What Elementary Educators Need to Know About

Encouraging Their Students to Think Mathematically

$10 + (8 + 2) + 31 =$
$2 \times 10 + 31 = 51$

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"You cannot teach a man anything, you can only help him find it within himself."
— Galileo

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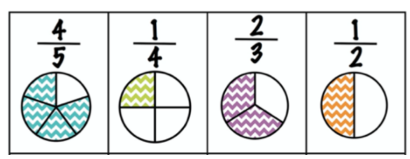
A hallmark of gifted education is to position students similarly to practicing professionals engaged in authentic work (Renzulli, 1977), including mathematics. Major publications in mathematics education have sought to clarify the kind of thinking espoused by mathematicians, such as the strands of mathematical proficiency identified in the National Research Council's *Adding It Up* (2001). Curriculum-related documents, including the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (PSSM; 2000) and the *Common Core State Standards for Mathematics* (CCSS-M; National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) respectively presented the process standards and standards for mathematical practice to help educators have their students engage in such mathematical thinking.

Nonetheless, the everyday curricular materials teachers rely on at times suggest these processes and practices are topics to cover rather than ways in which to engage with mathematical content. This suggests students are provided with a window into math rather than being immersed in a mathematical mindset. Put another way, mathematical thinking is a verb not a noun. Below are some actions educators can take to develop students' mathematical mindsets regardless of their math curricular materials.

Focus Your Planning and Instruction on Supporting Students' Mathematical Thinking Rather Than Just Teaching Them the Math



The intention of curricular documents like PSSM and CCSS-M was to highlight how students—not teachers—ought to think as they engage mathematically. When planning and teaching, consider what aspects students can discover themselves and what they need to be introduced to engage mathematically. For instance, students can figure out the meaning of the numerator and denominator when presented with fractions and accompanying visual representations, yet educators would need to share with them the names of the parts of the fraction. The meaning of symbolic notations (e.g., ÷, 3^2, =) and names of things and concepts (e.g., triangular prism, mean, area) are other examples of ideas educators eventually need to share with students.

Weave Mathematical Thinking Together Into All Math Content

3 +4	8 +7	6 +2
9 +4	2 +2	8 +0
2 +7	7 +8	9 +5
8 +1	5 +9	7 +4

The aforementioned curricular documents also highlight the notion that mathematical thinking ought to take place while students engage *with* mathematical content not separately from it. For instance, rather than having students first complete numerous addition problems then have them problem solve as they solve an addition problem, ensure they problem solve from the outset. Students could be tasked with finding patterns within the original list of problems, and could realize, for instance, you can break apart addends to double numbers, then add on to that initial sum (e.g., 3 + 4 = 3 + 3 + 1 = 6 + 1 = 7).

Let Students Determine How to Solve a Problem

Mathematicians decide how they will solve problems, so let students do the same. If published materials require students to, say, use a number line, draw a picture, or count by 5s, delete such directions. Be careful about “suggesting” the use of a strategy before students have the opportunity to grapple with problems because they likely will interpret that suggestion as a mandate.

Focus on Key Solution Strategies After Students Introduce Them

Keep an eye out for the mathematical concepts you seek to address as students work out their solutions. For instance, a pair might discuss how they realized that the sum of 9 + 5 is the same as 5 + 9. After sharing this with the class, they eventually should conclude that you can “turn around” the addends or add them in any order. Highlight how mathematicians realize the same idea and how they have named this phenomenon the “commutative property.” If the class does not suggest a major idea, present it as if it was an approach a former student shared previously to continue to encourage them to identify plausible solution strategies rather than relying on their teacher to provide one.

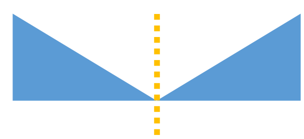
Have Students Determine the Validity of Their Answers—All of Them

Mathematicians regularly determine whether their answers are valid, and it is important for students to think similarly. Students can compare answers with peers, which particularly motivates them to determine why they may have gotten different and incorrect answers. They also should frequently defend solutions that are correct. Ask questions like “How do you know?” and “Why do you think that?” to get students to mathematically reason, argue, and justify.

Listen to—Don't Hear—Student Voices

Educators know what mathematics students should know for each lesson, unit, and year. It is easy to move to the next topic when we hear key ideas, yet if we listen more closely, students may need more time to solidify their understanding. Students may be able to state correct answers, particularly when there are few possibilities (e.g., greater than or less than), but not truly understand. As a check, present an invalid yet common misconception for the class to consider. For example, many students initially believe that a unit fraction with a greater denominator must be greater than one with a lesser one and incorrectly would state that $\frac{1}{3} > \frac{1}{2}$ even though a correct relationship would be recorded as $\frac{1}{3} < \frac{1}{2}$.

Trust That Students Can Think Mathematically, Even Young Ones



Elementary students are more than capable of thinking mathematically given the opportunity and support to do so. For instance, they can realize important geometric properties, such as reflections, rotations, and translations as they play with shapes and respectively talk about how you can flip, turn, and slide them around without changing the shape.

Give Students Time to Think Mathematically

Given that many assessments are more focused on ensuring math content has been mastered, the importance of fostering students’ mathematical thinking can be diminished. In the end, students’ ability to engage fully in a mathematical mindset helps ensure they will learn the mathematics at a level of understanding that will extend to future mathematical work and make this learning more efficient.

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