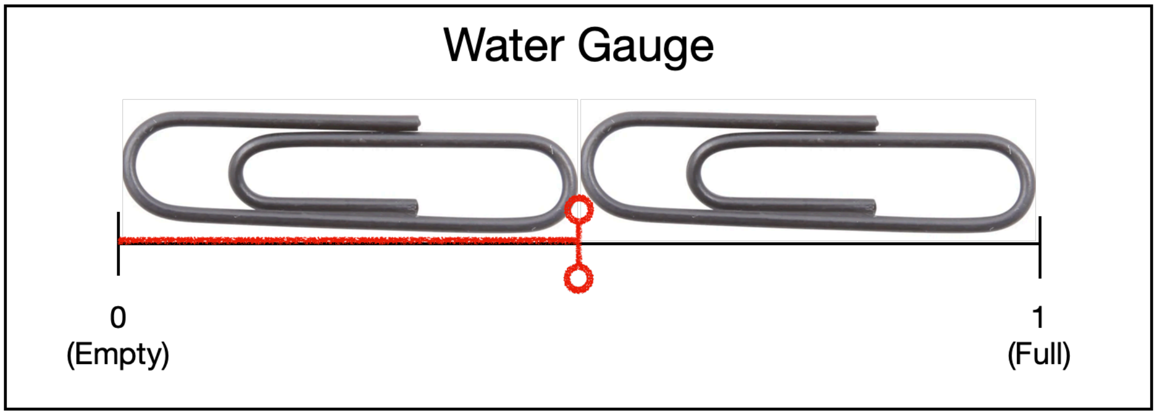
Space Walker ________________________________ Date __________________

Time for a Space Walk!

Peggy Whitson: Tier 1

Before you can go on your spacewalk, you need to make sure you have enough water in your suit to remain cool and hydrated. Again, Mission Control wants you to report on all your life support systems, but the markings have worn off. Li and Joe discuss using some items you have in the shuttle to describe how much water is in the tank. Joe placed paper clips on the gauge, like this:



Conversation

Mission Control: We cannot see your water gauge. Report on level of water remaining.

Joe: Water tank is 1 paper clip past empty.

Mission Control: Copy 1 paper clip past empty.

1. After this conversation, does Mission Control have a **precise** understanding of how much water is in the tank? Explain why or why not. What questions might they ask?

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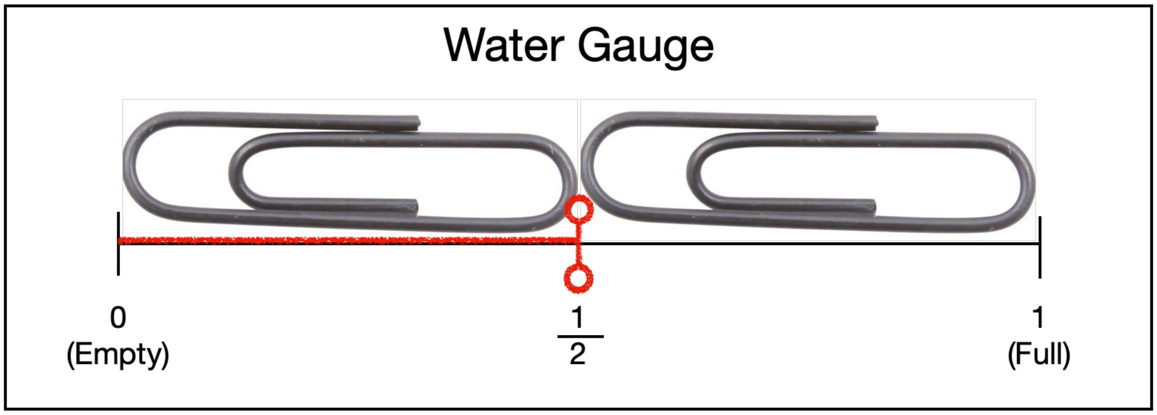
Li examined Joe’s paper clips, and then, she labeled a fraction:



Conversation

Mission Control: Could you report a precise fraction of your water levels?

Li: Yes, water tank is 1/2 full.

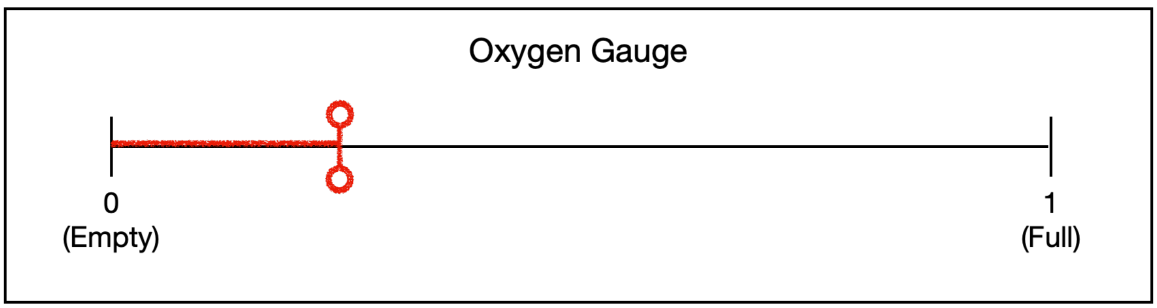
Mission Control: Copy 1/2 full.

2. Li reported they had 1/2 of a tank of water. Is she correct? Explain why or why not using the prompts below.

Guiding Questions	Your Response
Define: What does 1/2 communicate?	
Use of Tools: How did Li use the paper clips to determine the water tank is half full?	
Evaluate: How could you use other tools to check Li’s conclusion?	

3. Joe communicated the tank was 1 paper clip full. Li reported the tank was 1/2 full. Which astronaut was more precise? Explain. Why is being precise important in this situation?

4. Now, back to our opening task. Using the materials you have in the classroom, what is another way you might **precisely** communicate what fraction of the tank is full? How could you check?

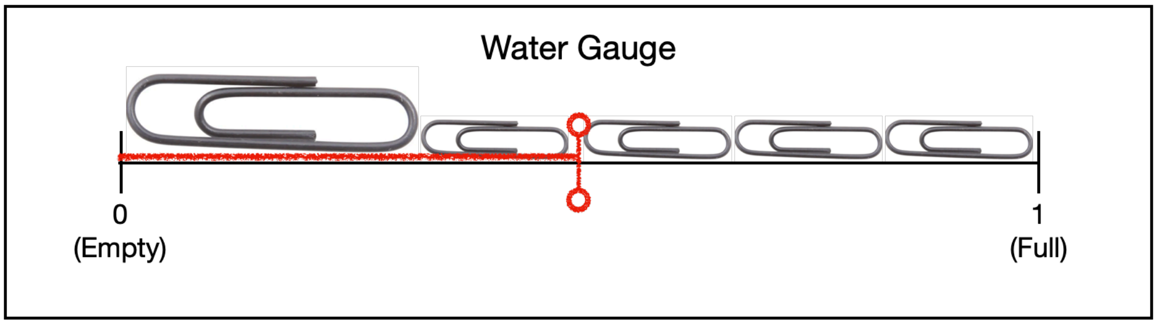


Space Walker ______________________________ Date ________________

Strolling in Space!

Guion Bluford: Tier 2

Before you can go on your spacewalk, you need to make sure you have enough water in your suit to remain cool and hydrated. Mission Control wants you to report on your water level, but the markings have worn off. Li and Joe discuss using some items you have in the shuttle to describe how much water is in the tank. Joe placed paper clips on the gauge, like this:



Conversation

Mission Control: We cannot see your water gauge. Report on level of water remaining.

Joe: Water tank is 2 paper clips past empty.

Mission Control: Copy 2 paper clips past empty.

1. After this conversation, does Mission Control have a **precise** understanding of how much water is in the tank? Explain why or why not. What question should they ask?

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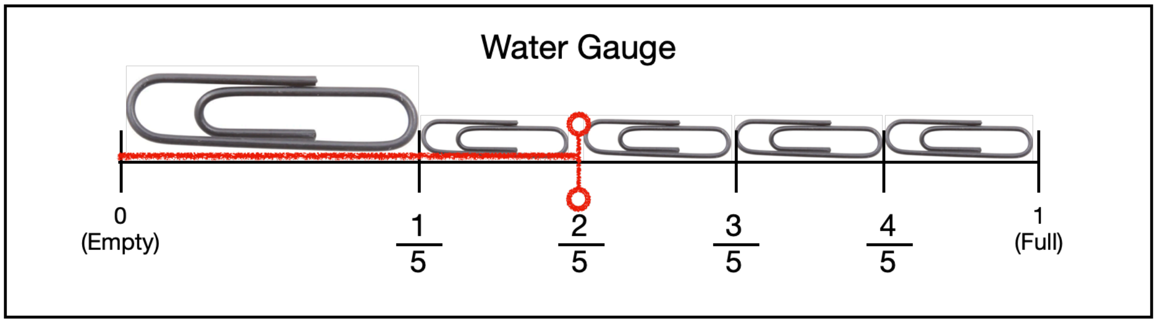
Li looked at Joe’s paper clips and added her own marks using fractions:



Conversation

Mission Control: Could you report a precise fraction?

Li: Yes, water tank is 2/5 full.

Mission Control: Copy 2/5 full.

2. She reported they had 2/5 of a tank of water. Is she correct? Explain why or why not. (Include a definition of a fraction.)

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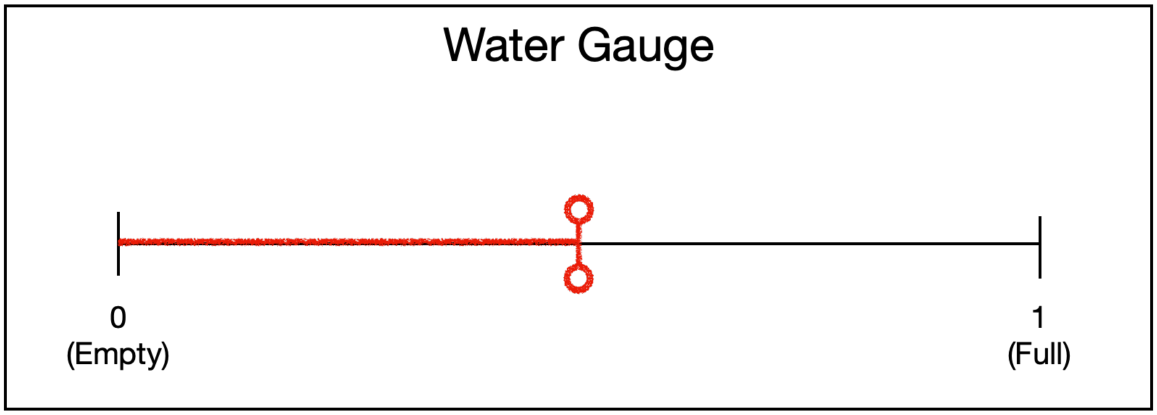
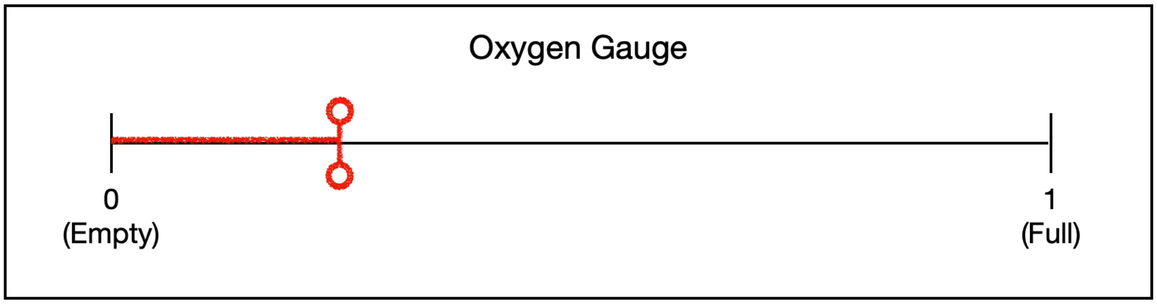
3. What is an additional **more precise** and **accurate** way you might communicate what fraction of the tank is full? What are 2 ways you can demonstrate your conclusion?



4. Now, back to our opening task. Using the materials you have in the classroom, what is another way you might **precisely** communicate what fraction of the tank is full? How could you check?



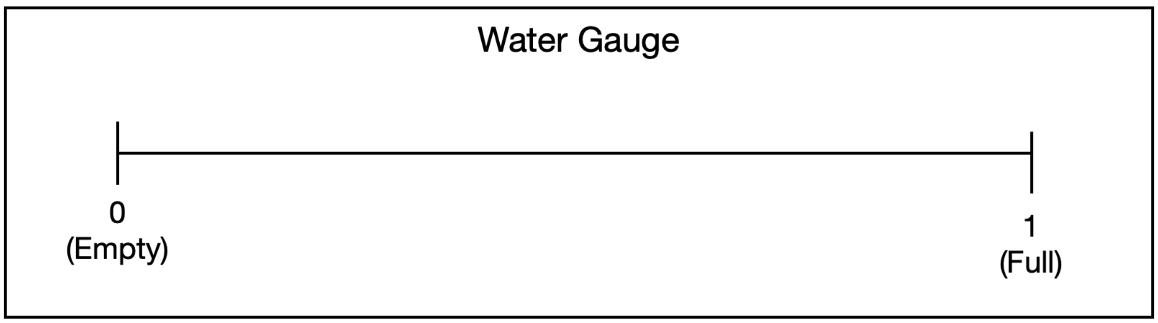
Mission Control Team Member _________________ Date _________________

Strolling in Space

Ellen Ochoa: Tier 3

Before the astronauts can go for a spacewalk, Mission Control and the astronauts need to know how much water is in the suit's water tank to help the spacewalkers stay cool and hydrated, but the markings have worn off.

Imagine you are in Mission Control. You know they have three sizes of paper clips available to them that they could use to determine how much water they have in their tank. You also know they have 20 paper clips in each size.



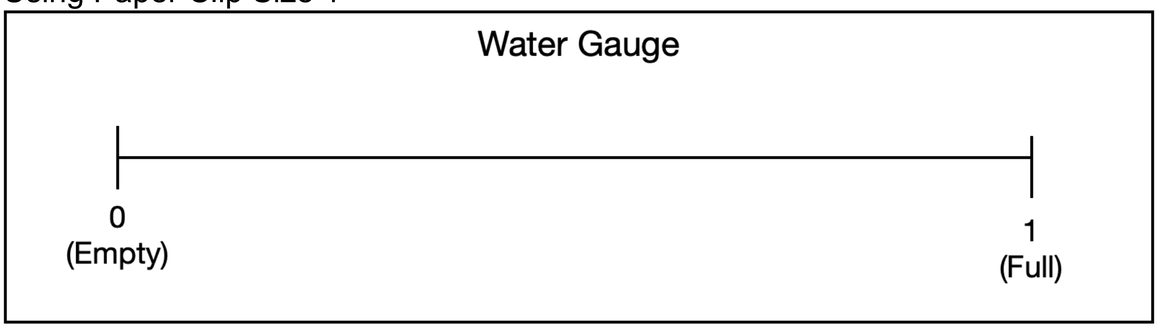
Paper Clip Size 1

Paper Clip Size 2

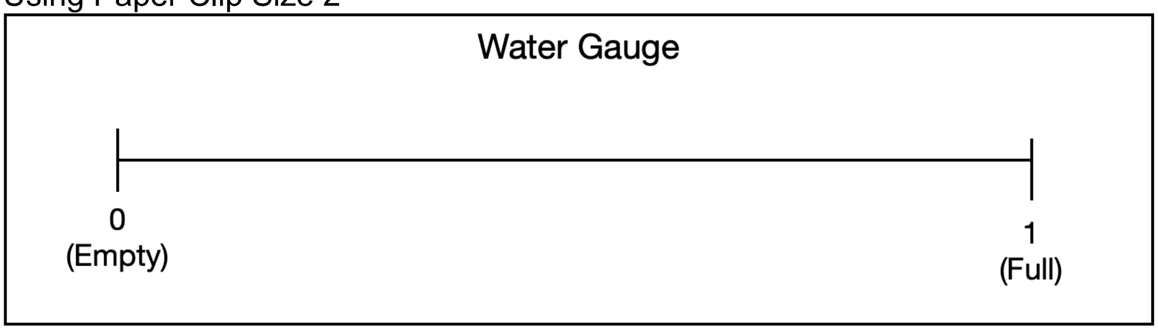
Paper Clip Size 3

1. The astronauts reported that they have 1/2 tank of oxygen. How could they have used any of the paper clip sizes to determine this amount? Explain. Draw each of their options and label the fractions.

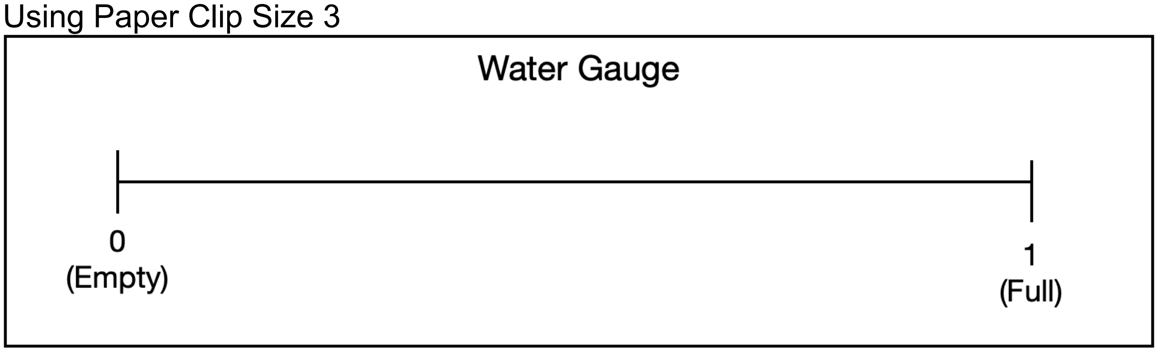
Using Paper Clip Size 1



Using Paper Clip Size 2



Using Paper Clip Size 3



The astronauts successfully completed their first spacewalk, but now they need to report back how much water they have left.

Mission Control Conversation

You: We need the astronauts to report on how much water they now have.

Mission Control, Tina: Tell the astronauts to use Paper Clip Size 3 to report how much water they have.

Mission Control, Devi: No! Wait! They should use Paper Clip Size 2. It is right in the middle.

Mission Control, Wallace: Why would that matter? Tell them Paper Clip Size 1 will be best.

2. As leader of Mission Control, you need to give the astronauts **precise instructions** on how to use their paper clips to report back on their water levels. Explain to the astronauts which paper clip size they should use and why.

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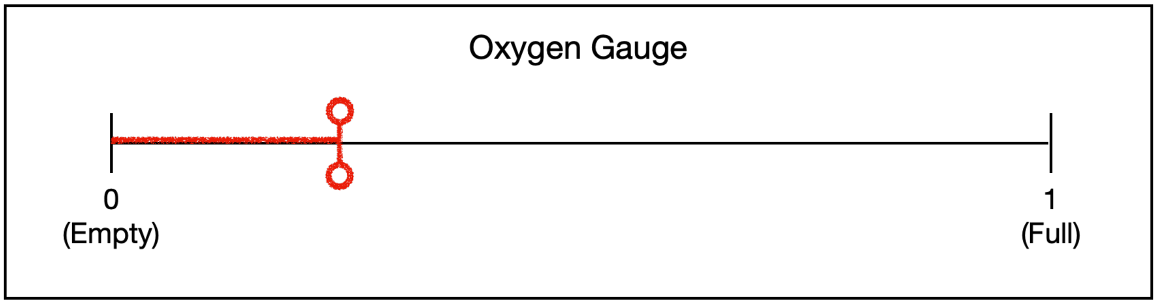
3. Now, back to our opening task. Using your classroom materials:
    a. What are 2 different ways you might **precisely** communicate what fraction of the tank is full?
    b. What makes one of your approaches better or worse than your other approach?



Mission to Mars Student Pages with Answer Keys

General Conceptual Framework
Throughout their responses, students need to incorporate the concept of equality. Fractions are only fractions when the portions are divided equally. The final full class discussion will synthesize across their experiences to demonstrate that fractions describe when a whole, in this case the space on a number line between 0 and 1, is divided into EQUAL parts.

Students should communicate precisely using definitions, examples, different representations, and various tools. They should use precise mathematical language of denominator and numerator to communicate, including the denominator is how many total EQUAL pieces are in the whole unit, and the numerator indicates how many of those pieces are present. They should precisely label the number line with equal pieces using a strategy or tool. They should recognize that fractions are helpful for their ability to communicate precise locations and amounts of parts of a whole.

Opening Task
Fraction Answer: 1/4

- **Fraction Concept:** The correct answer is 1/4 (or an equivalent fraction, like 2/8), but it is more important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine and communicate the fraction.
- **Mathematical Practice 6:** Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts.

Peggy Whitson: Tier 1

Tier 1 is differentiated as students only need to recognize the paper clips are already equal sizes. They still need to leverage those paper clips to communicate a fraction on a number line. They are also given additional scaffolding to craft their response.

1. *Answers may vary. Students should communicate that Mission Control only has a precise understanding IF they know how long the paper clip is AND how long the gauge is. If Mission Control has that information, then, the method could be precise. As it is, they do not know what "1 paper clip past empty" really means.*

2. *Answers may vary. Define-1/2 is a fraction that communicates the number line was broken into 2 equal pieces and that the tank is filling 1 of those 2 pieces. Use of Tools-Li used the paper clips to break the gauge into those 2 equal pieces and saw that 1 of the 2 were full, so she was able to communicate the tank was 1/2 full. Evaluate-students could check Li's reasoning using a ruler, folding, or any other consistent size tool, like tiles.*

3. *Li is more precise because it communicates the relationship of the part to the whole. Her response does not require knowing the length of the paper clip or of the gauge. It is consistently understood as a fraction. Being precise in this situation is important so Mission Control will be aware of how much water is available for the remaining parts of the spacewalk. Astronauts need water to cool their suits. If they are on their spacewalk without enough water, it may be a matter of survival.*

4. *Fraction Answer: 1/4*
    - ***Fraction Concept:*** *The correct answer is 1/4 (or an equivalent fraction, like 2/8), but it is more important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine and communicate the fraction.*
    - ***Mathematical Practice 6:*** *Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts.*

Guion Bluford: Tier 2

Tier 2 is differentiated as students need to recognize how when the paper clips are unequal sizes, the number of paper clips communicates nothing. Tier 2 does not have scaffolding in writing their responses, and they have to wrestle with fifths.

1. *Answers may vary. Students should communicate that Mission Control only has a precise understanding IF they know how long each paper clip is AND how long the gauge is. If Mission Control has that information, then, the method could be precise. As it is, they do not know what "2 paper clips past empty" really means. Further, they do not realize the paper clips are different sizes. If Joe insists on this method, Mission Control must ask for the lengths of the paper clips and the gauge.*

2. *Answers may vary. Define-2/5 is a fraction that communicates the number line was broken into 5 equal pieces and that the tank is filling 2 of those 5 pieces. Use of Tools-Li used the paper clips to break the gauge into those 5 pieces and saw that 2 of the 5 were full, so she was able to communicate the tank was 2/5 full. HOWEVER, this is overlooking the key idea that fractions communicate equal parts of a whole. The first paper clip is much larger than the others, so the whole is not equally divided and therefore, 2/5 is not accurate. Evaluate-students could check Li's reasoning using a ruler, folding, or any other consistent size tool, like tiles.*

3. *Answers may vary. The key idea is to break this line into equal parts and then communicate the filled portion using those equal parts. Students may use tiles, graph paper, rulers, or any number of other tools to demonstrate their response.*

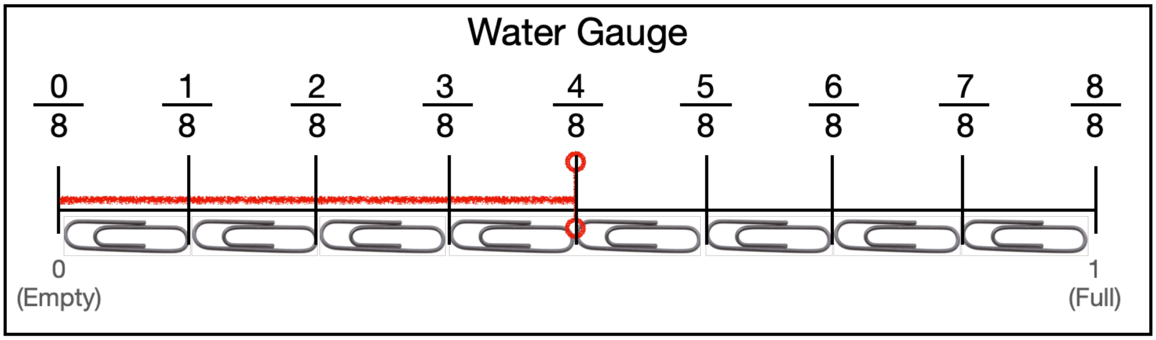
4. *Fraction Answer: 1/4*
   - ***Fraction Concept:** The correct answer is 1/4 (or an equivalent fraction, like 2/8), but it is more important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine and communicate the fraction.*
   - ***Mathematical Practice 6:** Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts.*

Ellen Ochoa: Tier 3

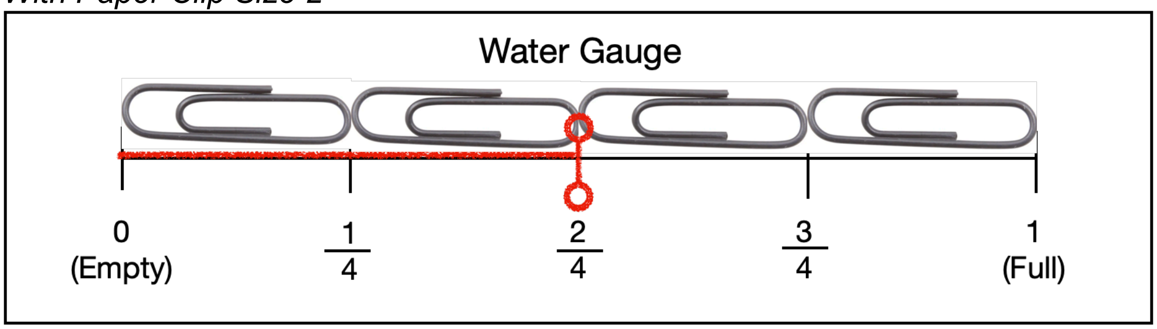
Tier 3 is differentiated as students wrestle with the idea that the smaller the parts, the more options for precision exists. They still need to construct number lines and understand fractions, but they start to uncover the benefit of breaking down the whole into smaller pieces for precision.

1. *Answers may vary. One example is as follows.*

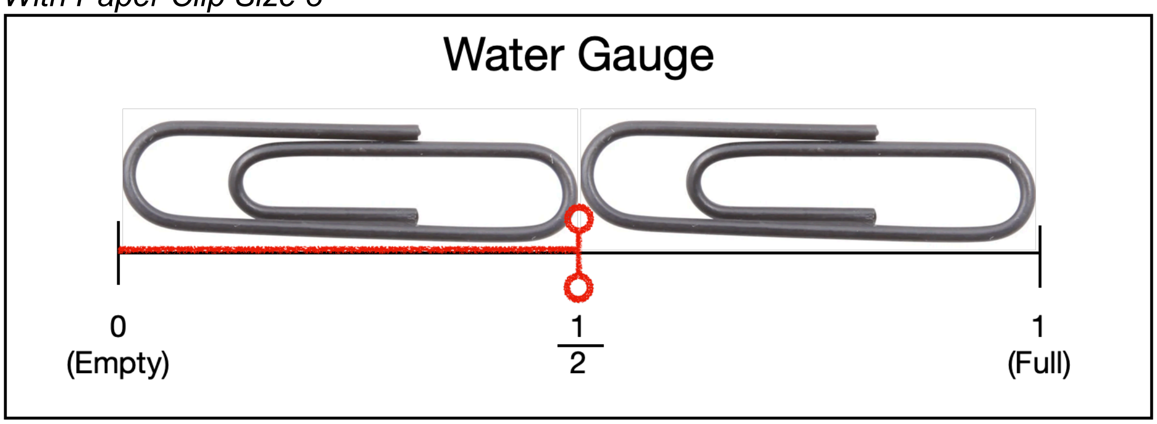
With Paper Clip Size 1



With Paper Clip Size 2



With Paper Clip Size 3



In general, students should be evaluated on breaking the parts into equal pieces. The labeled fractions could be equivalent fractions (e.g., 1/2 could be 2/4 or 4/8)

2. *Answers may vary. The smaller paper clip will be most helpful, as it can accurately determine distance down to the 1/8 of a tank, so it is able to determine 1/2, 1/4, and 1/8; whereas, the other options are not precisely able to determine 1/8.*

3. *Fraction Answer: 1/4*
    - ***Fraction Concept:*** *The correct answer is 1/4 (or an equivalent fraction, like 2/8), but it is more important that students display the concept of equal parts. They need to demonstrate that the number line needs to be broken into equal parts to determine and communicate the fraction.*
    - ***Mathematical Practice 6:*** *Students should demonstrate the precise use of tools and visual representations of equal parts. They may use a variety of tools, anything from a ruler, paper clips, or folding the paper, yet anything used must be used as a method for communicating equal parts.*
    - *Students must demonstrate two methods, but the key is to explore why one may be more precise than another. Size is one option. In this activity, students saw the smaller size of the paper clip would help communicate more precise results. However, other factors may also yield different degrees of precision, such as rigidity of the tool-if students use rubber bands, they may move between measurements, giving a less precise reading.*

Hint Cards for Communicating Precise Locations on a Number Line

Hint 1 What is the definition of a fraction?	**Hint 2** How can you divide the line into EQUAL parts?
Hint 3 What tools might you use to divide the line into equal parts? What is your process?	**Hint 4** How might you use folding to help divide the line into equal parts? What about tiles? Graph paper?

Challenge Cards for Fraction Understandings on Number Lines

Challenge 1 Are larger or smaller denominators more precise? How do you know?	**Challenge 2** Are there places on a number line when the size of the paper clip does not matter? Where? How do you know?

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This work was supported by Jacob K. Javits Gifted and Talented Students Education Program, United States Department of Education PR/Award # S206A170023, Thinking Like Mathematicians: Challenging All Grade 3 Students.