If Aliens Taught Algebra

Multiplication and Division Would be out of This World!

Grade 3

Teacher Manual

Third Edition

UCONN

June 2019

The National Research Center on the Gifted and Talented (2008-2013)
First Edition

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Mathematical language is a thinking tool that helps us use reason to create and communicate ideas about concepts, numbers, and shapes to solve problems in the real world.

- NRC/GT Research Team, University of Connecticut, 2008

If Aliens Taught Algebra

Multiplication and Division Would be out of This World!

Introduction

Third-grade students can do algebra and have fun in the process! Developing a solid understanding of arithmetic and prerequisite algebraic concepts at the elementary level can pave the way towards success in more advanced math classes throughout students' educational careers. At this early level, a foundation of core algebraic concepts helps ensure that students avoid developing misconceptions that could negatively impact their future success (Welder, 2012). This unit incorporates both educative materials and background information for teachers to support students' knowledge of algebra.

According to the algebra standards in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000), grades 3-5 students should

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts. (p. 158)

Given the emphasis on these algebra-focused standards, this unit includes lessons that build connections between mathematical operations, problem solving, and recognizing and working with visual patterns and numerical patterns with variables—all essential components of the foundation of prerequisite algebraic concepts.

To prepare students to work with algebraic concepts, the unit emphasizes understanding the concept of multiplication in the context of problem solving and in relation to other operations. Traditionally, students are required to memorize multiplication tables and then use these memorized facts to solve related problems. This process leaves out an important goal highlighted in the NCTM standards (2000), which states that students should know not only the procedure for multiplying numbers, but also understand multiplication's various meanings and interrelationships with other operations.

In this unit, the idea that multiplication is an extension of addition and the inverse of division is presented in a student friendly manner with the help of Planet Nine aliens. Students view multiplication through the lens of addition to increase the number of available strategies they have as they encounter complex word problems. For example, students may begin a problem by saying, “We have to add 8 sixes to solve this problem.” Students may begin by adding then realize that they can multiply 6 by 8 or 8 by 6 and get the same result. One goal of this unit is for students to develop their own efficiencies in problem solving.

The unit consists of lessons intended to build a conceptual framework for the structure and uses of multiplication. Students are provided with paper manipulatives to visualize what happens in the multiplication process. In addition to solving problems utilizing these pictures, students are asked to create their own problems. This allows students to consider the context of multiplication and enables teachers to assess students’ abilities to think backwards. Shifting the role of the student from consumer to creator, a process Haylock (1987) refers to as “problem posing,” is an essential component in developing students’ mathematical creativity.

The unit also develops students’ abilities to arrange data in increasingly complex tables. Utilizing tables gives students the opportunity to examine patterns that emerge in rows, columns, or both. Teachers can take this opportunity to discuss how removing a row would disrupt the pattern. For teachers wishing to differentiate tasks by enriching material for mathematically talented students, giving tables with more difficult patterns is a fairly easy means of providing additional challenge. Student work pages, which are organized in the accompanying Student Mathematician Notebook (SMN), include activities with these tables. For example, in the table from *Lesson 7: Getting to Know Planet Nine Aliens* Student Page **[SMN page 143]**, the first column increases by one each time. Creating a table that increases by three each time would provide additional challenge and variation in response leading to more meaningful classroom discussions.

The unit continues with a shift to the commutative property of multiplication in Lesson 10. Students will be relieved to discover that the commutative property allows them to memorize half as many facts! Like its addition counterpart, multiplication is an extremely flexible operation. In the context of a problem, 2 x 5 might make more sense to a student than 5 x 2. For example, having 2 Planet Nine aliens with 5 eyes each is different than having 5 Planet Nine aliens with 2 eyes each, though the overall number of eyes in both cases would be 10. NCTM (2000) highlights the need for students to work flexibly and solve problems using different mathematical representations.

Finally, the unit concludes by emphasizing more complex patterning, which incorporates an understanding of algebraic functions. In Lessons 14 and 15, students have an opportunity to work with variables to represent increasing and decreasing patterns in the context of alien-themed word problems. In the final lesson, Lesson 16, students will help the leader of the Planet Nine alien visitors find his spaceship crew at the local amusement park by solving mathematical problems to locate the crew members. Additionally, students will open a time capsule that contains artifacts exemplifying how they learned to think and act like mathematicians throughout the unit. This final lesson allows students to apply their knowledge of concepts and skills from the unit in a fun way!

Rationale

Providing equal access to educational opportunities for all students is an important democratic philosophy across schools in the United States. Unfortunately, factors, such as insufficient time and lack of resources, can create barriers for teachers to generate appropriately rigorous curricula. Finding effective means for facilitating differentiated instruction is particularly imperative in light of the U.S. Department of Education National Mathematics Advisory Panel's push for all 8th graders to take algebra, which has been historically viewed as a gateway to college. The rationale is that earlier placement in algebra will support students' math literacy and increase the number of students who take advanced math classes (Spielhagen, 2006).

What does this mean for mathematics at the elementary level? High-quality exposure to prerequisite algebraic concepts in earlier grades, such as those presented in this unit, is vital for students' success in future coursework. The unit was purposefully designed to provide background content information for teachers. The information allows teachers to support student development and provide exposure to developmentally appropriate algebraic concepts that are challenging for young mathematicians. In addition, teachers recognize that students have a continuum of skills influenced by prior knowledge; thus, the need for differentiation exists in any diverse classroom (Tomlinson, 2005). This curricular unit provides guidance in responding to students' diverse needs as well as recommendations for classroom management when implementing differentiation practices, like flexible grouping and tiered activities.

In response to the issue of global competitiveness in mathematics, President Bush created the National Mathematics Advisory Panel in 2006. The panel's final report in March 2008 recommended ways to improve mathematics achievement for all students in the United States. Table 1, on the following pages, indicates how this unit addresses recommendations of the National Math Advisory Panel as well as the Common Core State Standards.

The Common Core State Standards Initiative provides a clear set of educational standards for students in grades K-12. Table 2 displays the connections between Operations and Algebraic Thinking, Number and Operations in Base Ten, and this unit.

The Partnership for 21st Century Learning collaborated with teachers, education experts, and business leaders to create a Framework for 21st Century Learning. The framework identifies and promotes the knowledge and skills students need for success in education, work, and citizenship. The Learning and Innovation Skills component of the framework is comprised of the 4Cs: creativity, critical thinking, collaboration, and communication. Because the 4Cs are central to students' long-term success, educators should strive to foster 4Cs development in the classroom. Students demonstrating creativity engage in processes by which they generate novel, worthwhile ideas or products. Students with strong critical thinking skills analyze, synthesize, and evaluate information. Students collaborating with others work cooperatively with diverse groups while coordinate teammates' ideas or efforts. Finally, students exhibiting effective communication skills actively listen and utilize multiple modes of communication for a variety of purposes. Table 3 illustrates the connections between the 4Cs and the lessons in this unit.

Table 1

NCTM Curriculum Standards & Focal Points

NCTM Curriculum Standards & Focal Points	Lesson 1: Decomposition—Preparing for Blast Off	Lesson 2: Rounding—The ALIEN-R2200, A Wonderful Invention	Lesson 3: Mental Math—Planet Nine Aliens Go on a Shopping Spree!	Lesson 4: Flexible Numbers—Equal Sides	Lesson 5: 100s Charts—Where Did I Leave My Treasure?	Lesson 6: 100s Charts—Searching for Patterns	Lesson 7: Multiplication Madness—Getting to Know Planet Nine Aliens	Lesson 8: Multiplication Madness—Seeing Stars	Lesson 9: Arrays—Arranging Planet Nine Aliens	Lesson 10: Multiplication Madness—Meet Multiplication's Friend, Division	Lesson 11: Orbiting Oberon on the Oneida Rocket Ship—A Pre-boarding Task for Ms. Oort's Class	Lesson 12: Perplexing Visualizations—Unlocking the Code	Lesson 13: Repeating Patterns—Teaching Planet Nine Aliens Mathematical Terms	Lesson 14: Growing Patterns—Coasting at the Amusement Park!	Lesson 15: Growing Patterns—Cookies That Are Out of This World!	Lesson 16: An Intergalactic Top-Secret Mission—Find the Planet Nine Alien Spaceship Crew!
ALGEBRA																
1. Students describe, extend, and make generalizations about geometric and numeric patterns.		X			X	X	X					X	X	X	X	X
2. Students represent and analyze patterns and functions, using words, tables, and graphs.		X			X	X	X					X	X	X	X	X
3. Students identify such properties as commutativity, associativity, and distributivity, and use them to compute with whole numbers.			X	X			X	X	X	X	X					
4. Students represent the idea of a variable as an unknown quantity using a letter or a symbol.	X													X	X	
5. Students express mathematical relationships using equations.	X		X	X			X	X	X	X	X			X	X	X
6. Students model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.	X		X	X	X	X	X	X	X	X	X	X	X	X	X	X
7. Students investigate how a change in one variable relates to a change in a second variable.		X	X	X											X	X

Table 2
Common Core State Standards

Common Core State Standards	Lesson 1: Decomposition—Preparing for Blast Off	Lesson 2: Rounding—The ALIEN-R2200, A Wonderful Invention	Lesson 3: Mental Math—Planet Nine Aliens Go on a Shopping Spree!	Lesson 4: Flexible Numbers—Equal Sides	Lesson 5: 100s Charts—Where Did I Leave My Treasure?	Lesson 6: 100s Charts—Searching for Patterns	Lesson 7: Multiplication Madness—Getting to Know Planet Nine Aliens	Lesson 8: Multiplication Madness—Seeing Stars	Lesson 9: Arrays—Arranging Planet Nine Aliens	Lesson 10: Multiplication Madness—Meet Multiplication's Friend, Division	Lesson 11: Orbiting Oberon on the Oneida Rocket Ship—A Pre-boarding Task for Ms. Oort's Class	Lesson 12: Perplexing Visualizations—Unlocking the Code	Lesson 13: Repeating Patterns—Teaching Planet Nine Aliens Mathematical Terms	Lesson 14: Growing Patterns—Coasting at the Amusement Park!	Lesson 15: Growing Patterns—Cookies That Are Out of This World!	Lesson 16: An Intergalactic Top-Secret Mission—Find the Planet Nine Alien Spaceship Crew!
Operations & Algebraic Thinking																
Represent and solve problems involving multiplication and division							X	X	X	X	X			X	X	X
Understand properties of multiplication and the relationship between multiplication and division							X	X	X	X	X			X	X	X
Multiply and divide within 100										X	X					X
Solve problems involving the four operations, and identify and explain patterns in arithmetic					X	X		X						X	X	X
Number & Operations in Base Ten																
Use place value understanding and properties of operations to perform multi-digit arithmetic	X	X	X	X	X											X

Source: http://www.corestandards.org/Math/Content/3/OA

Table 3

21st Century Skills (4Cs: Creativity, Critical Thinking, Collaboration, Communication)

21st Century Skills: 4Cs	Lesson 1: Decomposition—Preparing for Blast Off	Lesson 2: Rounding—The ALIEN-R2200, A Wonderful Invention	Lesson 3: Mental Math—Planet Nine Aliens Go on a Shopping Spree!	Lesson 4: Flexible Numbers—Equal Sides	Lesson 5: 100s Charts—Where Did I Leave My Treasure?	Lesson 6: 100s Charts—Searching for Patterns	Lesson 7: Multiplication Madness—Getting to Know Planet Nine Aliens	Lesson 8: Multiplication Madness—Seeing Stars	Lesson 9: Arrays—Arranging Planet Nine Aliens	Lesson 10: Multiplication Madness—Meet Multiplication's Friend, Division	Lesson 11: Orbiting Oberon on the Oneida Rocket Ship—A Pre-boarding Task for Ms. Oort's Class	Lesson 12: Perplexing Visualizations—Unlocking the Code	Lesson 13: Repeating Patterns—Teaching Planet Nine Aliens Mathematical Terms	Lesson 14: Growing Patterns—Coasting at the Amusement Park!	Lesson 15: Growing Patterns—Cookies That Are Out of This World!	Lesson 16: An Intergalactic Top-Secret Mission—Find the Planet Nine Alien Spaceship Crew!
Creativity																
Use a wide range of idea creation techniques (such as brainstorming)		X		X			X	X			X		X	X	X	
Create new and worthwhile ideas	X				X							X				X
Elaborate, refine, analyze, and evaluate their own ideas in order to improve and maximize creative efforts					X		X				X					
Demonstrate originality and inventiveness in work and understand the real world limits to adopting new ideas		X	X						X						X	
Critical Thinking																
Use various types of reasoning as appropriate to the situation	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Analyze how parts of a whole interact with each other to produce overall outcomes in complex systems	X		X	X			X	X		X			X			
Analyze and evaluate alternative points of view	X			X	X				X	X	X				X	X
Synthesize and make connections between information and/or arguments			X			X								X	X	X

Table 3 *(continued)*

21st Century Skills (4Cs: Creativity, Critical Thinking, Collaboration, Communication)

21st Century Skills: 4Cs	Lesson 1: Decomposition—Preparing for Blast Off	Lesson 2: Rounding—The ALIEN-R2200, A Wonderful Invention	Lesson 3: Mental Math—Planet Nine Aliens Go on a Shopping Spree!	Lesson 4: Flexible Numbers—Equal Sides	Lesson 5: 100s Charts—Where Did I Leave My Treasure?	Lesson 6: 100s Charts—Searching for Patterns	Lesson 7: Multiplication Madness—Getting to Know Planet Nine Aliens	Lesson 8: Multiplication Madness—Seeing Stars	Lesson 9: Arrays—Arranging Planet Nine Aliens	Lesson 10: Multiplication Madness—Meet Multiplication's Friend, Division	Lesson 11: Orbiting Oberon on the Oneida Rocket Ship—A Pre-boarding Task for Ms. Oort's Class	Lesson 12: Perplexing Visualizations—Unlocking the Code	Lesson 13: Repeating Patterns—Teaching Planet Nine Aliens Mathematical Terms	Lesson 14: Growing Patterns—Coasting at the Amusement Park!	Lesson 15: Growing Patterns—Cookies That Are Out of This World!	Lesson 16: An Intergalactic Top-Secret Mission—Find the Planet Nine Alien Spaceship Crew!
Interpret information and draw conclusions based on the best analysis		X				X						X				
Reflect critically on learning experiences and processes					X											X
Solve different kinds of non-familiar problems in both conventional and innovative ways	X	X					X	X				X	X	X		
Collaboration																
Demonstrate ability to work effectively and respectfully with diverse teams			X													
Exercise flexibility and willingness to be helpful in making necessary compromises to accomplish a common goal			X		X											
Assume shared responsibility for collaborative work	X	X	X	X	X	X			X	X	X	X	X	X	X	X
Value the individual contributions made by each team member					X	X										X

Table 3 *(continued)*

21st Century Skills (4Cs: Creativity, Critical Thinking, Collaboration, Communication)

21st Century Skills: 4Cs	Lesson 1: Decomposition—Preparing for Blast Off	Lesson 2: Rounding—The ALIEN-R2200, A Wonderful Invention	Lesson 3: Mental Math—Planet Nine Aliens Go on a Shopping Spree!	Lesson 4: Flexible Numbers—Equal Sides	Lesson 5: 100s Charts—Where Did I Leave My Treasure?	Lesson 6: 100s Charts—Searching for Patterns	Lesson 7: Multiplication Madness—Getting to Know Planet Nine Aliens	Lesson 8: Multiplication Madness—Seeing Stars	Lesson 9: Arrays—Arranging Planet Nine Aliens	Lesson 10: Multiplication Madness—Meet Multiplication's Friend, Division	Lesson 11: Orbiting Oberon on the Oneida Rocket Ship—A Pre-boarding Task for Ms. Oort's Class	Lesson 12: Perplexing Visualizations—Unlocking the Code	Lesson 13: Repeating Patterns—Teaching Planet Nine Aliens Mathematical Terms	Lesson 14: Growing Patterns—Coasting at the Amusement Park!	Lesson 15: Growing Patterns—Cookies That Are Out of This World!	Lesson 16: An Intergalactic Top-Secret Mission—Find the Planet Nine Alien Spaceship Crew!
Communication																
Articulate thoughts and ideas effectively using oral, written, and nonverbal communication skills	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Listen effectively to decipher meaning, including knowledge, values, attitudes, and intentions		X			X		X									
Use communication for a variety of purposes (e.g., to inform, instruct, motivate, and persuade)					X										X	X

Source: http://www.p21.org/storage/documents/docs/P21_Framework_Definitions_New_Logo_2015.pdf

If Aliens Taught Algebra: A Model-based Unit

This unit was designed with specific modifications and differentiation for high ability or gifted learners. Additionally, it was designed to be responsive to academic diversity in talent pool students and students in general education classrooms. Elements of three well-known curricular models in the field of gifted and talented education were combined and utilized to develop this unit: the Differentiation of Instruction Model from Carol A. Tomlinson (2001), the Depth and Complexity Model from Sandra N. Kaplan (1998), and the Schoolwide Enrichment Model from Joseph S. Renzulli and Sally M. Reis (1997). These three research-based models support and promote qualitative differentiation of learning; the ways it is pursued; the nature and extent of student engagement; the active, investigative roles assumed by students; and the quality of student products. The Differentiation of Instruction Model is designed to provide rich, engaging curriculum matched to the diverse interests, readiness levels, and learning profiles of individual students. The model assumes that there is no distinct, single curriculum appropriate for gifted learners, but rather that all students, including gifted students, require educational experiences suited to their individual needs (Tomlinson, 1996).

The Depth and Complexity Model emphasizes the importance of rich, deep, and complex content in appropriately serving gifted learners (Kaplan, 1998). This model emphasizes the benefit of higher-level thinking skills, elaborate product development, and more advanced resources in a curriculum for gifted students; however, redefining the nature of the content is the crux of the model's curriculum equation. The redefinition is based on the premise that appropriate, high-level content is synonymous with the dimensions of depth, complexity, novelty, and acceleration (Kaplan, 1998).

The Schoolwide Enrichment Model identifies a talent pool of 15 to 20% above average ability, and/or students with the potential for high ability, who will be served through a variety of options; the options include learning experiences geared toward students' interests and learning styles, curriculum compacting, and enrichment experiences (Renzulli & Reis, 1985, 1997, 2014). Over several decades, the Schoolwide Enrichment Model research demonstrated its effectiveness across a broad range of school socioeconomic levels, program organization patterns, and gifted learners (Baum, 1985, 1988; Olenchak & Renzulli, 1989; Reis & Renzulli, 2003).

This curriculum unit reflects the commonalities of these well-known models in the field of gifted and talented education. This unit tailors essential content, process, and products to the academic needs of students in academically diverse classrooms; emphasizes conceptual thinking, real-world disciplinary inquiry, and problem solving; assesses specific and developing learning needs of talent pool students and all students in general education classrooms; helps students acquire increasing levels of expertise; and encourages student involvement with problem solving and product development with real-world utility.

If Aliens Taught Algebra Pacing Chart

Unit Pretest	**Norm-referenced Pretest**	**Lesson 0:** Introducing the Unit	**Lesson 1:** Decomposition—Preparing for Blast off	**Lesson 2:** Rounding—The ALIEN-R2200, A Wonderful Invention **Tiered Student Pages** Fibonacci, Diophantus, & Kovalevsky
Lesson 3: Mental Math—Planet Nine Aliens Go on a Shopping Spree! **Tiered Student Pages** Diophantus & Kovalevsky	**Lesson 4:** Flexible Numbers—Equal Sides **Tiered Student Pages** Fibonacci, Diophantus, & Kovalevsky	**Lesson 5:** 100s Charts—Where Did I Leave My Treasure? **Tiered Student Pages** Diophantus & Kovalevsky	**Lesson 6:** 100s Chart—Searching for Patterns **Tiered Student Pages** Fibonacci, Diophantus, & Kovalevsky	**Lesson 7:** Multiplication Madness—Getting to Know Planet Nine Aliens
Lesson 8: Multiplication Madness—Seeing Stars	**Lesson 9:** Arrays—Arranging Planet Nine Aliens **Tiered Student Pages** Fibonacci, Diophantus, & Kovalevsky	**Lesson 10:** Multiplication Madness—Meet Multiplication's Friend, Division	**Lesson 11 (OPTIONAL):** Orbiting Oberon on the Oneida Rocket Ship—A Pre-boarding Task for Ms. Oort's Class	**Lesson 12:** Perplexing Visualizations—Unlocking the Code **Tiered Student Pages** Fibonacci, Diophantus, & Kovalevsky
Lesson 13: Repeating Patterns—Teaching Planet Nine Aliens Mathematical Terms **Tiered Student Pages** Fibonacci & Diophantus	**Lesson 14:** Growing Patterns—Coasting at the Amusement Park! **Tiered Student Pages** Fibonacci, Diophantus, & Kovalevsky	**Lesson 15:** Growing Patterns—Cookies That Are out of This World! **Tiered Student Pages** Fibonacci, Diophantus, & Kovalevsky	**Lesson 16 (Day 1):** An Intergalactic Top-Secret Mission—Find the Planet Nine Alien Spaceship Crew! **Tiered Student Pages** Fibonacci, Diophantus, & Kovalevsky	**Lesson 16 (Day 2)** Debrief: Thinking Like Mathematicians
FLEX TIME—Four flex days are built in for lessons that take more than one class period to complete.			**Unit Posttest**	**Norm-referenced Posttest**

If Aliens Taught Algebra Pacing Chart Example Over Four Months

Month 1:

- ☐ **Unit Pretest**
- ☐ **Norm-referenced Pretest**
- ☐ **Lesson 0:** Introducing the Unit
- ☐ **Lesson 1:** Decomposition—Preparing for Blast off
- ☐ **Lesson 2:** Rounding—The ALIEN-R2200, A Wonderful Invention
  - **Tiered Student Pages:** Fibonacci, Diophantus, & Kovalevsky
- ☐ **Lesson 3:** Mental Math—Planet Nine Aliens Go on a Shopping Spree!
  - **Tiered Student Pages:** Diophantus, & Kovalevsky

Month 2:

- ☐ **Lesson 4:** Flexible Numbers—Equal Sides
  - **Tiered Student Pages:** Fibonacci, Diophantus, & Kovalevsky
- ☐ **Lesson 5:** 100s Charts—Where Did I Leave My Treasure?
  - **Tiered Student Pages:** Diophantus, & Kovalevsky
- ☐ **Lesson 6:** 100s Chart—Searching for Patterns
  - **Tiered Student Pages:** Fibonacci, Diophantus, & Kovalevsky
- ☐ **Lesson 7:** Multiplication Madness—Getting to Know Planet Nine Aliens
- ☐ **Lesson 8:** Multiplication Madness—Seeing Stars
- ☐ **Lesson 9:** Arrays—Arranging Planet Nine Aliens
  - **Tiered Student Pages:** Fibonacci, Diophantus, & Kovalevsky
- ☐ **Lesson 10:** Multiplication Madness—Meet Multiplication's Friend, Division

Month 3:

- ☐ **Lesson 11:** Orbiting Oberon on the Oneida Rocket Ship—A Pre-boarding Task for Ms. Oort's Class
- ☐ **Lesson 12:** Perplexing Visualizations—Unlocking the Code
  - **Tiered Student Pages:** Fibonacci, Diophantus, & Kovalevsky
- ☐ **Lesson 13:** Repeating Patterns—Teaching Planet Nine Aliens Mathematical Terms
  - **Tiered Student Pages:** Fibonacci & Diophantus
- ☐ **Lesson 14:** Growing Patterns—Coasting at the Amusement Park!
  - **Tiered Student Pages:** Fibonacci, Diophantus, & Kovalevsky
- ☐ **Lesson 15:** Growing Patterns—Cookies That Are out of This World!
  - **Tiered Student Pages:** Fibonacci, Diophantus, & Kovalevsky

Month 4:

- ☐ **Lesson 16 (Day 1):** An Intergalactic Top-Secret Mission—Find the Planet Nine Alien Spaceship Crew!
  - **Tiered Student Pages:** Fibonacci, Diophantus, & Kovalevsky
- ☐ **Lesson 16 (Day 2):** Debrief: Thinking Like Mathematicians
- ☐ **Unit Posttest**
- ☐ **Norm-referenced Posttest**

Organization of the Unit

Unit Breakdown

This unit was designed to develop students' conceptual pre-algebraic understanding by emphasizing numerical computations in the context of problem solving, decomposing and recomposing numbers, and comfort working with numerical and visual patterns. This unit is divided into three main sections. In the first section, the lesson objectives focus on rounding, addition, subtraction, and mental math. The second section focuses on multiplication and division. The third section emphasizes numerical and visual patterns. The following is a list of the topical sections throughout the unit:

- **Mental Math** skills support the development of fluency in numerical computations. Mental math abilities will also provide students with the confidence to calculate complex computations and build fluency alternating between different operations. Lessons in the unit first emphasize decomposing and recomposing numbers to facilitate rounding and estimation skills.
- **Charts with Patterns** support students' understanding of multiples in the 100s chart. In Lesson 5, students learn to navigate through the 100s chart and discover patterns. Then, in Lesson 6, students explore the 100s chart to connect multiples on the chart with multiplication.
- **Patterns and Sequences** are incorporated throughout the unit, including patterns that increase or decrease in sequence. Some lessons focus on visual patterns versus numerical patterns. For numerical patterns that emphasize alternating between different operations, students will predict which number comes next in the pattern. Additionally, students are challenged to determine formulas or functions to represent various numerical patterns.
- **Evaluating** classmates' responses provides opportunities for students to reflect on their work. Furthermore, it allows them to agree or disagree on mathematical ideas and discuss common misconceptions. The Examine and Elaborate section of the lesson often gives opportunities for such discussions. Students share the work they have done and receive feedback from other students and the teacher. Allowing students to evaluate one another fosters conceptual understanding of ideas that may have been previously unknown or unclear.

Student Mathematician Notebook

The Student Mathematician Notebook (SMN) allows students to explore and record their mathematical ideas while giving them opportunities to work as young mathematicians. By recording their thinking in the Student Pages of the SMN, students may use their notebooks to participate in classroom discussions or work through investigations in the Explore sections.

Student Pages in the SMN are also intended to elicit students' responses to open- and closed-ended questions. The questions frequently prompt students to record their answers, explain their thinking, or reflect on their problem-solving in written form. Writing in mathematics allows students to strengthen their understanding of content while developing their reasoning and communication abilities; however, some students may be resistant to writing or may struggle to produce written work. Teachers should consider providing additional supports or adaptations for these students. For instance, teachers might provide writing resources, like sentence starters (e.g., "I found my answer by. . . .") or a list of relevant mathematical terms and phrases. Depending on students' needs, teachers can also adapt the product. For example, students could type their answers, write responses on a whiteboard, use images or bulleted lists, compose responses with a partner, or give their answers orally. By providing necessary supports or adaptations, teachers can make the mathematical content and unit activities more accessible to resistant or struggling writers.

Finally, the SMN may be used to assess student learning and thinking. By offering students feedback on their progress in the SMN, students may learn from their mistakes. Throughout the Teacher Manual, teachers are guided in how to use the Student Pages. For instance, each lesson lists several possible responses to Share and Discuss questions; sample dialogues are also included to support students' understanding of the concepts. These possible responses highlight the notion that students come with different ideas and teachers should incorporate several different responses (i.e., both correct and incorrect reasoning) to build a mathematics community. The discussion provides students with opportunities to make sense of mathematics and engage in mathematicians' behaviors. Additionally, the Answer Key book includes the Student Pages with sample answers. The corresponding page numbers for the SMN are included at the beginning of each lesson. Checkpoints are incorporated periodically to review previously learned skills and concepts.

Resources for Teachers and Students

Throughout the unit, teachers and students may access resources on the flash drive. The flash drive includes links to websites, videos, and articles that extend learning opportunities. Additionally, teachers can access online vocabulary flashcards and games created through Quizlet to accompany the Unit.

Unit Pretest and Posttest

The Unit Test, *A Test for all Humans*, is designed to assess students' problem solving abilities. The test is used as the pretest and posttest for the unit. Since the test assesses students' problem solving abilities and *not* the ability to multiply, scoring the assessment requires teachers to consider students' thinking related to each item. The goal of the unit is not to make students proficient multipliers, but rather to increase students' flexibility in problem solving using strategies, such as repeated addition, multiplication, representation, and counting techniques. These are the types of solution strategies teachers should look for on the assessment.

To support differentiated instruction, teachers should administer the Unit Test as a pretest to determine students' existing knowledge and readiness for the unit. Student performance on the Unit Test is only one indicator of readiness for different tasks and grouping levels. Teachers should also remember that students' readiness for specific concepts may vary throughout the unit; therefore, both formal and informal assessments should be used to inform grouping and assigning tasks. Using the same Unit Test as the pretest and posttest will allow teachers to evaluate individual student growth.

Following the Unit Test in this manual is an Answer Key and Unit Test Rubric that includes the question, the number of points allocated to each question, and sample student answers. The Unit Test Rubric is provided following the Answer Key in this manual to aid teachers in forming flexible groups of students throughout the unit lessons.

Lesson Design

As teachers begin to use the lessons, they will notice that all lessons follow the same format.

The following information is provided at the beginning of each lesson:

- **Big Ideas**
- **Lesson Objectives**
- **Materials**
- **Mathematical Terms**
- **Selected Mathematical Practices**
- **Differentiation**

The following sections are also included in each lesson:

Lesson Preview	The "Lesson Preview" section provides a lesson overview.
Launch	The "Launch" phase introduces new mathematical content while allowing students to consider the direction of the lesson in relation to their prior knowledge and/or experiences. The Launch also highlights mathematical practices, which are essential for students to begin "thinking like mathematicians."
Explore	The "Explore" section encourages students to pursue ideas and curiosities that arise during the Launch. Activities are often student-centered and exploratory in nature, allowing students to draw their own conclusions about mathematical ideas and discuss them with classmates.

Examine and Elaborate	Students solidify new knowledge from the lesson, communicating what they have learned in a variety of ways.
Debrief and Look Ahead	The “Debrief and Look Ahead” reviews the lesson’s main ideas. Students also reflect on how they thought and acted like mathematicians during the lesson by creating artifacts for a time capsule. A preview the next lesson’s purpose is provided to make connections between lessons.
Assess	Both formal and informal assessment options are included: • **Student Pages:** The Student Pages that accompany the explorations offer opportunities for students to think critically about important concepts. After the lesson-related questions, many of these pages include practice problems designed to extend the big ideas or help gauge student progress. Teachers can decide whether to formally assess students on these sections of the student pages or use them as a guide in determining which students need targeted assistance. All of these pages are accompanied by answer pages, which include possible student responses and student difficulties. Many pages are tiered for varied levels of prior knowledge or learner readiness. • **Homework Pages:** Activities or worksheets students can complete at home are also included in the SMN. Homework pages are designed for students at any level who completed the lesson. Some tiered activities from the lessons may be used as homework and may be assigned to students based upon the Grouping Guides or student performance during that lesson. • **Checkpoints:** Periodically, checkpoints are included as a means of assessing student progress on unit concepts and regular curriculum foundational skills. • **Group Discussion:** Suggested questions are provided to stimulate group discussions that offer opportunities for students at all levels to share their mathematical strategies and learn from each other. Discussions can also focus on topics that arise from student communication during the Examine and Elaborate section or questions posed on the Student Pages. • **Direct Inquiry:** This occurs when teachers directly ask students comprehension questions or ask students to demonstrate understanding.

Differentiation

This unit is grounded in research-based curricular models. Essentially, the unit asks teachers to respond proactively to students’ needs as determined by students’ prior knowledge or learner readiness. The teachers’ instructional decision making is guided by general principles of differentiation (see Table 4). These may include respectful tasks, on-going assessments and adjustment, clear communication of the learning goals, appropriate challenge and scaffolding, and flexible grouping.

Table 4
Thinking Like Mathematicians: Challenging All Grade 3 Students
Differentiation Strategies Within the Unit

Content

- learning goals
- prior knowledge or learner readiness
- tiered activities
- formative assessment
- varied levels of challenge
- "teaching up" (aim high, provide scaffolding)
- know (information, facts, vocabulary), understand (concepts, big ideas, connections), apply (skills, processes)
- real-world application

Process

- questioning strategies
- 4Cs (21st Century Skills)
    - creativity
    - critical thinking
    - collaboration
    - communication
- hands-on activities/manipulatives
- connections

Product

- oral, visual, and written opportunities
- multiple ways to demonstrate knowledge, understanding, and skills
- multiple models and representations
- meaningful and respectful tasks
- summative assessment

Learning Environment

- flexible grouping
- whole group/small group/individual instruction
- growth mindset
- learning community

This mathematics unit presents differentiated content that will engage and challenge students appropriately. Most lessons are differentiated based on students' readiness levels. The content presents greater complexity and goes into greater depth for students who are more familiar with the concepts.

Throughout the unit, students will have many opportunities to work together in flexible groups. When grouping students, teachers will often do so based on readiness for that lesson (using the unit pretest as a guide). Teachers may also consider grouping students because they used different methods, which encourages students to focus on conceptual understanding. Groups throughout the unit should be flexible and purposeful to support prior knowledge or learner readiness. It is important that each group member is challenged to think during lessons.

Multiple lessons guide teachers to proactively respond to students' varied readiness levels. Some lessons include a Grouping Guide that informs how students might work collaboratively in groups on tiered Student Pages using information about students' prior knowledge from the unit pretest (see Table 5). For instance, in the Grouping Guide chart below from Lesson 2, groups are formed based upon the number of points students earned on specific questions from certain Unit Test items.

Table 5
Grouping Guide for Lesson 2

Groups Formed by Scores on Question 2 on the Pretest		
Fibonacci Lab Group	Diophantus Lab Group	Kovalevsky Lab Group
Scored 0 points on item, other observations	Scored 1 point on item, other observations	Scored 2 points on item, other observations
Student Names	Student Names	Student Names
Student Pages for Lessons		
Understanding the ALIEN-R2200—Fibonacci Student Page **[SMN pages 25-27]**	*Understanding the ALIEN-R2200—Diophantus* Student Page **[SMN page 29]**	*Understanding the ALIEN-R2200—Kovalevsky* Student Page **[SMN page 31]**

Lesson 2 is intended to help students describe how to round numbers to the nearest 10; however, the Fibonacci Student Page provides students with more visual supports and repetition than the Diophantus and Kovalevsky pages. The Kovalevsky Student Page challenges students to articulate how to round numbers when there is a 5 in the ones place, which is a special case for rounding that sometimes confuses students.

Teachers may also group students by assessing students' levels of prior knowledge informally with a brief discussion about the lesson content. Student groups should be assigned flexibly, using the Grouping Guide **as a starting point**; however, teachers' knowledge of their students should also help determine student grouping for each tiered lesson. Teachers have the flexibility to move students among the tiers during any lesson if a student is progressing quickly or if a student is placed in a group that is too difficult. Students can also choose to complete the more advanced tiers if they are initially assigned to a lower tier and they would like to challenge themselves.

NOTE: It is not necessary to assign students to all tiers available in a lesson. For instance, if all students in a class are all ready for the Kovalevsky tier (the most advanced level), then the whole class can work on that tier.

Mathematical Practices

In addition to new content, each lesson includes opportunities for direct instruction in mathematical practices that will encourage students to "think like mathematicians." The eight Common Core State Standards for Mathematical Practice are:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

These practices, or habits of mind, are closely associated with the 21st Century Skills 4Cs (creativity, critical thinking, collaboration, and communication). Each lesson contains a mathematical practice question (e.g., How do mathematicians support their answers/solutions?), which is explored and modeled during the lesson Launch. Throughout the lesson, teachers prompt and encourage students to use these mathematical practices.

During the Debrief and Look Ahead sections, students are given opportunities to reflect on their use of the Mathematical Practices by creating artifacts for a time capsule. Artifacts might include items like stories, drawings, work samples, audio recordings, videos, or other ideas that students generate. At the end of the unit, the class will invite a guest to watch students open the time capsule; students will use the artifacts to talk about the importance of thinking and acting like mathematicians.

Instructional Discourse

In her 1998 work, Wood categorized teachers' patterns of questioning as either "funneling" or "focusing." The major difference between the two types of questioning lies in who is responsible for the mathematical thinking. In funneling, this duty is the responsibility of the teacher, where a student is guided through a series of questions built around how the teacher would solve the problem. Focusing, on the other hand, is a type of questioning that enables the students to justify responses based on their own thinking. Focusing questions help students engage in the Mathematical Practices (MP) and 4Cs.

Each lesson provides multiple features to help teachers anticipate how they might facilitate whole-class discussions to give students the opportunity to further experience thinking—and communicating—like mathematicians. *It's important to realize that the ideas presented are plausible ones, and teachers are encouraged to focus on their own students' contributions and needs.* The Collaborate and Communicate section within the Explore phase identifies a question the entire class will eventually examine. To prepare, small groups will share their ideas on large chart paper. Teachers can implement Smith and Stein's *5 Practices for Orchestrating Productive Mathematics Discussions* (2011), including "anticipating," "monitoring," "selecting," "sequencing," and "connecting." Sample student responses are provided that address the anticipating phase and teachers can record additional strategies they observe for future reference. Sample responses also help teachers plan for the Share and Discuss phase, where they can incorporate the other four practices.

Additionally, teachers can utilize "talk moves" that are described in Chapin, O'Connor, and Anderson's (2009) book, *Classroom Discussions: Using Math Talk to Help Students Learn*, to ask focusing versus funneling questions. Talk move tips also are noted throughout the lesson to suggest places where they might be useful during activities. Examples of talk moves are also included in the sample dialogues in the Share and Discuss section. The five talk moves include:

1. **Revoicing**—This talk move involves having the teacher restate a student's answer to clarify the teacher's understanding of the student's response. This is especially helpful if the student's response is not clear. An example would be, "You said that the numbers in this pattern are increasing by three's. Is this right?"
2. **Repeat/Rephrase**—With this talk move, teachers ask other students to repeat or rephrase what a classmate just stated. For example, "Could someone repeat what Sally just said in your own words?" This allows other students to hear what the first student stated and it gives them a chance to process the information.
3. **Reasoning**—After a student responds, the teacher might ask: "Do you agree with Sally's explanation or disagree? Why?" as a way to promote mathematical reasoning among students based on each other's responses.

4. **Adding On**—Teachers can encourage participation among more students in the class by asking students "Would someone else like to add more to Sally's explanation?"
5. **Wait Time**—In this move, the teacher waits and gives students an opportunity to think about a question that has been asked or think about a classmate's answer. This gives students a chance to process and formulate a response. The teacher might say, "Take your time, Joe, and we will wait for you to think."

Finally, insights about how to support students who are struggling or need further challenge are included to help teachers anticipate how the whole-class discussion might transpire in their own classes.

Super Challenges

An effective differentiation strategy for students who finish their work early is an anchor activity. This unit provides teachers multiple resources for anchor activities. For example, to further differentiate for students who finish their work early or for those who need additional challenge, Super Challenges are included in some lessons. These challenges are located in the SMN. To illustrate, in Lesson 15, students can choose from four Super Challenge word problems related to the objectives of the lesson. The Super Challenges in Lesson 15 ask students to identify and solve growing pattern problems. Some lessons have Super Challenges in which students create their own problems for classmates to solve. Super Challenges encourage students to challenge each other. Teachers should encourage students who try the Super Challenges to persevere even if the problems are slightly challenging for them. In addition, students should be provided with opportunities to share their work and receive teacher feedback on the processes they used to solve the problems.

Lessons Tiered by Mathematician Name

Throughout the unit are tiered Student Pages or tiered groups, with different levels of challenge. The tiered groups are named after famous mathematicians who made significant contributions to the field. The selected groups and their descriptions are detailed below, followed by brief mathematician biographies. These biographies should be shared with students (see Flash Drive for copy of the Famous Mathematicians page). Alternatively, students could research the mathematicians as an interdisciplinary connection.

Fibonacci—Least challenging of the 3 levels. Designed for students with little to no prior knowledge of a topic.
Diophantus—Medium level of challenge. Designed for students with some understanding of a topic.
Kovalevsky—Most challenging. Designed for students who already grasp the topic and are ready for more challenging applications with less scaffolding.

Famous Mathematicians

Leonardo Fibonacci (pronounced fee-bow-NAWH-chee) c. 1170 - c. 1250
Leonardo Fibonacci was born in Pisa, Italy, the same place as Galileo Galilei. His father was a merchant who sold goods to North Africa. Leonardo watched the merchants carefully and learned much about money and numbers from them. Leonardo traveled with his father to North Africa. This is where Leonardo learned a new system of numbers that was much easier than the system of numbers he had learned in Europe. He wrote a book about these numbers and introduced them to Europe. Leonardo loved animals. He also studied the patterns on the outside of pineapples. Because of his love of studying patterns, Leonardo discovered a series of numbers that is now called the Fibonacci Sequence. He also studied spiders and dogs and wrote mathematics problems about them. Leonardo wrote 6 books in all and came up with many theories, or ideas, about numbers. Leonardo became so famous that he became friends with the Emperor, and a statue was built of him that still exists to this day. Today there is a magazine about mathematics named after Fibonacci.

Diophantus of Alexandria (pronounced dy-oh-FAN-tuhs)
Diophantus of Alexandria was born in Greece over 1,800 years ago. He lived in Alexandria, Egypt, one of the mathematics centers of the ancient world. Here he studied numbers and came up with many theories about equations. He even invented symbols to represent numbers. He is known as the Father of Algebra. Diophantus wrote 13 books about these theories and inspired many great future mathematicians. One of his books survived over 1,500 years and taught many Europeans about algebra. It is considered the greatest Greek mathematics book in history. A Greek mathematician wrote a book about Diophantus that contained number games and strategy puzzles. One of these riddles is famous and is still difficult to solve, even to this day. Diophantus was married and had a son. Diophantus lived to be 84 years old when most men in his day only lived to be 40 years old.

Sonya Kovalevsky (pronounced koh-vuh-LEHV-skee) 1850 - 1891
Sonya Kovalevsky was born in Moscow, Russia. She is considered one of the brightest female mathematicians since Hypatia. She always loved math. Her father did not believe girls should study math, and he made her leave school at 13 years old. Sonya studied secretly. She borrowed a math book from a neighbor. Sonya showed her neighbor what she learned easily. The neighbor convinced Sonya’s father to let her return to school. Sonya learned geometry and calculus in a few months. She was so good at math that she was one of the first women to earn the highest degree from the local university. Sonya is a true example of an expert mathematician.

Physical and Affective Learning Environment

The **physical learning environment** of the classroom is a key component to consider when preparing for the differentiated mathematics classroom and to develop a community of mathematical thinkers and practicing junior professional mathematicians.

The physical learning environment is defined as ***the concrete structure and layout of the learning space, as well as the placement of classroom furniture, tools, manipulatives, books, posters, and any other materials to support student learning***. The physical learning environment is not restricted to the classroom, as it could include the school library, science lab, computer room, resource room, or other spaces within the school that would further promote student learning of mathematics.

In addition, the goal of this unit is to promote talent development among all students; therefore, modifications to the learning environment should mirror the diversity of the classroom. The physical learning environment can also contribute to the **affective environment**, **or the social and emotional climate or culture of the classroom**, in that when students see concrete evidence that they are represented and connected to the class, in turn, they will feel emotionally safe to take risks, explore new ideas, think creatively, and communicate their ideas. The affective learning environment includes the positive learning space in which students' beliefs, values, ideas, feelings, mindsets, and personalities are experienced and shared. It includes the respectful relationships between and among the students, teacher, and support staff. A positive and healthy affective learning environment should be established to promote a successfully differentiated classroom.

Essential guiding questions teachers should ask themselves before a new group of students arrive for the school year include the following:

- How can I create a physical and affective environment that supports students' ability to think, feel, and act like young practicing mathematicians?
- How can I design the physical and affective environment that will help create a safe classroom culture that is open to student discovery, collaboration, and ownership of their learning?

Here are some practical suggestions:

Tip #1: Promoting the Affective Learning Environment

As mentioned previously, the physical learning environment can support the development of the affective environment, when students see and feel that they are an important member of the classroom. What happens in the first few weeks of school can be crucial in setting the stage for a safe and positive emotional climate and successful school year for both the teacher and the students in the differentiated classroom. Before expecting students to work in groups, take on challenges, or complete tiered tasks, start by getting to know students and planning activities for students to know each other. Consider conducting activities that communicate, “This classroom is safe for students to have different interests, strengths, and weaknesses.” Celebrate when students learn from mistakes by creating an interactive bulletin board that displays new learning as a result of unanticipated mistakes.

Tip #2: Flexible Grouping of Desks

This curriculum unit includes tiered lessons in which students are grouped by prior knowledge or learner readiness. Students may work on the tiered lessons independently and/or gather with a small group to discuss what they learned. In many lessons in this unit, students from all groups come together to share their learning as a whole class. Therefore, the classroom should include options for flexibly grouping students. For example, rather than placing desks into straight rows, consider grouping them to facilitate small group discussions. Small u-shaped arrangements of desks allow a teacher to place a chair in the open space, thus communicating to students that the teacher is a part of the discussion (see Figure 1). This also allows easy access to students to monitor, support, and extend student learning. A common area in the classroom for all students to gather, such as a rug for meeting spaces, would support a larger group discussion. Another important consideration is safety. The placement of desks and tables in the room should allow students to move safely and freely from one space to another.

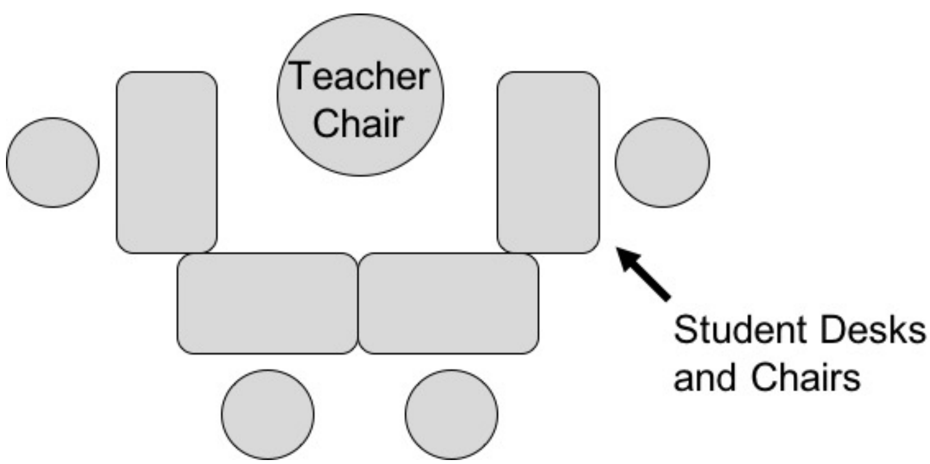

Figure 1. Example desk arrangement to facilitate small group discussions.

Tip #3: Accessibility of Materials

In the differentiated classroom, the materials, or tools, to support student learning, communication, thinking, processing, and discovery of mathematics should be accessible and varied in complexity.

The placement of the materials should be intentional. Consider the height of your 3rd graders. Where can the materials be stored so that they can easily and safely be reached and returned by students? Materials should also be visible, if space permits, and clearly labeled, so the students will be more likely to use them when needed. For example, plastic, gallon-sized storage bags are a convenient option to store a variety of materials, needed for a single lesson or unit of study, for each student or small group. These can be placed in small storage bins on the lower shelves of a bookcase in plain sight for the students. Alternatively, materials might be placed on a countertop, at a mathematics station, or on a movable cart. Materials may include, but are not limited to the following: highlighters, sticky notes, manipulatives, computers, laptops, or other mobile devices.

Tip #4: Introducing Students to the Physical Learning Environment
To support classroom management, students should know the protocols and procedures for moving about the classroom, as well as accessing, using, and returning the materials. These routines should be explicitly communicated and practiced early in the school year and reviewed periodically as needed. A fun way to introduce students to the materials is to allow them to informally explore each learning tool, and then share with each other what they learned and experienced. In addition, have students practice moving about the classroom from one learning space to another. Doing so will allow you to communicate the classroom protocols for moving safely and when they are allowed to move about the classroom freely.

Tip #5: Making Mathematics Visible
Be intentional about the visuals on walls or what is displayed in different areas throughout the room. These visuals can communicate to students the importance of mathematics in your classroom. For example, to build a language rich classroom, display the authentic language of mathematicians that you want students to use. For classrooms with English learners, include mathematical language both in English and their home language, along with visual cues. Google Translate (https://translate.google.com) or a similar online translation tool can help you prepare visuals such as word walls or posters to display mathematical terms in different languages (see Figure 2). Additional resources are available in many different places. There is a glossary in the Teacher Manual and the Student Mathematician Notebook. Vocabulary words, definitions, examples, and visual representations are also available on the Flash Drive to support students who are continuing to develop their mathematical language skills.

Another way to be responsive to the diversity of your students is to include images, books, or other reading materials of mathematicians from different cultural or ethnic backgrounds. In this way, you will help to develop students' self-efficacy, or belief system that they, too, have the potential to become creative mathematical thinkers. Further, to promote an inclusive physical learning environment, display original student work or student-made learning posters.

Word Bank – Lesson 1

English	Spanish	Symbol/Símbolo
Addend	Sumand	4 + 5 = 9
Benchmark	Punto de Referencia	...19 20 21 22....
Decomposition	Descomposición	27 = 25 + 2 27 = 30 - 3
Equation	Ecuación	44 - 6 = 38 59 = 50 + 9 20 + 2 = 4 + 18
Inequality Sign	Signo de Desigualdad	≠
Number Sentence	Frase Numerica	42 ≠ 45 - 7 40 + 2 = 42

Figure 2. Example word poster for Lesson 1.

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Human ______________________________ Date on Earth _____________

If Aliens Taught Algebra—A Test for all Humans

An alien species has arrived on Earth from the Planet Nine. They would like to find out how much math you know! They want you to show them how you solve problems on your planet. ***Please show your work using words, numbers, and/or pictures*** to help the Planet Nine aliens understand our math.

1. Two spaceships have arrived with 10 Planet Nine aliens altogether. There are different numbers of Planet Nine aliens in each ship. How many Planet Nine aliens could be in each ship? Find two different combinations. (Lesson 1)

Ship 1: _______________ + ________________ = 10 Total Planet Nine Aliens

Ship 2: _______________ + ________________ = 10 Total Planet Nine Aliens

2. What pattern do you notice in the following Planet Nine Alien Input—Output Machine? ***Hint: Read the table from left to right. The number 53 goes into the Machine and then out comes the number 50! Can you figure out what happened?*** (Lesson 2)

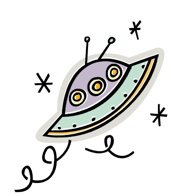
Input	Output
53	50
25	30
67	70

3. The Planet Nine aliens decided to go on a picnic but needed groceries at the store. Bippie spent $26 and Vi-vi spent $13. Use estimation to find about how much money they spent in all. Describe how you solved the problem. (Lesson 3)

4. To lock the spaceship, the Planet Nine aliens typed in a secret code. Fill in the missing secret numbers in the equations below. (Lesson 4)

9 + 5 = ___ + 9

3 + 6 + 2 = (___ + 6) + 2



5. We shaded in multiples of 2 on the chart below.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Now, use the chart below and shade in all the multiples of three.
(Lesson 6)

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

6. In the last problem, you shaded in multiples of three. Describe two different patterns that you see in this chart. Use words such as vertical, horizontal, and diagonal to help you describe the patterns.
(Lessons 5 and 6)

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

____

____

____

____

____

Another spaceship has landed on Earth! As the Planet Nine aliens step out, you see that they only have 4 fingers on each hand.

7. If each Planet Nine alien has 3 hands, how many fingers does each have? Show your work. (Lesson 7)

____

____

____

____

____

____

____

Today is Adorna's birthday. There are 4 humans from Earth and 5 Planet Nine aliens at the party.

8. How could you show the number of eyes on the Planet Nine aliens below using a multiplication number sentence? (Lesson 8)

____

____

____

9. Soland baked 19 large cookies for the birthday party. There were 5 Planet Nine aliens and 4 humans at the party. If he places 2 cookies in each bag for his guests, how many cookies will not be in a bag? Describe how you solved the problem. (Lesson 11)

____

____

____

____

____

10. Solve the following if x = 5. (Lessons 14, 15, and 16)

2x + 6 = ______

Pies for the Planet Nine Alien Party!

Use the following table to answer the questions below.
(Lessons 14, 15, and 16)

Number of Planet Nine Aliens	1	2	3	4	5	6
Number of Pies	4	8		16		

11. If the pattern in the table continues, how many pies will there be if 5 Planet Nine aliens come to the party? (Lessons 14, 15, and 16)

____

____

12. If 20 Planet Nine aliens come, how many pies will there be altogether? (Lessons 14, 15, and 16)

____

____

If Aliens Taught Algebra—A Test for all Humans

Note: Any question that the student skips is worth 0 points.

Q 1: Decomposing Numbers

Points	Answer
2	Provides two combinations that total to 10.
1	Provides one correct combination that totals to 10.
0	Answers are both incorrect.

Q 2: Rounding

Points	Answer
2	States that the numbers from the input column are rounded to the nearest ten in the output column.
1	States that the numbers from the input column are rounded up in the output column.
0	Explanation is mathematically inaccurate. OR States the numbers in both the input and output columns increase in numerical order.

Q 3: Mental Math

Points	Answer
2	Estimates the sum between \$35 and \$45 and shows work.
1	Rounds numbers but sum is incorrect OR sum is correct but does not round the numbers.
0	Sum is incorrect and explanation is mathematically inaccurate.

Q 4: Balancing Equations

Points	Answer
2	The missing number is 5 for the first equation and 3 for the second equation.
1	Solves one equation correctly.
0	Both equations are incorrect.

Q 5: 100s Chart

Points	Answer
2	Shaded in all 8 multiples of 3 (i.e., 3, 6, 9, 12, 15, 18, 21, 24)
1	Shaded in 1 or 2 numbers that are not multiples of 3.
0	Shaded in 3 or more numbers that are not multiples of 3.

Q 6: 100s Chart

Points	Answer
2	Provides two accurate descriptions of patterns in the chart. Descriptions should all be *unique* (i.e., a vertical, horizontal, and diagonal pattern).
1	Provides two descriptions of patterns but one is inaccurate OR Provides only one accurate description of a unique pattern in the chart.
0	Provides two inaccurate descriptions of patterns.

Q 7: Repeated Addition

Points	Answer
2	Provides correct answer of 12 fingers all together and accurately shows the work.
1	Provides a correct answer but does not show the work. OR Provides an incorrect answer but shows work accurately. OR Provides a correct answer and does not show work accurately.
0	Provides incorrect answer and does not show work. OR Provides incorrect answer and an inaccurate explanation.

Q 8: Multiplication

Points	Answer
2	States 5 x 3 = 15 eyes OR 3 x 5 = 15 eyes
1	Provides a number sentence with the correct factors but an incorrect product OR provides a number sentence with one incorrect factor and a correct product.
0	Student provides an incorrect number sentence.

Q 9: Division

Points	Answer
2	States that 1 cookie will remain and provides an accurate explanation.
1	States that 1 cookie will remain but does not show the work. OR Provides a correct answer with an inaccurate explanation. OR Provides an incorrect answer with an accurate explanation.
0	Provides an incorrect answer and inaccurate explanation.

Q 10: Growing Patterns

Points	Answer
1	Answer is 16.
0	Provides an incorrect answer.

Q 11: Growing Patterns

Points	Answer
1	20 pies
0	Provides an incorrect answer.

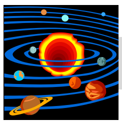
Q 12: Growing Patterns

Points	Answer
1	80 pies
0	Provides an incorrect answer.

Introducing the Unit—

If Aliens Taught Algebra: Multiplication and Division Would Be out of This World!

LESSON 0



Big Ideas

To prepare students for active engagement throughout the unit, this lesson explores the concept of "thinking like mathematicians," the meaning of mathematics and algebraic thinking, and real-world applications of mathematics. Lesson 0 also introduces the unit theme, structure, and time capsule activity while establishing important classroom norms.

Learning mathematical content and skills is most successful in intentionally designed learning environments. Establishing classroom norms, or the routines, activities, rules, and expectations, at the start of the unit will promote a positive physical and affective learning environment. Recall from the introduction, that the physical learning environment is the concrete structure and layout of the learning space, as well as the placement of classroom furniture, tools, manipulatives, books, posters, and other materials supporting student learning. The affective environment refers to the social and emotional climate of the classroom.

Depending on students' needs and prior experiences, components in this lesson can be implemented with flexibility.

Lesson Objectives 	• Students will be able to explain that mathematics can be used in different careers. • Students will be able to explain the theme, "thinking like mathematicians." • Students will be able to describe and apply the routines, rules, and expectations for the unit.
Materials 	• Time capsule container (e.g., plastic liter bottle, large water bottle, metal box, glass jar) • Chart paper • Crayons

Mathematical Terms	• **Algebra:** A topic studied as part of mathematics; the study of mathematical symbols (e.g., numbers; equal, addition, or multiplication signs) and the rules for working with these symbols. • **Astronomy:** The scientific study of space, stars, planets, and other celestial bodies. • **Astronomers:** People who study astronomy. • **Mathematics:** The study and use of numbers, patterns, and shapes. • **Mathematicians:** People who study or use mathematics in their work.
Selected Mathematical Practices	• MP5: Use appropriate tools strategically • MP6: Attend to precision

Lesson Preview

In this lesson, students will explore how mathematics fit in the real world, key mathematical terms, and the unit structure and theme. Students will also be introduced to the routines, activities, and norms they will experience in this unit.

Launch

1. **Thinking Like Mathematicians**

As a whole group or in small groups, briefly discuss what students know about mathematics and mathematicians. Ask the students:

- What do we know about the meaning of the word “mathematics”?
- Why is it important for us to study mathematics?
- What do we call people who study mathematics?
- What does it mean to think and act like mathematicians in this unit?

If students struggle with these questions, introduce the definitions of **mathematics** and **mathematicians** listed above. Alternatively, students may explore these questions through the optional activities.

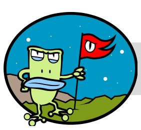
[OPTIONAL]: Have students draw pictures of what mathematicians might look like or how they might look as mathematicians. Have students share their work with the class. To extend their knowledge, share more information about mathematicians through biographies, websites, videos, etc. (See Flash Drive for suggested resources.)

[OPTIONAL]: Have students read biographies of well-known mathematicians from diverse backgrounds, including ones that are similar and different from students in the class. Students might also enjoy learning about the 3 mathematicians, Fibonacci, Diophantus, and Kovalevsky, featured in the unit. (See Flash Drive for suggested resources.)

After students discuss the questions or complete an activity, have them share what they learned with the whole group. Consider recording students' ideas on a whiteboard or chart paper. If necessary, establish routines and rules for sharing ideas with the whole class (e.g., giving different answers is ok; actively listen to others; think about and learn from others). Consider adding the following thoughts about mathematics and mathematicians:

- Mathematics is important because it helps us solve problems and better understand the world around us.
- Mathematicians can be any gender and come from any background.
- Thinking like mathematicians means that you solve problems, ask questions, think about how numbers work, or use different tools and strategies.

Content Note: *It is important to note that although the lessons provide guidelines about how to help students understand what it means to think mathematically, each class and group of students will inherently develop this meaning according to their current understanding, contributions, and interactions during each lesson. Therefore, teachers should build off their students' ideas and use suggestions in the unit to augment them over time.*



Explore

2. Careers in Mathematics

To build off of Part 1, explain to the whole class that mathematics and developing mathematical thinking skills are important in many careers. Ask the students:

- What types of jobs do you think mathematicians have?

Consider writing students' ideas on a whiteboard or chart paper. Possible careers that apply mathematics include: animated film-making, business, cartography (map making), climate study, computer programming, engineering, quantitative research (research using primarily numbers), robotics, and space exploration and astronomy. Alternatively, have students complete an optional activity to explore these careers.

[OPTIONAL]: Have students read about different careers in mathematics to generate a list similar to the one above. Provide them with reading materials (see Flash Drive for suggestions) or have them locate materials online and/or at the library. After students learn about mathematics careers, have the students discuss their findings with the class.

Next, explain to students that this mathematics unit includes an outer space theme. To investigate how this theme connects to mathematics and “thinking like mathematicians,” they will learn about how astronomers used mathematics to discover Planet Nine.

Linking Mathematics to Astronomy

Introduce students to astronomers’ groundbreaking announcement about a hypothetical planet, called “*Planet Nine*,” which may be located in the outskirts of our solar system (see Flash Drive for factual news articles and images). Tell students that **astronomy** is the study of stars, planets, and space; therefore, **astronomers** are special scientists who observe stars, planets, and space. Explain that astronomers often record what they learn with numbers and mathematics. For example, astronomers “found mathematical evidence suggesting there may be a ‘Planet X’ deep in the solar system” (NASA, 2018[*]). This means they used advanced mathematics and computers to potentially discover another planet in our solar system. Because astronomers have not seen Planet Nine, they are now using powerful telescopes to look for the planet and confirm it exists.

[OPTIONAL]: Share a video about Planet Nine with students (see Flash Drive of resources for two examples appropriate for students).

Ask the students:

- How do you think astronomers use mathematics or think like mathematicians in their work?

Build students’ background knowledge as necessary while discussing their ideas with the whole group. Tell students that astronomers work together to use new discoveries and background knowledge to better understand our solar system, galaxy, and beyond! For example, the astronomers learning about Planet Nine collaborated with others and persevered to solve problems. They also used geometry, physics, simulations, and advanced mathematics to study unusual orbits of objects (dwarf planets and icy objects) in the Kuiper Belt. Based on their findings, they made hypotheses about Planet Nine:

- Planet Nine may be 4.5 billion years old.
- Planet Nine’s mass may be 10 times that of Earth and 5,000 times that of Pluto.
- Planet Nine’s orbit around the sun may take 10,000-20,000 Earth years.

[*] Erickson, K. (2018). *NASA science solar system exploration: Hypothetical Planet X.* Retrieved from https://solarsystem.nasa.gov/planets/hypothetical-planet-x/in-depth/

Explain that astronomers shared this information about Planet Nine because communication is important in mathematics. By communicating their Planet Nine discoveries, astronomers inspire others to learn more about this planet and the information they collected. During this unit, students will learn to communicate their ideas like mathematicians to express what they know and to learn from each other. Thinking and acting like mathematicians could also mean that students work together, persevere through challenging problems, answer real-world problems in different ways, and understand that some problems have multiple solutions.

Introducing the Alien Theme

Share the unit title, *If Aliens Taught Algebra: Multiplication and Division Would be out of This World!*, with the students. Explain that **algebra**, an important topic in mathematics, is the study of mathematical symbols (e.g., numbers; equal, addition, or multiplication signs) and the rules for working with these symbols. Algebra helps mathematicians solve real-world problems because the symbols can represent real-world things, like the number of Planet Nine alien friends that fit on a rollercoaster car!

Tell students that they will be thinking like mathematicians to solve real-world and out-of-this world problems that will introduce them to algebra! They will have a bit of help along the way from some friendly (fictional) Planet Nine aliens that they will meet in the word problems.

Unit Structure and Classroom Norms

Share the unit structure with students and discuss classroom norms. Below are some major components of the unit along with suggestions for establishing classroom norms that promote a safe physical learning environment and positive affective culture. Tailor your discussion to the needs of your class. Consider writing students' ideas on a whiteboard or chart paper.

- There are 16 lessons in the unit.
- Students will have their own Student Mathematician Notebooks (SMN). Have students flip through their SMN and discuss routines for accessing/using their SMN.
- Students will learn about rounding, estimation, patterns, multiplication, division, and algebraic thinking. Discuss the importance of (a) perseverance when learning new content, (b) learning from and celebrating mistakes, and (c) taking risks.
- Some lessons have manipulatives and other resources to support their learning. Discuss the rules, routines, and expectations for safely accessing, using, and returning the materials.
- Students will sometimes work on different assignments, but they will learn the same lesson objectives. The differentiated student

pages in the SMN are named after famous mathematicians: Fibonacci, Diophantus, and Kovalevsky. Explain that (a) mathematicians have strengths and weaknesses, (b) we can all learn something new, (c) it is ok to work on different tasks because we are all learning, and (d) learning is a process.
- Sometimes students may work in groups, with partners, or independently. Discuss (a) rules, routines, and expectations for working collaboratively as a group; (b) rules, routines, and expectations for moving safely from one working area in the classroom to another; and (c) how the desks might be arranged for facilitate group work (students could also contribute to designing the classroom set up).

Debrief and Look Ahead

3. Time Capsule Culminating Project

Explain to students that, at the end of every lesson, they will create artifacts to exemplify how they were like mathematicians during the lesson. Artifacts might include items like stories, drawings, work samples, audio recordings, videos, or other ideas that students generate. Depending on the lesson, artifacts could be created by individuals, pairs, small groups, or the whole class. Students will then place the artifacts in a time capsule or designated container (e.g., plastic liter bottle, large water bottle, metal box, glass jar) until the end of the unit.

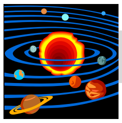
At the end of the unit, the class will invite a guest (e.g., principal, math specialist, other school staff member, community member with a math-related career, etc.). The class will open the time capsule and students will use the artifacts to share the importance of thinking and acting like mathematicians with their guest.

[OPTIONAL]: Have students get into small groups, and work on one component of creating and decorating the time capsule. For example, one group can work on space themed decorations, and another group could create mathematical themed decorations.

Decomposition—

Preparing for Blast off

LESSON 1



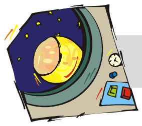
Big Ideas

Decomposition is the process of breaking numbers down into usable pieces. It is a strategy that can be used to help students improve their mental computation, speed, and accuracy. It is also the first step in understanding equality within individual numbers. For example, 45 = 40 + 5. Ten is an especially valuable number to know how to decompose. When students understand how to put different numbers together to equal 10, they will be able to add and subtract faster.

Lesson Objectives 	• Students will be able to list all the possible ways to decompose 10. • Students will be able to make 10 from other numbers. • Students will be able to use the decomposition strategy to perform addition. • Students will be able to explain their mathematical thinking.
Materials 	• Student Page—*Spaceship Seats* [SMN pages 1-3] • Student Page—*Spaceship Seats Challenge* [SMN pages 5-7] • Student Page—*Packing Party for Earth* [SMN pages 9-13] • Student Page—*Super Challenges* [SMN page 15] • Student Page—*Triangles for Super Challenge 3* [SMN page 17] • Student Page—*Breaking it Down Practice* [SMN page 19] • Student Page—*Breaking it Down Practice 2* [SMN page 21] • Student Page—*The Magical Number 10* [SMN page 23] Remind students that manipulatives are available to use strategically and appropriately.

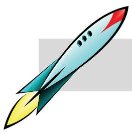
Mathematical Terms	• **Addend:** A number that is being added to another number. (e.g., 4 + 5 = 9; 4 and 5 are addends). • **Benchmark:** A point of reference; in this lesson we are using numbers ending in 0 as benchmarks. • **Decomposition:** Breaking down a number to make a problem mentally easier to calculate. • **Equation:** A mathematical sentence that contains an equal sign. • **Inequality Sign:** A sign used to represent a number sentence that is not equal (≠). • **Number Sentence:** A mathematical sentence that contains any sign (equality or inequality); an open number sentence that contains a variable or missing number.
Selected Mathematical Practices	• MP1: Make sense of problems and persevere in solving them • MP2: Reason abstractly and quantitatively

Differentiation	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• know (information, facts, vocabulary), understand (concepts, big ideas, connections), apply (skills, processes)
	Process
	• questioning strategies
	• 4Cs (21st Century Skills)
	  ○ creativity
	  ○ critical thinking
	  ○ collaboration
	  ○ communication
	Product
	• oral, visual, and written opportunities
	Learning Environment
	• whole group/small group/individual instruction
	• learning community



Lesson Preview

In this lesson, the students will group Planet Nine aliens to make 10, discuss how many different ways 10 can be broken down into two numbers, and apply this knowledge to addition problems. The inequality sign will also be introduced during the discussion of the investigation in the Explore section of this lesson (part 3).



Launch

1. **Thinking Like Mathematicians**

As a whole group, discuss how mathematicians solve problems. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask students:

- How do mathematicians solve problems?
- Let's pretend that the Planet Nine aliens are coming to visit us. When they arrive, how might they know we are mathematicians by the way we solve problems?

If students struggle to generate ideas, share the following examples. When mathematicians solve problems, you might notice that they use many different tools and strategies, they work together, they ask questions, they take their time, they use different approaches to check their answers, and they persevere even when the problems are difficult.

Spaceship Seats

Tell the students their first task is to help the Planet Nine aliens prepare for a trip to Earth. Some Planet Nine aliens have already begun to board several space crafts. Instruct students to turn to *Spaceship Seats* Student Page **[SMN pages 1-3]**. Read the poem out loud.

Spaceship Seats

The Planet Nine aliens are getting ready for their long trip.
They are boarding and packing their ship.
They need your help to know . . .
How many Planet Nine aliens are ready to go?

Ask the students:

- How many Planet Nine aliens are on the S.S. (Spaceship) Hailey Bop 907?
- What are different ways to figure out how many Planet Nine aliens are on the S.S. Hailey Bop 907?

Talk Move: *Use Wait Time. Wait Time encourages teachers to give students a longer period of time to think about the question posed. Wait Time was first explored by an educational researcher, Mary Budd Rowe (1972), and she found that typical classroom teachers only give students 1.5 seconds of Wait Time before asking another question or calling on a student. When that time was increased to at least 3 seconds, students' answers were longer and more likely to be correct.*

Most students will count each individual Planet Nine alien. That is a good place to start, but then challenge students to figure out the number of Planet Nine aliens on board using different methods. If the students need a hint, ask the students:

- Could you think about how many windows are in each row?

You should discuss with students how the rows can be visually combined. If students do not share this strategy, present it to the class as another student's idea. For example, you can tell the students that another third-grade student found that the first 2 rows could be visually combined to fill in the whole row making 10 Planet Nine aliens in all. The student used the same strategy for the third and fourth rows. Finally, the student added 10 + 10 = 20 Planet Nine aliens.

***Talk Move**: Use Reasoning. Teachers ask students if they agree or disagree with another student's strategy and why. This talk move requires students to make their mathematical reasoning explicit. Additionally, the Reasoning talk move encourages students to listen to others' ideas and engage in respectful discussion.*

Next, ask the students:

- How many more Planet Nine aliens could get on board for question # 3 on *Spaceship Seats* Student Page **[SMN page 3]**?

If they realize there are 10 windows per row and 4 rows in all, they can figure out that there are 40 possible seats in all. So, they could subtract the 20 Planet Nine aliens from 40 to figure out how many more Planet Nine aliens could get on board. They could also count the empty windows individually or visualize the empty Planet Nine alien seats in one row joining the empty seats in the next row to make a group of 10.

[OPTIONAL]: There is a second Spaceship to try their new strategies on *Spaceship Seats Challenge* Student Page **[SMN pages 5-7]**.

As a whole class, have students share their thinking and answers. Then discuss the importance of 10.

To make a real-world connection, tell the students to think about how long it takes to count to 100. Ask students:

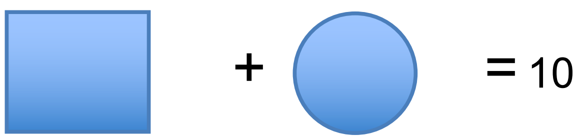
- What is one way to count to 100 really fast without having to say every number between 1 and 100?

They may answer that you can count by 5s or 10s. The big idea is that using **benchmark** numbers can make counting numbers and adding easier and quicker. Tell students that in this lesson, 10 is an example of a benchmark number.

Explore

2. Packing Party for Earth

For this first investigation, all students will complete the same investigation *Packing Party for Earth* Student Page **[SMN pages 9-13]**. Place them into small groups of 2 or 3 students. This first part of the investigation asks students to think of all the possible combinations of two numbers that will equal 10. Explain to students that **decomposition** means breaking down a number like 10 to make a problem mentally easier to calculate. The goal of this investigation is for students to decompose 10 to become familiar with numbers that can be paired to total 10. This will help students recall a number's fact family when adding or subtracting, which will build fluency (e.g., 4 + 6 = 10, 6 + 4 = 10, 10 – 4 = 6, 10 – 6 = 4). This investigation also implicitly introduces the idea of an **equation**. To highlight this idea, you may want to record on the board a square added to a circle equals 10. This is the equation the students will work with in the first part of the investigation.



Here is an example of a conversation with students about this equation.

Teacher: Sometimes when we are not sure what number belongs in an equation, we put a shape or a letter in its place. Today, we will examine numbers that fit in this equation to make it true. What number could the square be? Remember it needs to be a number that can be added to another number to equal 10.

Marcus: I think the square could be 3.

Teacher: Let's record the 3 under the square and copy the rest of the equation below. Now what should the circle be to make the **number sentence** true?

Avis: The circle should be 7.

Teacher: [Teacher records answer.] Does anyone want to agree or disagree? (*Reasoning talk move*)

Belinda: I want to agree.

Teacher: Why do you agree?

Belinda: Because when you add 3 to 7 you get 10.

Teacher: Good explanation and thinking. Is there any other number that the circle could stand for that would equal 10?

Joe: I don't think so.

Teacher: Could someone add on to Joe's thought? (*Adding On talk move*)

Sara: Well, if you try 6, 3 + 6 is not 10. If you try 8, 3 + 8 is also not 10. So, I don't see another way to make the statement true.

Teacher: Sara gave us numbers that would make the statement false. We could represent that using this symbol **≠**. This is called an **inequality** sign. Does anyone have another way we could make this number statement false?

The conversation could continue to explore ways to make the equation true or false. The second part of the investigation increases the challenge level by asking students to complete given equations. The students are given two pieces of the equation, the sum and one **addend**, and they need to calculate the last addend. Teach students that an addend is a number that is added in an addition **number sentence**. Explain what a number sentence is to students. If students are struggling, you may encourage them to look back at their first list for guidance.

Collaborate and Communicate

Have students record their ideas for *Packing Party for Earth* **[SMN pages 9-13]** on their individual worksheet or one for the small group. They then should focus on the question, “How do you know if you have all the possible combinations?” and record their explanation on a whiteboard or large chart paper. Help them clarify their ideas by asking questions like, “What do you mean here?” and “How might you share that idea with the rest of the class?” Point out that mathematicians use various representations to help explain their thoughts and use precise language to do so. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group randomly came up with equations and did not organize them.*	**B.** [Possible response] *This group systematically organized their equations just using numbers and did not use the commutative property.*	**C.** [Possible response] *This group systematically organized their equations, used sticky notes to denote the square Wombles and circle Snufplets, and utilized the commutative property.*

Examine and Elaborate

3. Highlight Students' Mathematical Thinking

Mathematicians think about possible solutions in a variety of ways. Therefore, it is important for students to realize that they, too, can approach problems using different strategies. Ultimately, students need to understand that a possible solution should be judged by the correctness of the mathematics, and there might even be some valid ideas within a solution when a student has an incorrect answer.

Share and Discuss

It is therefore important for students first to clearly share their ideas with others so their validity can be determined by the class. In this sample dialogue, the teacher decides to first call on a small group that did not systematically organize their questions. This helps the rest of the class realize that using an organized list is an effective strategy.

Teacher: We've heard from two groups already. Marcus' group (see response A) thinks there are six different combinations: 3 Wombles and 7 Snufplets, 5 and 5, 8 and 2, 6 and 4, 0 and 10, as well as 7 and 3. Belinda's group (response B) thinks there also are six combinations. Is that what you meant to say? (*Revoicing talk move*)

Marcus: [Students from both groups nod in agreement.] Yeah, but we have different ones.

Teacher: Can someone add onto Marcus' observation? (*Adding On talk move*)

Avis: Me and Marcus and Janell all shared our ideas, but I think we forgot some. But the other group, Melinda's group, forgot some too.

Teacher: Ok, let's compare your equations. Do you think we should start with the list that Marcus' group has or with Belinda's? Why?

Belinda: I think ours. We put it in order.

Teacher: Belinda thinks we should start with her group list. Who agrees or disagrees with this idea, and why? (*Reasoning talk move*) Maddie?

Maddie: I agree. Because it's in order.

Teacher: Belinda, can you repeat what Maddie just said in your own words? (*Repeat/Rephrase talk move*)

Belinda: Yeah, we started with zero, then went to one, then to two, then three. . . .

Teacher: Maddie, is that what you said?

Maddie: Yeah, that's what I meant. I noticed that the first numbers start at zero, then go up one each time.

Teacher: Ok, work with your group to compare both lists. Let's try out Belinda's idea to start with the list her group put together, then check to see if Marcus' group has each equation.

Students soon saw the benefits of using an organized list to make sure they had identified all possible equations. Someone then pointed out that they had found even more equations—11 to be exact—and shared their ideas. While they also used an organized list, they too used "turn around facts," which are properly known as the **commutative property of addition**. In fact, Marcus' group recorded 3 + 7 and 7 + 3, but they did not use this strategy to identify other equations.

Differentiate Further as Needed

In the previous section, students were asked to be sure they found all of the combinations to make 10. Some students might need additional support to understand the lesson concepts. For students who randomly wrote number sentences on their pages, you can have them re-write their number sentences on sticky notes and ask them if they have any ideas on how they might check to see if they have all the possible combinations. Then, introduce any ideas appropriate for those who need extra challenge with this sentence. You can also extend the challenge to find all the combinations of numbers that make 12, 15, or 20.

Some students may benefit from further challenge of the lesson concepts. As students work on the Lab, you can ask students who have previously demonstrated deep understanding of the commutative property to find all the combinations if a third group of Planet Nine aliens was also going to board the Spaceship. Super Challenges are also available for this lesson (see the next section).

The use of the number 10 was purposeful. Using a simpler number will help to teach the complex concept of decomposition and help students to engage in mathematical communication. If you believe additional challenge is needed, you can have each alien symbol on the student pages represent 10 aliens. Then, you can have students find the all ways to make 100 using multiples of 10 (e.g., 10 + 90, 20 + 80).

I'm All Done, Now What Do I Do?

If students finish any investigation or assignment in this unit early, they may work on a Super Challenge. In this lesson, the challenges increase in difficulty, so give the students Super Challenge 1 first on *Super Challenges* Student Page **[SMN pages 15-17]**. Since this is possibly the first opportunity for students to work with the Super Challenge cards, take a moment to explain the purpose of the cards. The Super Challenges are not designed to be graded; rather, they are for any students to challenge themselves if they finish their work early.

Additional Breaking it Down Practice

This lesson has an additional practice for students which will prepare students for the assessment. Complete the first few problems of the *Breaking it Down Practice* Student Page **[SMN page 19]** together as a class. This activity applies the big ideas of this lesson into addition equations. Start by talking through the strategy:

> If I am adding 8 and 6 and I forget what the answer is, and I really don't want to use my fingers, I could use what we learned today about 10s. I might start with the 8 and try to make it 10 because I know how to add numbers to make 10. If I start with the 8, then I need a 2 from the 6, which will leave 4. The problem becomes 10 + 4, which equals 14.

Have students complete the rest of the page independently. Next, discuss with students how creating 10s makes it easier to add.

Debrief and Look Ahead

4.

Debrief Content and Skills

Ensure that students have a solid understanding of decomposition. Ask students:

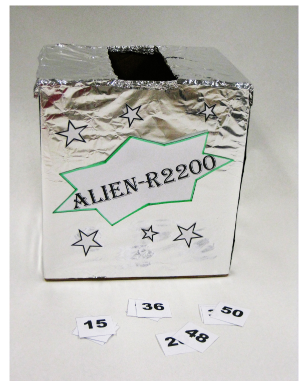
- How might a mathematician explain decomposition?

- What strategies did you use to find different numbers to equal 10?
- How does decomposing the number 10 help us solve problems?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on how mathematicians solve problems. Review some of the ideas students brainstormed at the beginning of class and offer examples of how students solved problems like mathematicians during class. Give students time to create artifacts for the time capsule that demonstrate how they solved problems like mathematicians.



Look Ahead

In Lesson 2, the students will learn another strategy, *rounding*, to make adding easier. As an optional activity, you will want to prepare for this lesson by collecting 2 boxes and decorating 1 of them with foil or any other way you see fit. Then, in Lesson 3, the students will put the strategies together and practice mental addition.

Assess

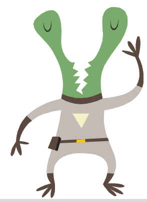
5.

Practice Decomposing Numbers

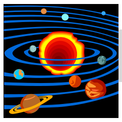
Breaking it Down Practice 2 Student Page **[SMN page 21]** will serve as an individual assessment. There is also an independent practice sheet, *The Magical Number 10* Student Page **[SMN page 23]**, that could be sent home as homework or given to students during independent work time.

Rounding—

The ALIEN-R2200, A Wonderful Invention

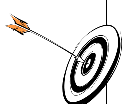


LESSON 2



Big Ideas

A solid foundation in understanding the concept of rounding is an important real-world skill. It will also support students' ability to work with larger numbers in algebra. Rounding can be used to calculate either an exact answer or an estimate. Decomposing numbers and rounding skills will strengthen students' abilities to mentally calculate the estimated and exact costs of items.

Lesson Objectives 	• Students will determine and explain the rules when working with function machines. • Students will describe how to round a number to the nearest 10. • Students will understand visually the concept of rounding.
Materials 	• Teacher Materials: Number Cards • Function Box (**OPTIONAL**) • Planet Nine Alien Invention Box: This is a simple cardboard box, decorated however you see fit. An easy way would be to simply wrap it in foil. If you have any type of electronic white board, you could also create an electronic version. • Student Page—*Understanding the ALIEN-R2200—Fibonacci* [SMN pages 25-27] • Student Page—*Understanding the ALIEN-R2200—Diophantus* [SMN page 29] • Student Page—*Understanding the ALIEN-R2200—Kovalevsky* [SMN page 31] • Student Page—*Super Challenges* [SMN page 33] • Student Page—*Understanding Rounding (Extra Practice)* [SMN pages 35-37] Remind students that manipulatives are available to use strategically and appropriately.

Mathematical Terms	• **Function:** A rule for calculating sums or differences when using input and output boxes. • **Rounding:** Altering a number so that it is easier to use in calculations.
Selected Mathematical Practices	• MP7: Look for and make use of structure • MP8: Look for and express regularity in repeated reasoning

Differentiation	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• tiered activities
	• varied levels of challenge
	• "teaching up" (aim high, provide scaffolding)
	• formative assessment
	Process
	• questioning strategies
	• 4Cs (21st Century Skills)
	  ○ critical thinking
	  ○ creative thinking
	  ○ collaboration
	  ○ communication
	• hands-on activities/manipulatives
	Product
	• oral, visual, and written opportunities
	• multiple ways to demonstrate knowledge, understanding, and skills
	• multiple models and representations
	Learning Environment
	• flexible grouping
	• whole group/small group/individual instruction
	• learning community

Lesson Preview

In this lesson, students discover the rules of rounding numbers to the nearest 10 by using input and output boxes.

Launch

1. **Thinking Like Mathematicians**

As a whole group, discuss how mathematicians work together. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask students:

- How do mathematicians work together?
- When and/or why might mathematicians work together?
- If Planet Nine aliens arrive in the future to visit us, how might they know we are mathematicians by the way we work together during math class?

If students struggle to generate ideas, explain that mathematicians collaborate to share ideas and solve problems. When mathematicians work together, you might notice that they listen to others, value others' ideas, respectfully disagree, offer their own ideas, are flexible and helpful, and share responsibility for solutions.

Pre-initiation (OPTIONAL)

This activity is necessary if students have not been previously introduced to function machines. To prepare for this part of the lesson, obtain a plain box and cut out the numbers on the Teacher Materials: Number Cards at the end of this lesson. Show the students a plain box and explain that this is a function machine. Numbers enter into this device and come out differently. To begin, use a simple rule like +2 (add 2), but don't tell the students that this is the rule. Hold up the number (34) so students can see it, and then pull out the sum of that number plus 2 (36). Ask students:

- What happened in the function machine to turn the number 34 to 36? How do you know?

Tell students that this function machine will compute the same **function**, or rule, each time a number is placed into it until the function is changed. Demonstrate by inputting two other numbers into the box and showing the students the output sums. Explain to students that the numbers that you put into the function box are the **input**, and the numbers that come out of the function box are the **output**. On the board, create a two-column table to record the input and output numbers like the one in step 2 below. Ask students:

- What is happening in this function box? How do you know?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Explain that they have discovered the missing addend for the equations as well as the rule for the function box. If students need additional practice, demonstrate the process again, but this time put the number 20 into the box and take out the number 16. Then put in the number 15, and take out the number 11. Students should discover the rule is –4 (minus 4).

Planet Nine Alien Invention Box

Explain the following to the students:

The Planet Nine aliens heard about these function machines and decided to improve upon them. They created the ALIEN-R2200, which is a very unique machine!

Show the students the fancy box. (Feel free to decorate this box, which you may already have used in the pre-initiation activity, any way that you see fit. Foil would be sufficient.) Using the numbers provided, demonstrate how the Planet Nine alien's invention works. After you put each number into the machine and show the output number, record it in the table on the board. It should look something like this:

IN	OUT
48	50
32	30
49	50

Ask students:

- What do you think is the function or rule of this new fancy function machine? How do you know?

After you pull out three output numbers, give the students a chance to think and talk to a partner about what could be occurring. You may want to remind students to listen actively and respectfully, and to respond to a partner's response. When students are finished discussing, have students give you a silent, secret signal (e.g., thumbs up) to show they are ready to share with the whole group. As a class, have students share their ideas about what the function might be. Encourage students to explain their thinking.

At this point, do not tell students if they are correct. Instead, explain to students that they will be working on an investigation that will continue to explore the function of the ALIEN-R2200 machine.

***Talk move:** Use Revoicing. Revoicing encourages teachers to restate a student's response. By rephrasing what a student shares, teachers can clarify a confusing response for themselves and other students in the class. Revoicing also provides repetition of ideas and additional processing time, which allows students to better understand the mathematical concepts being discussed.*

***Talk Move:** Use Reasoning. Ask students if they agree or disagree with the other student's strategy and more importantly why they disagree or agree.*

Explore

2.

Understanding the ALIEN-R2200

In this investigation, students will be working on one of the *Understanding the ALIEN-R2200* Student Pages based upon their differentiated groups. The groups are based on the results of the unit pretest and other observations of students' understanding.

You may make a list of students for each group, write names on a sticky-note, and place them on the appropriate column on the table below.

Groups Formed by Scores on Question 2 on the Pretest		
Fibonacci Lab Group	Diophantus Lab Group	Kovalevsky Lab Group
Scored 0 points on item, other observations	Scored 1 point on item, other observations	Scored 2 points on item, other observations
Student Names	Student Names	Student Names
Student Pages for Lessons		
Understanding the ALIEN-R2200—Fibonacci Student Page **[SMN pages 25-27]**	*Understanding the ALIEN-R2200—Diophantus* Student Page **[SMN page 29]**	*Understanding the ALIEN-R2200—Kovalevsky* Student Page **[SMN page 31]**

Remind students of the rules or routines for working in math groups as discussed in Lesson 0, if necessary. Also, remind students that they are to work with their group members to figure out the function of the ALIEN-R2200.

If students finish their Explore activity early, they may work on the *Super Challenges* Student Page **[SMN page 33]**.

Collaborate and Communicate

Bring students back together to discuss the results of their investigation. Students will have investigated rounding concepts in each of the *Understanding the ALIEN-R2200* differentiated worksheets. Have the class discuss the following question to solidify their understanding: "What are the mathematical rules for the ALIEN-R2200?” Share some of the in and out numbers from each worksheet for all students to consider during the discussion and have them predict what might happen if other numbers are placed into the ALIEN-R2200.

IN	OUT
19	20
23	20
32	30
35	40
48	50
67	70
64	60

Mathematicians look for patterns on their own, and the quest to uncover patterns is intriguing for them. To help students engage similarly, first ask questions that are rather open, such as, “What patterns do you notice in this in and out table?” If students themselves do not offer more specific patterns, ask them questions based on the ideas they share, “I see that you are talking differently about the ‘in’ and ‘out’ columns. What is different about those numbers?” “You seem to be focusing on patterns across the rows. Why did you group the numbers that way?”

Have small groups share their explanation on a whiteboard or large chart paper. As students share, remind them that mathematicians learn from each other. If they hear other groups sharing interesting or new ideas, they could add them on a whiteboard or large chart paper.

Below are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes in the following table:

A. [Possible response]	**B.** [Possible response]	**C.** [Possible response]
This group counted up from each ALIEN-R2200 “in” number to the “out number and noticed that some were closer than others.	*This group thinks the ALIEN-R2200 must be broken. Each “out” number should be the “in” number with zero replacing any digit in the ones place.*	*This group uses a number line to show how each pair of numbers rounds down or up to the nearest 10.*

Examine and Elaborate

3. Highlight Students’ Mathematical Thinking

Share with students that mathematicians relish the time that they spend trying to figure out patterns and, since they enjoy this challenge, they feel a sense of accomplishment when they finally uncover it. With this activity, students ultimately should understand that the **function**, or the rule, is that the number is rounded to the closest 10.

Explain to students that “rounding” means that you change a number so that it is easier to use in calculations. When rounding to the nearest 10, numbers that are four or less away from a 10 round to that 10 (e.g., 16, 17, 18, 19, 21, 22, 23, and 24 round to 20). Devoid of any context, numbers that are five away round to the next 10 (e.g., 15 rounds to 20, 45 rounds to 50). Ask students what would happen if the number 40 was placed into the ALIEN-R2200. Logically, the number would stay the same.

Share and Discuss

The following sample dialogue showcases how students may make sense of rounding. A role the teacher takes is to draw out students’ ideas.

Teacher: Let’s start with (the group that shared Response B). What’s your thinking?

Ali: Well, we noticed that the 'out' numbers all have a zero at the end. So, we think we should just change the numbers in the ones place to zero.

Jayna: Yeah, but if you do that with 67, you would get 60 not the 70 that came out. Same with the 48.

Ali: Oh, yeah. It's not 40, it says 50. Maybe the machine is broken.

Teacher: That is one possible idea; however, let's assume that the machine is not broken. We have just not yet figured out the pattern. Jacob, what did your group find?

Jacob: We counted from each 'in' number to the 'out' number. See, like this (see Response A): 19, then 20. Next one goes 23, 22, 21, 20. Then 32, 31, 30. We only had to count a few times before we got to the 'out' one.

Teacher: Can someone please repeat what Jacob just said in your own words? Nyah? *(Repeat/Rephrase talk move)*

Nyah: I think he said that each time you count from the 'in,' you don't have to count too much to get to the 'out.'

Teacher: Jacob, is that what you said?

Jacob: Yeah, it's kinda close for all of them.

Teacher: Let's have someone add to that idea. I notice that (the group who produced Response C) included **representations**, which are visuals and symbols, to share their thinking. This is an important way mathematicians share their ideas and figure out answers. I wonder if that will help us see the pattern. Cindy, can you tell us more about that and how your pictures might be connected to Jacob's group's ideas? *(Adding On talk move)*

The group that produced Response C presented their number line and students noted how the numbers close to a 10 would round to it. They added on some other in/out numbers that the ALIEN-R2200 would produce. Some numbers were to the left of the 10 (e.g., 18 to the left of 20 and 46 to the left of 50) and some were to the right of the 10 (e.g., 22 to the right of 20 and 51 to the right of 50). Name the function, or rule, students are explaining—that the number is rounded to the closest 10—if they do not use this language themselves.

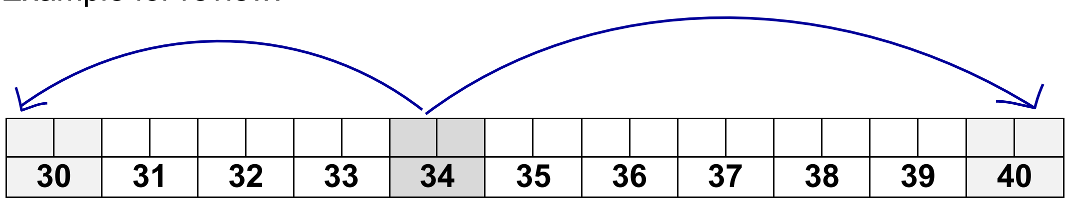
Next, discuss with students what would happen if the number 40, 20, 80, 30, etc. were placed into the ALIEN-R2200. Logically, the numbers would stay the same.

Finally, students need to understand rounding numbers with fives in the ones place. This may be confusing for students because the numbers with a 5 in the ones place are directly in the middle of two 10s (e.g., 35 is in between 30 and 40). Some students may already know that numbers ending in a 5 are typically rounded up and can explain this to the rest of the class. Share real-life instances when it is better to round up or **overestimate**. For example, it is typically better to overestimate the total cost when shopping or the total number of people attending a party. One might also overestimate the size of an object to fit in a room. It is important to make it clear that students have freedom in the real world to decide how best to round.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. If the students do not understand the ALIEN-R2200 function, go over the following example. Tell students that they are going to round 34 to the nearest 10 by placing the 34 in the ALIEN-R2200. Have students trace the arrows from the 34 to the 30 and then from the 34 to the 40. This will help students to understand that the distance is the shortest from the "in" number to the "out" number compared to the distance from the "in" number to the other multiple of 10. This could also be illustrated by placing a number line on the floor and having the students walk the paths from the 34 to the 30 and 40 to see which path is the shortest.

Example for review:

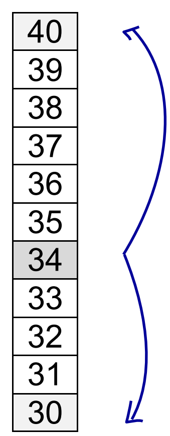


IN	OUT
34	30

Also, a rhyme that may help students remember how to round is:

1 through 4, fall on the floor
5 through 9, climb the line

This rhyme is helpful if students see or draw a vertical number line such as the following:



Some students may benefit from further challenge of the lesson concepts, such as realizing that more numbers round up than down in between two sequential 10s (e.g., the five numbers 40, 41, 42, 43, and 44 all round to

40 whereas six numbers, 45 through 50, all round to 50). In fact, there is a type of rounding called Bankers' Rounding where the number is always rounded to an even number. For example, 22.5 is rounded to 22, but 23.5 is rounded to 24. This may be of interest to some students who realize if you always round up your calculations will increasingly be off. Using the Bankers' Rounding strategy distributes the rounding up and down more evenly.

Debrief and Look Ahead

4.

Debrief Content and Skills

Ensure that students have a solid understanding of rounding as a concept. Ask students:

- How would a mathematician explain "rounding"?
- What strategies do you use when rounding?
- How could you use rounding in the real-world?

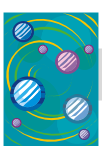
Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians

Remind students that this lesson focused on how mathematicians work together. Review some of the ideas students brainstormed at the beginning of class and have students offer examples of how they acted like mathematicians while they worked together during the lesson. Give students a few minutes to create artifacts for the time capsule that demonstrate how they worked together like mathematicians.

Look Ahead

In Lesson 3, students will build upon rounding by applying this skill to mental math computations.



Assess

5.

Practice Rounding Numbers

The extra practice page *Understanding Rounding (Extra Practice)* Student Page **[SMN pages 35-37]** may be used as additional support for students who are struggling with the concept of rounding. It may also be used as seatwork or homework.

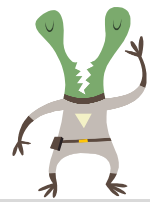
As an extension to this investigation, you could also ask students to write a letter to a Planet Nine alien friend explaining what they learned about how the ALIEN-R2200 works and when it would be useful in the real world.

Teacher Materials: Number Cards

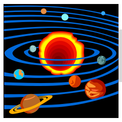
11	15	16
20	30	32
34	36	48
49	50	

Mental Math—

Planet Nine Aliens Go on a Shopping Spree!

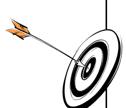


LESSON 3



Big Ideas

Using benchmarks to make groups of 10 is also very helpful when it comes to working with money amounts. Estimating sums of money is a real-world skill. Estimation is an important skill to check the accuracy of one's mental math computations. Rounding is used to estimate a "close enough" answer in situations that do not require an exact answer. For example, when students are shopping, they may wish to estimate the sum to see if they have enough money to purchase certain items. In this case, knowing the exact total is not as important as knowing if they have enough money. To calculate the exact answer, a student may wish to round when faced with an addition number sentence like 49 + 8. A student may round 49 to 50, add 50 to 8 to get 58, and finally subtract the 1 that was initially added to get the exact answer of 57.

Lesson Objectives 	• Students will understand how to decompose $10. • Students will apply decomposition and rounding to estimate money amounts and find exact answers. • Students will explain when estimating is used in real-world situations.

Materials	• Student Page—*A Planet Nine Alien Shopping Spree!—Directions* [SMN page 39] • Student Page—*Number Cube—Diophantus* [SMN page 41] • Student Page—*Number Cube—Kovalevsky* [SMN page 43] • Student Page—*Tommy's Toy Store* [SMN page 45] • Student Page—*A Planet Nine Alien Shopping Spree!—Diophantus* [SMN pages 47-49] • Student Page—*A Planet Nine Alien Shopping Spree!—Kovalevsky* [SMN pages 51-53] • Student Page—*Toy Store Wish List—Homework* [SMN page 55] Remind students that manipulatives are available to use strategically and appropriately.
Mathematical Terms	• **Associative Property of Addition:** The grouping of addends in a number sentence does not change the sum. • **Estimate:** An educated guess for the answer to an algorithm. • **Estimating:** Rounding numbers to calculate an answer such as a sum or difference. • **Recompose:** Putting numbers back together after decomposing them to make a problem mentally easier to calculate. • **Rounding:** Altering a number so that it is easier to use in calculations.
Selected Mathematical Practices	• MP4: Model with mathematics • MP7: Look for and make use of structure

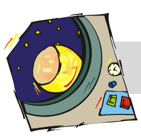
Differentiation

Content
- learning objectives
- prior knowledge or learner readiness
- tiered activities
- formative assessment
- varied levels of challenge
- know (information, facts, vocabulary), understand (concepts, big ideas, connections), apply (skills, processes)
- real-world application

Process
- questioning strategies
- 4Cs (21st Century Skills)
  - creativity
  - critical thinking
  - collaboration
  - communication
- hands-on activities/manipulatives
- connections

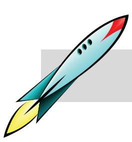
Product
- oral, visual, and written opportunities

Learning Environment
- flexible grouping
- whole group/small group/individual instruction
- learning community



Lesson Preview

In the previous two lessons, students explored ways to add using decomposition strategies and rounding numbers. In this lesson, they will practice selecting a strategy to either estimate or find the exact answer to an addition problem that works best for them or for a specific problem.



Launch

1. **Thinking Like Mathematicians**
As a whole group, discuss how mathematicians communicate. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask students:
    - How do mathematicians communicate?
    - Why do mathematicians use different methods for communicating their ideas?

- If Planet Nine aliens arrive in the future to visit us, how might they know we are mathematicians by the way we communicate our ideas?

If students struggle to generate ideas, share that mathematicians communicate by (a) explaining their thinking to others, (b) listening to others explain their thinking, and (c) asking questions. When mathematicians explain their thinking to others, they often use mathematical vocabulary, pictures, tools, manipulatives, or examples of their work. When mathematicians are listening to others, they face the speaker and make eye contact, they try to understand others' methods, and they consider the similarities or differences between their ideas and other mathematicians' ideas. Finally, mathematicians ask thoughtful questions to clarify ideas and gain a better understanding of the topic.

Review

Begin the lesson by briefly discussing the following review questions. Ask the students:

- What are all the ways that we have learned to make 10?
  The order of answers will vary but should include:
  0 + 10
  3 + 7
  1 + 9
  4 + 6
  2 + 8
  5 + 5

Next, ask the students:

- If we wanted to figure out different ways to make $10 instead of 10, how would this change our thinking?
  Instead of decomposing whole numbers, students would decompose $10 into smaller whole dollar amounts:
  $0 + $10
  $3 + $7
  $1 + $9
  $4 + $6
  $2 + $8
  $5 + $5

Students should be able to explain that when you decompose money amounts, you use the same process as with regular whole numbers. For example, with whole numbers, 10 decomposes into 7 and 3. Similarly, $10 decomposes into $7 and $3. Ask the students:

- How would you round 42, 37, and 65 to the nearest 10?
  Answers: 40, 40, 70

- If the numbers were changed to money amounts ($42, $37, and $65), how would this change how the numbers are rounded?
  Answers: $40, $40, $70
  Instead of rounding each number to the nearest group of 10, they would be rounding to the nearest group of $10.

Explain to the students the goal of this lesson is to apply what they have learned about decomposing 10 from Lesson 1 and rounding from Lesson 2 to problems that young mathematicians might encounter in the real world.

A Real-world Shopping Problem

Next, read the following real-world problem to the students:

> *Soland would like to go on a shopping spree! She has saved $33 in her vacation piggy bank. Also, she earned $17 helping her Planet Nine alien relatives unpack their luggage after arriving on Earth. Before going to the toy store, Soland needs to know how much money she can spend.*

As a class, in small groups, or in pairs, have students discuss different ways to estimate how much money Soland has to spend at the toy store using rounding. If students work with a partner or in small groups, have the students share their strategies with the whole group.

Talk Move: *Use Repeat/Rephrase. Ask a student to restate what a classmate shared. Asking students to repeat other students' contributions, or to rephrase it in their own words, gives students more time to process the information. This also provides evidence that other students were listening and heard what their classmate said. All of these contribute to the likelihood that other students are participating in the discussion (Chapin et al., 2009).*

If the students did not discuss it, share the following strategy by explaining that another third-grade class used these steps.

Mental Estimation:

Step 1: Mentally ***round*** *each money amount.*
$33 rounds down to $30, and $17 rounds up to $20.
Step 2: Estimate the sum of the rounded money amounts.
$30 + $20 = $50

Talk Move: *Use Reasoning. Ask students if they agree or disagree with the other student's strategy and more importantly why they disagree or agree.*

Explain to students that **estimating** is different from **rounding** numbers. When mathematicians estimate a sum, they mentally calculate an approximate solution to the algorithm. An **estimate** is also an educated guess for the range of the sum. In this context, rounding is an important step before estimating a solution. Next, ask the students:

- How is estimating helpful in a real-world situation?
  Sample answer: *When you need to know about how much money you have to spend at a store.*

You can also use your estimated sum to check how accurate your mental math is when adding money amounts. If you estimated that you have about $50, then when you use mental math to find the exact amount, it should be either close to $50 or exactly $50. Ask the students:

- How would you figure out the exact amount of money that Soland has to spend at the toy store by using mental math?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

After students have an opportunity to think through this question independently or with a partner, have them share their ideas with the class. Explain to students how another group of third-grade students used mental decomposition to find the exact money amount that Soland has in her vacation piggy bank using the following possible response:

Mental Decomposition:

Step 3: Decompose each money amount.
$33 decomposes into $30 + $3, and $17 decomposes into $10 and $7.

Remind students that this is only one way that the money amounts could be decomposed. Ask the students:

- Can you think of a different way to decompose the numbers to find the sum?

Step 4: **Recompose** the numbers. This means to regroup the $3 and $7 into one group of 10 and then add the three addends together.
$30 + ($3 + $7) + $10
$30 + $10 + $10

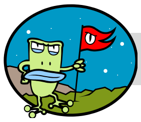
This step also illustrates the **associative property of addition**—grouping addends together differently in the number sentence will not change the sum.

Step 5: Use mental math to find the sum.
$30 + $10 + $10 = $50

Ask the students:

- What did you notice about the estimated sum and the exact sum?

They should be able to note that both money amounts are the same. Explain that this is not always the case. Sometimes the estimate will be greater or less than the exact sum.



Explore

2. In this investigation, students should work with partners in differentiated groups based on their addition skills, mental math skills, and performance in Lessons 1 and 2. Students may also be grouped based upon their scores on the unit pretest.

Groups Formed by Scores on Question 3 on the Pretest	
Diophantus Lab Group	Kovalevsky Lab Group
Scored 0 points on item	Scored 1 or 2 points on item
Student Names	Student Names
Student Pages for Lessons	
Number Cube—Diophantus Student Page **[SMN page 41]** *Tommy's Toy Store* Student Page **[SMN page 45]** *A Planet Nine Alien Shopping Spree!—Diophantus* Student Page **[SMN pages 47-49]**	*Number Cube—Kovalevsky* Student Page **[SMN page 43]** *Tommy's Toy Store* Student Page **[SMN page 45]** *A Planet Nine Alien Shopping Spree!—Kovalevsky* Student Page **[SMN pages 51-53]**

Students will be working with a partner to help out Soland on a shopping spree. To begin, read the following description to the students. Have students follow along on *A Planet Nine Alien Shopping Spree!—Directions* Student Page **[SMN page 39]**.

Soland mathematician would like to go on a shopping spree at Tommy's Toy Store! The toy store is located at the Amusement Park, so this is a special kind of toy store. Each customer gets to roll a die (number cube) to see how many toys he or she can buy at one time. Soland would like to spend less than $100 for each turn. Soland needs help from you and your partner.

Explain the following directions to the students:

1. First, you will cut out and make your *Number Cube* **[SMN page 41** or **43]**. Roll the die to see how many toys Soland can buy at *Tommy's Toy Store* **[SMN page 45]**. Each partner will have a turn to roll the die.
2. Help Soland decide which items she should buy that total less than $100.
3. Estimate the cost of the items so Soland does not go over $100 for each roll.
4. Next, use mental math to figure out the exact total for the items.
5. Show your work on the *A Planet Nine Alien Shopping Spree* Student Page **[SMN pages 47-49** or **51-53]**.

Remind students to use decomposition to find groups of 10 to make mental addition easier.

Depending on your students' background knowledge working with money, you might consider adding more complexity to the lesson by:

- using larger money amounts for the prices of the items in the toy store;
- changing the money amounts to include values in the decimal places (e.g., $3.94);
- having Soland spend more money at the Toy Store; and
- having students find multiple ways to decompose the cost of the items to find the estimates or exact total costs.

Collaborate and Communicate

Students will reflect on their use of mental math to solidify those strategies. Have the class consider how to add $47 and $51 in their heads. Small groups will answer, "What strategies can you use to mentally add?" They should record their explanation on a whiteboard or large chart paper chart paper. Mathematicians use representations to communicate their ideas. Build off students' initial ideas by asking them how others might more clearly understand their thinking. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group subtracted 1 from $51 and added it to $47 to compute $50 + $48.*	**B.** [Possible response] *This group rounded both dollar amounts to $50 and then subtracted 3 and added 1.*	**C.** [Possible response] *This group added from left to right.*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**

Remind students that mathematicians communicate their ideas for solving problems using visuals and symbols, which are called **representations**. In addition to learning about the various strategies that can be used to mentally add numbers, such as $47 and $51, it is important for students to represent their ideas in ways that others can clearly understand.

Share and Discuss

The following discussion focuses on clearly communicating the variety of ways that students can mentally add. The teacher helps students realize the importance of using representations, like symbols, and ways to present them, such as with color-coding, to convey ideas in their minds.

Teacher: We have heard a couple of groups present their ideas so far. Anthony's group (see Response A) came up with one possible solution: subtracting 1 from $51 and adding it to $47 to compute $50 + $48. Amari's group (Response C) added from left to right. Can you explain to the class any strategies that you used while adding the problems in your mind?

Amari: First we knew that 40 plus 50 equals 90, then we knew that 7 plus 1 equals 8, so we did 90 plus 8 and got our answer of 98.

***Anthony*:** I don't think I would be able to do all of that in my head.

Jessica: That's why we rounded the numbers (Response B) because it was easier for us to do in our heads.

Teacher: Alright, so all of the groups figured out this problem in a different way, does everyone agree that we all used different strategies, and some were easier for us and some were more difficult for us? Why might this have happened? *(Reasoning talk move)*

Amari: Yes, because our group believed that our way of solving the problem was the easiest way, but I liked the rounding idea too!

Teacher: Did anyone try using some pictures, symbols, or manipulatives in this classroom to help mentally add?

Anthony: We used money to help us answer this question. We thought of a $50 bill and then having 1 extra dollar. So we wanted to get rid of that dollar and add it to our two $20 dollar bills, one $5 dollar bill, and 2 dollar bills so now we would have 3 dollar bills, one $5 dollar bill, and two $20 dollar bills making a total of $98. [Student shows a picture to the class.]

Jessica: Oh, I never thought of solving the problem like that, I like it!

Teacher: Let's brainstorm other ways we can think of solving this problem using images or symbols. *(Wait Time talk move)*

Students soon saw the benefits of using representations while trying to mentally add. At first, students might be intimidated by mental math, but with this recent realization they can start to use real-world representations, such as money, to solve mental addition or subtraction problems. Some students might believe adding left to right mentally might be too difficult, for instance. If extra time is needed, emphasize the use of another student's example of images or symbols for students who had difficulty generating strategies for solving this problem.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts of rounding numbers as one way to help them estimate sums. To help students keep track of how they rounded their numbers, you might have students divide the "Cost of the Items" column into two sections that so the students can write the exact cost and the rounded cost.

Some students might need additional help with seeing how they can decompose and recombine numbers to add mentally. To help them visualize the decompositions, you might have them use a strategy such as branching (i.e., writing the numbers in expanded form and recombining the numbers to make them easier to add—see below) to note how they decomposed and recomposed the numbers.

$47 + $51

$40 + $7 + $50 + $1

$90 $8

Having money manipulatives available may also be helpful. Finally, having the *Packing Party for Earth* Student Page **[SMN pages 9-13]** from Lesson 1 can be used to help students see all the possible ways to make 10 to see how they might decompose/recompose the numbers.

Some students may benefit from further challenge of the lesson concepts. To encourage these students to think flexibly about numbers, you can ask "What is the most number of items you can purchase and stay under $100" and/or "What is the least number of items you can purchase and stay under $100?" Have students justify their answers.

Debrief and Look Ahead

4.

Debrief Content and Skills

Ensure that students have a solid understanding of estimation. Ask the students:

- How would a mathematician explain estimation?
- How are estimation and rounding alike? How are they different?
- What strategies can help you with mental addition?
- When might you use estimation and rounding in the real-world?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on how mathematicians communicate. Review some of the ideas students brainstormed at the beginning of class and offer examples of how students communicated like mathematicians during class. Give students time to create artifacts for the time capsule that demonstrate how they communicated like mathematicians.

Look Ahead

In Lesson 4, students will continue to build their mental computation skills and conceptual understanding of decomposing and rounding numbers as they learn to balance equations.

Assess

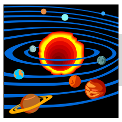
5. Practice Mental Math

Examine students' work on *A Planet Nine Alien Shopping Spree!* Student Page **[SMN pages 47-53]** to determine if students are applying the skills discussed in this lesson. Students' work should show evidence of using benchmarks to make groups of 10 to mentally add numbers together to calculate sums.

For homework, students can complete the *Toy Story Wish List—Homework* Student Page **[SMN page 55]** for additional practice, which can also serve as an assessment of students' progress with mental addition. Students will need to take home a copy of *Tommy's Toy Store* Student Page **[SMN page 45]** to complete the homework. You may also challenge your advanced students by having them use only double-digit amounts. Struggling students may use only single-digit amounts.

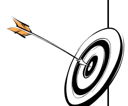
Flexible Numbers—
Equal Sides

LESSON 4



Big Ideas

Decomposing numbers, the use of benchmarks, and regrouping are strategies used to simplify adding and strengthen mental computation skills. Balancing a scale is similar to balancing an equation and it requires flexible thinking about numbers. A flexible understanding of the relationship between both sides of an equation (i.e., the concept of equivalency) is important for building a foundation for algebra. A common student misconception is that the equal sign means “the answer is” and “the answer (a single number) should be on the right of the equal sign.” This lesson supports student understanding that the equal sign is used to indicate that the values on each side of the equal sign are equivalent. For example, all of the following are accurate representations of equivalency (e.g., 5 = 2 +3, 2 + 3 = 4 + 1, 2 + 3 = 5, 1 + 2 + 2 = 1 + 4).

Lesson Objectives 	• Students will compute mental math skills using grouping and ordering strategies to solve addition problems. • Students will solve addition problems using the decomposition of numbers strategy. • Students will be able to explain that the order and grouping of addends does not affect the answer. • Students will apply the concept of equivalency.
Materials 	• Student Page—*Introduction to Equal Sides* [SMN page 57] • Student Page—*Equal Sides—Fibonacci* [SMN pages 59-63] • Student Page—*Equal Sides—Diophantus* [SMN pages 65-69] • Student Page—*Equal Sides—Kovalevsky* [SMN pages 71-75] • Student Page—*Equal Sides—Homework* [SMN page 77] • Pan Balance (**OPTIONAL**) Remind students that manipulatives are available to use strategically and appropriately.

Mathematical Terms	• **Commutative Property of Addition:** The order that addends are added does not change the sum. • **Property:** A math rule.
Selected Mathematical Practices	• MP3: Construct viable arguments and critique the reasoning of others • MP4: Model with mathematics

Differentiation	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• tiered activities
	• formative assessment
	• varied levels of challenge
	• real-world application
	Process
	• questioning strategies
	• 4Cs (21st Century Skills)
	  ○ creativity
	  ○ critical thinking
	  ○ collaboration
	  ○ communication
	• hands-on activities/manipulatives
	• connections
	Product
	• oral, visual, and written opportunities
	• multiple ways to demonstrate knowledge, understanding, and skills
	• multiple models and representations
	• meaningful and respectful tasks
	Learning Environment
	• flexible grouping
	• whole group/small group/individual instruction

Lesson Preview

In this lesson, the students will apply the commutative property, the associative property, and the decomposition of numbers to make a scale balance. This is a real-world application of balancing equations.

Launch

1. **Thinking Like Mathematicians**

As a whole group, discuss how mathematicians support their answers. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask the students:

- How do mathematicians support their answers?
- What type of information do mathematicians give in support of their answers?
- If Planet Nine aliens arrive in the future to visit us, how might they know we are mathematicians by the way we support our answers?

If students struggle to generate ideas, share that mathematicians support their answers by explaining the steps they used to arrive at a solution. They also use precise mathematical language, pictures, charts, tools, manipulatives, or other examples.

Introduction to Balancing Scales

This investigation allows students to think about making sides of a scale balance. Ask the students to think about a seesaw or a pan balance:

- When is the seesaw or balance completely straight?
- What happens if there is more weight on one side of the balance?

Demonstrate how it works using a pan balance. Then, make the connection between balances and math equations. When the scale is completely balanced, the one side of the equation is exactly equal to the other side. Ask the students:

- Does it matter what order you put the weights on each side? Why or why not?

Remind the students that they know how to break a number down into its pieces and they will be using this knowledge during the lesson.

Introduction to Equal Sides

Instruct students to examine the Introduction to Equal Sides in *Introduction to Equal Sides* Student Page **[SMN page 57]**. Go over the questions with the students as a class. Make sure they understand that both sides need to equal each other to be balanced. Explain to students that they cannot take away or move any of the printed weights hanging from the scale. The numbers on the scale refer to the amount of weight hanging from that location. For example, a weight under the 7 is equal to 7 pounds. In small groups or pairs, give them a few minutes to try out the problem and then discuss the different strategies used with the whole class.

***Talk Move:** Use Reasoning. Ask students if they agree or disagree with the other student's strategy and more importantly why they disagree or agree.*

Explore

2. Explain the Lab

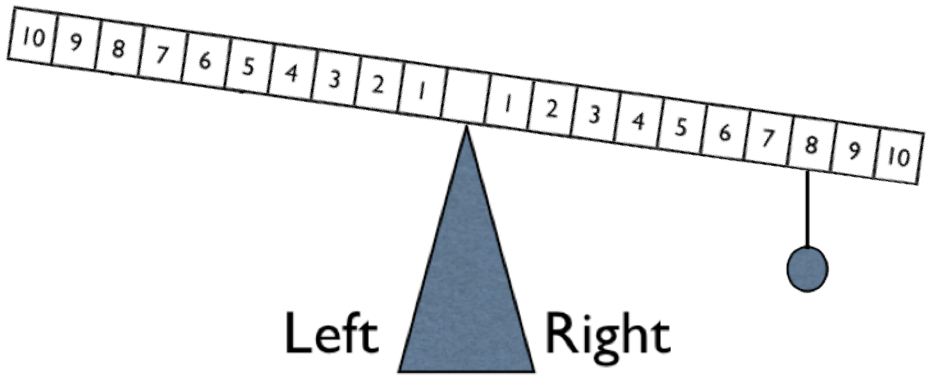
In this investigation, students will be working together in groups to explore how to balance scales that have different weights on them. Scales will be balanced by making both sides equal. Each group has an empty scale to create its own problem at the end of the investigation if there is extra time.

Using the table below, form three lab groups for this investigation—the Fibonacci, Diophantus, and Kovalevsky groups. Students will be completing *Equal Sides* Student Page **[SMN pages 59-75]**. The differences between the groups are in the progression of difficulty. Base your decision on how your students performed on the unit pretest. Allow for flexibility in meeting the needs of your particular students.

Groups Formed by Scores on Question 4 on the Pretest		
Fibonacci Lab Group	Diophantus Lab Group	Kovalevsky Lab Group
Scored 0 points on item	Scored 1 point on item	Scored 2 points on item
Student Names	Student Names	Student Names
Student Pages for Lessons		
Equal Sides—Fibonacci Student Page **[SMN pages 59-63]**	*Equal Sides—Diophantus* Student Page **[SMN pages 65-69]**	*Equal Sides—Kovalevsky* Student Page **[SMN pages 71-75]**

Collaborate and Communicate

After students finish the lab, bring them together for a discussion that addresses the importance of thinking flexibly about numbers. Have small groups collaborate on the following question: “What are different strategies you can use to balance the scales?” Mathematicians justify their answers, and one way to do this is to provide examples. Have students rely on the scales they worked on in small groups in addition to a new problem where there is one weight hanging on the 8 on the right side of the balance scale.



Have small groups record their explanation on a whiteboard or large chart paper. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response]	**B.** [Possible response]	**C.** [Possible response]
This group used doubles on either side of the balance scale and equal sign. To solve the new problem, they noted that 4 + 4 = 8.	*This group decomposed numbers. They showed how 1, 2, and 5 add to 8.*	*This group relied on the commutative property, knowing that 8 + 2 = 2 + 8.*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**
To think flexibly with numbers, mathematicians use different number properties or rules along with the equal sign that signifies both expressions to either side have the exact same value. They further justify their ideas, and one way to do this is by using examples.

Share and Discuss

This sample dialogue highlights some ideas that might come up in other classes. The role of the teacher is to encourage students to explain their thinking using examples.

There are three strategies that the students should be able to use and explain:

- They should understand that reversing the order in which they add the numbers will give them the same sum (e.g., 5 + 7 = 7 + 5). Teach students that this is called the **commutative property**. A **property** is a math rule.
- Different ways of grouping the addends will also produce the same sum [e.g., (5 + 7) + 4 = 5 + (7 + 4)]. Remind students that this is called the **associative property**.
- They should also understand that they could break down a number to make it easier to work with (e.g., 8 is the same as 5 + 3). Remind students that this is called **decomposition**.

Teacher: So Autumn (see Response C), can you explain the strategies you used to balance this scale?

Autumn: We realized that we could write the number sentence in two different ways. So we wrote 8 + 2 equals, or is the same, as 2 + 8.

Teacher: Ok, so you wrote 8 + 2 = 2 + 8. Is that what you mean? *(Revoicing talk move)*

Autumn: Yes!

Teacher: Could you explain your thinking while you solved this problem?

Autumn: We balanced the scale by putting a weight on 2 for the right side of the scale and then one weight on 2 on the left side of the scale and one weight on 8 to make both sides equal 10 in total.

Teacher: Does anyone know what mathematical strategy Group C used when they wrote their number sentence out like that? (Reference the mathematical terms)

Byran: The commutative property?

Teacher: Does anyone agree with Byran? Why or why not? *(Reasoning talk move)*

Alanah: Yes, I think it is the commutative property because both sides of the equation equal the same number.

Teacher: I agree too! Let's think of another way that you could solve this problem, Sarida can you explain what your group did (see Response B) to balance this scale?

Sarida: Yeah, we broke down the number 8 because we thought it would be easier.

Byran: What do you mean you broke down the number Sarida?

Sarida: We broke down 8 to make it 1, 2, and 5 which all adds up to 8. Then we put one weight on 1, 2, and 5 to make the balance equal.

Teacher: Sarida, does anyone from your group want to come up and show the class what you mean?

After Group B explains their response, guide students to understand that Group B used decomposition to solve this problem. If the students do not use this language themselves, name the rule, and have students expand as to how this group used decomposition.

Differentiate Further as Needed

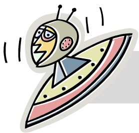
Some students might need additional support to understand the lesson concepts. Consider having students use different colors on each side of the balance on the worksheet to highlight the nature of the pan balance. Virtual pan balances may also be helpful if you have access to them. If not, the NCTM Illuminations website has a free virtual pan balance for students to manipulate (https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Pan-Balance----Numbers/).

Some students may benefit from further challenge of the lesson concepts; the Kovalevsky lab asks the students to create their own balance with 8 weights that totals 30 on each side, which challenges students to mentally add while considering equivalent number sentences. The additional restrictions require students to think creatively. You might ask students from any of the groups to complete this page if they show they are decomposing numbers larger than 10 comfortably.

The use of one and two-digit numbers was purposeful in the lesson. Using a simpler number will help to teach the complex concept of equivalency and can help students to engage in mathematical communication. If you still feel additional challenge is needed, you can instruct students to add additional weights to their scale balances to have them work with three-digit numbers (e.g., they might have 12 weights hanging from the number 10 on the scale balance on one side and then they might show 5 weights on the number 10 and 14 weights on the number 5).

Review

Summarize and conclude with the ideas the students developed in the discussion. Remind students that the order (commutative property) and grouping (associative property) of the addends does not affect the sum.



Debrief and Look Ahead

4.

Debrief Content and Skills

Ensure that students have a solid understanding of the strategies used to balance an equation. Ask the students:

- How would a mathematician explain the commutative property?
- How would a mathematician explain the associative property?
- What strategies could you use to balance an equation?

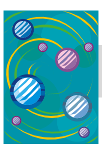
Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on how mathematicians support their answers. Review some of the ideas students brainstormed at the beginning of class and offer examples of how students communicated like mathematicians during class. Give students time to create artifacts for the time capsule that demonstrate how they supported their answers as mathematicians would.

Look Ahead

The next lesson moves into an examination of the 100s chart. First, students get familiar with the 100s chart, and the following lesson introduces multiples as repeated addition on the 100s chart.



Assess

5.

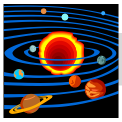
Practice Balancing Equations

Collect all of the *Equal Sides* Student Page **[SMN pages 59-75]** to assess students' understanding of balancing equations. Check to see if students were able to explain their thinking and show evidence of using decomposition, the associative property, and the commutative property to balance the equations. The *Equal Sides—Homework* Student Page **[SMN page 77]** may be assigned for students to complete at home as an additional assessment.

100s Charts—

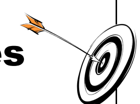
Where Did I Leave My Treasure?

LESSON 5

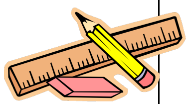


Big Ideas

This lesson continues to promote flexible mathematical thinking by encouraging students to find and use numerical patterns in the 100s charts. As students play with the 100s charts in this lesson, they will be exposed to the inverse relationships of addition and subtraction. They will also communicate these patterns to their classmates, which promotes mathematical discussion. The ability to discuss mathematical patterns is a skill that all practicing mathematicians use frequently.

Lesson Objectives 	• Students will explore and generalize patterns in a 100s chart. • Students will communicate mathematical thinking about patterns.

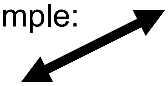
Materials



- Student Page—*Introduction to 100s Charts* [SMN page 79]
- Student Page—*Where Did I Leave My Treasure?* [SMN pages 81-83]
- Student Page—*Treasure Hunt Directions for All Groups* [SMN page 85]
- Student Page—*Treasure Hunt—Diophantus* [SMN pages 87-89]
- Student Page—*Treasure Hunt Questions—Diophantus* [SMN page 91]
- Student Page—*Treasure Hunt—Extra Diophantus* [SMN pages 93-95]
- Student Page—*Treasure Hunt—Kovalevsky* [SMN pages 97-99]
- Student Page—*Treasure Hunt Questions—Kovalevsky* [SMN pages 101-103]
- Student Page—*Treasure Hunt—Extra Kovalevsky* [SMN pages 105-107]
- Student Page—*Extra 100s Chart* [SMN page 109]
- Student Page—*Super Challenges* [SMN page 111]
- Student Page—*Pattern Mysteries—Homework* [SMN page 113]
- Folders for students' papers
- Laminated 100s Chart

Remind students that manipulatives are available to use strategically and appropriately.

<table>
<tr>
<td>Mathematical Terms</td>
<td>

Column: A vertical arrangement of items or numbers in a list. Example: 3
4
5
Diagonal: A line that is on a slant compared to the top and sides of a page. Example: 
Horizontal: Parallel to the horizon. Example: 
Row: A horizontal arrangement of items or numbers in a list. Example: 3 4 5
Vertical: At a right angle to the horizon (up and down). Example: 

</td>
</tr>
<tr>
<td>Selected Mathematical Practices</td>
<td>

MP7: Look for and make use of structure
MP8: Look for and express regularity in repeated reasoning

</td>
</tr>
</table>

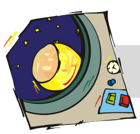
Differentiation

Content
- learning objectives
- prior knowledge or learner readiness
- tiered activities
- varied levels of challenge
- know (information, facts, vocabulary), understand (concepts, big ideas, connections), apply (skills, processes)
- real-world application

Process
- questioning strategies
- 4Cs (21st Century Skills)
    - creativity
    - critical thinking
    - collaboration
    - communication

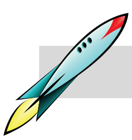
Product
- oral, visual, and written opportunities
- multiple ways to demonstrate knowledge, understanding, and skills

Learning Environment
- flexible grouping
- whole group/small group/individual instruction



Lesson Preview

Students will analyze 100s charts to find patterns. Then, they will use those patterns to give directions to a partner to help find hidden treasure. Finally, students will use these same patterns to fill in missing spaces cut from 100s charts.



Launch

1. **Thinking Like Mathematicians**
As a whole group, discuss why mathematicians use mathematical language. Consider writing a list of students' ideas on a whiteboard or chart paper. Ask the students:
    - Why do mathematicians use mathematical language?
    - How might mathematical language help mathematicians communicate?
    - If Planet Nine aliens arrive in the future to visit us, how might they know we are mathematicians by the language we use?

If students struggle to generate ideas, explain that mathematicians use mathematical language to be precise. Using mathematical language allows mathematicians to be clear and specific when they communicate their ideas. Mathematicians might use specific mathematical vocabulary (e.g., equation, commutative property) or provide details in mathematical terms (e.g., giving an answer with the units of measurement).

Exploration of 100s Charts

Tell students to examine the blank 100s chart on *Introduction to 100s Charts* Student Page **[SMN page 79]**. Ask the students:

- What do you notice about the 100s chart?

The students can turn and talk to a partner. Then, ask the students to share their partners' best ideas.

Talk Move: *Use Adding On. Adding On encourages students to contribute more comments related to other students' ideas. This talk move is intended to increase student participation and engagement in the class discussion.*

Most of the time students will begin to notice the patterns in the 100s chart. If students are struggling to describe patterns, tell them to pick any number on the chart and ask the students:

- What happens if you move 1 box to the right or left?

Systematic Introduction to Mathematical Terms

Introduce the Mathematical Terms that will help students more clearly define the patterns they see. Ask the students:

- When you move **diagonally** in the chart, do you notice any patterns?
- If you move **horizontally**, what patterns do you see in the **rows**?
- If you move **vertically**, what patterns do you see in the **columns**?

Content Note: *If students are struggling to learn these mathematical terms, have students stand up and show the directional terms using their arms. (e.g., Vertical—students put both arms straight up or down). You can also connect to real world examples. Columns are used to hold up buildings so they are vertical. You can help students visualize rows by showing a video/image of someone rowing a boat* ***across*** *a lake.*

Where Did I Leave My Treasure?

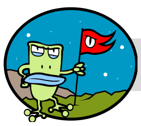
Tell the students that Toxo accidently left his treasure map in their Student Mathematician Notebook, *Where Did I Leave My Treasure?* Student Page **[SMN pages 81-83]**. He would like you to call him and give him directions on how to find it. Have the students discuss the following questions in small groups or pairs:

- What is one way you could describe how to find the treasure?
- What are some sample directions for Toxo?
- Did anyone do it a different way?

The students may give examples that use directions such as go down 5 rows and then to the left 6. This is a good starting point. Then, encourage students to think about the directions in terms of addition and subtraction.

Some possible solutions would include adding 50 and then subtracting 6 or subtracting 6 and adding 50. Another way would be to add 10, subtract 1; add 10, subtract 1; add 10, subtract 1; add 10, subtract 1; add 10, subtract 2. Students may also say add 44. When you hear a solution, use a 100s chart on the board to show the students what the directions look like on the chart. You might also have students go up to the chart to demonstrate the directions.

If students are ready for additional challenge, ask them to create directions to the treasure that include at least 4 directions.



Explore

2. **Explain the Lab**

In this investigation, students will be in differentiated groups based on the results of the unit pretest. Form two lab groups, the Diophantus and Kovalevsky groups. These groups are formed based upon student performance on one of the Unit Test questions; however, allow for flexible grouping to meet the needs of your particular students.

Groups Formed by Scores on Question 6 on the Pretest	
Diophantus Lab Group	Kovalevsky Lab Group
Scored 0-1 points on items	Scored 2 points on items
Student Names	Student Names
Student Pages for Lessons	
Treasure Hunt—Diophantus Student Page **[SMN pages 87-89]**. *Treasure Hunt Questions—Diophantus* Student Page **[SMN page 91]**. *Treasure Hunt—Extra Diophantus* Student Page **[SMN pages 93-95]**.	*Treasure Hunt—Kovalevsky* Student Page **[SMN pages 97-99]**. *Treasure Hunt Questions—Kovalevsky* Student Page **[SMN pages 101-103]**. *Treasure Hunt—Extra Kovalevsky* Student Page **[SMN pages 105-107]**.

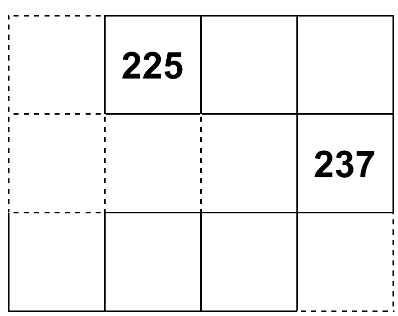
The main difference between the two groups is the type of chart given to the students. The Diophantus group continues to work with typical 100s charts. The Kovalevsky group, however, will look at a 100s chart with rows of 5. Both groups have questions that instruct them to reflect upon their treasure hunts and apply their knowledge of patterns to other situations. Instruct the students to work together to complete these questions.

Within the two main groups, students should be further paired into groups of two. The two main groups have the same directions that can be found on *Treasure Hunt Directions for All Groups* Student Page **[SMN page 85]**. In each group, there will be a treasure finder and a treasure hider. The treasure hider will place an “X” on the chart where he/she wants to hide the “treasure.” Remind the students it is important for the treasure hider to not show the treasure finder where the treasure is located. Instruct the students to put a folder up between them. If folders are not convenient, you could also have them sit on the floor back to back. Once the treasure hider hides the treasure, the treasure finder tells the hider his/her starting location. Then the treasure hider gives directions from that starting location to the treasure. These directions should include adding and subtracting to ensure students engage in a challenging treasure hunt, give the class or groups of students a minimum number of directions appropriate for their readiness level (e.g., 3-10 directions).

Collaborate and Communicate

The class will discuss part of a new chart that will reinforce the patterns on the other charts they investigated in groups and will allow them to be focused on the same ideas. The new chart is a 200s chart that has 201 in the top left corner and, given the ten columns, ends with 210 on the first row. Each of the 9 nine rows count by 10s. Ask small groups to answer the questions, “What are the missing numbers? How do you know?” You should record their explanation on a whiteboard or large chart paper.

201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
291	292	293	294	295	296	297	298	299	300



Note that those students in the Diophantus group worked with a similar chart, just that it began with 1 rather than 201, and they eventually will realize the patterns are the same. The Kovalevsky group will notice that the patterns differ according to the number of columns. They originally worked with multiples of five, now they will do so with 10. Realizing such patterns often is a focus of mathematician's thinking.

Based on their experiences, here are some possible student responses. You can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group relied on the idea that each row jumps by 10s.*	**B.** [Possible response] *This group highlighted how you subtract 11 when going up one row diagonally and add 11 when going down in this direction.*	**C.** [Possible response] *This group used the fact that each column differs by one to start counting at 225, then around the chart while realizing the 247 can be included.*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**
Mathematicians use precise language to communicate their ideas. Students ultimately should use such language to describe their own strategies to identify the missing numbers.

Share and Discuss
This sample dialogue focuses on the bottom left box of the chart, number 244. The shaded boxes and the bolded numbers are the ones from the original problem. The numbers in regular font are the ones that belong in those cells. In the ensuing discussion, the teacher emphasizes the importance of mathematicians being precise with their language.

	225	226	227
			237
244	245	246	

Teacher: Let's start with the number 244, Brock (see Response C) can you explain how your group got to 244?

Brock: Our group realized that if you look at each column they are different by 1 number, so we started counting at 225 then kept going around the chart. We kept doing that until we got to 237. Then we skipped a couple of numbers and started at 244 and kept going when we realized that 247 was the next number.

Teacher: Thank you, did any other group use this strategy?

Autumn: We kind of used that strategy, but we counted by 10s *(see Response A).

Teacher: Ok, can you explain what your group did more specifically?

Autumn: Well, we realized that each row jumps by 10, so we found 241, and then looked for 244 to see what would come after 244.

Jakob: I'm confused, Brock your group counted columns but Autumn your group counted rows. . . . How did you both get the same answer?

Teacher: That's a great question! Does anyone want to clarify by demonstrating on the chart how you can get the same answer by counting rows or counting columns?

Autumn: I will!

Teacher: Thank you Autumn.

After students demonstrate to each other how they found their answers, have students reflect on the importance of being clear and concise when explaining their answers so that other individuals will understand their process.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. You might include some sentence frames on notecards to help "treasure hider" come up with ways to explain how to get to the treasure.

"Jump vertically ↑ ↓___ rows." "Jump horizontally← →____ columns."

If students complete their assignments early or they need a challenge, they may work on the *Super Challenges* Student Page **[SMN page 111]**. The Super Challenges focus on understanding the number system and how going up or down a row represents a change of 10 and moving left or right in a column is a change of 1.

An *Extra 100s Chart* **[SMN page 109]** is provided for students in case they need another chart.

Summarize the Addition and Subtraction Patterns

You may want to take the laminated 100s chart and write in the patterns as the students say them. For example, you would draw a diagonal arrow to the left going down, and write +9 on it. If you want to extend the summary, you may also use the 100s chart that has 5 columns and write in the patterns that are on that chart. Another possible extension would include discussing what effect the number of columns has on the patterns.

Debrief and Look Ahead

4.

Debrief Content and Skills

Ensure that students have a solid understanding of 100s charts. Ask the students:

- What important patterns did you notice in the 100s chart?
- What strategies did you use to find the missing numbers in the new 200s chart?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on why mathematicians use mathematical language. Review some of the ideas students brainstormed at the beginning of class and offer examples of how students used mathematical language during class. Give students time to create artifacts for the time capsule that demonstrate how they used mathematical language as mathematicians would.

Look Ahead

In the next lesson, students will move beyond the addition and subtraction patterns that can be found in the 100s chart to the multiplication patterns. Students will explore various multiples on the 100s chart and discuss when certain numbers are multiples of several numbers.

Assess

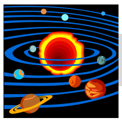
5. **Practice Recognizing Patterns**

The *Pattern Mysteries—Homework* Student Page **[SMN page 113]** may be used as a separate assessment. You may choose to send it home, use for morning work, or give students time in class to complete the assessment.

100s Charts—

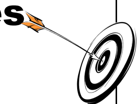
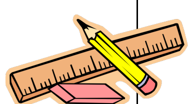
Searching for Patterns

LESSON 6



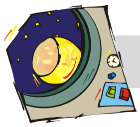
Big Ideas

The most important concept of this lesson is “multiples.” A strong foundation of multiples will anchor future lessons when multiplication is introduced in more depth. The students will see this concept throughout their mathematical lives. For example, multiples are used when calculating the cost of attending a movie for a family of four with an $8 ticket price.

Lesson Objectives 	• Students will understand the relationship between repeated addition and multiples. • Students will identify patterns of multiples on the 100s chart.
Materials 	• Student Page—*Multiples of Three* [SMN page 115] • Student Page—*100s Chart Pattern Language* [SMN page 117] • Student Page—*Searching for Patterns—Fibonacci* [SMN pages 119-121] • Student Page—*Searching for Patterns—Diophantus* [SMN pages 123-127] • Student Page—*Searching for Patterns—Kovalevsky* [SMN pages 129-131] • Student Page—*Extra 100s Chart* [SMN page 133] • Student Page—*500s Chart* [SMN page 135] • Student Page—*Super Challenges* [SMN page 137] • Student Page—*Searching for Patterns—Homework* [SMN pages 139-141] • Laminated 100s Chart • Crayons Remind students that manipulatives are available to use strategically and appropriately.

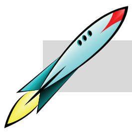
Mathematical Terms	• **Generalization:** Stating a conclusion based on a small amount of information, instances, or items. • **Multiple:** The product of a whole number and any other whole number. Example: The multiples of 3 are 0, 3, 6, 9, 12, 15, . . .
Selected Mathematical Practices	• MP7: Look for and make use of structure • MP8: Look for and make use of regularity in repeated reasoning

Differentiation	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• tiered activities
	• formative assessment
	• varied levels of challenge
	• know (information, facts, vocabulary), understand (concepts, big ideas, connections), apply (skills, processes)
	• real-world application
	Process
	• questioning strategies
	• 4Cs (21st Century Skills)
	    ○ critical thinking
	    ○ collaboration
	    ○ communication
	• hands-on activities/manipulatives
	• connections
	Product
	• oral, visual, and written opportunities
	• multiple ways to demonstrate knowledge, understanding, and skills
	• meaningful and respectful tasks
	Learning Environment
	• flexible grouping
	• whole group/small group/individual instruction
	• learning community



Lesson Preview

Individually, in groups, and as a whole class, students will use the concept of skip counting and multiplication to explore and compare patterns of multiples in a 100s chart.



Launch

1. **Thinking Like Mathematicians**

In small groups, have students review how mathematicians work together. Consider having them record their ideas or share them aloud with the whole class. Ask the students:

- How do mathematicians work together?
- How have we been working together like mathematicians?
- What other ways should we be working together to ensure that the Planet Nine aliens know we are mathematicians?

If students struggle to generate ideas, remind students that mathematicians collaborate to share ideas and solve problems. When mathematicians work together, you might notice that they listen to others, value others' ideas, respectfully disagree, offer their own ideas, are flexible and helpful, and share responsibilities for solutions.

Multiple Madness: Finding Multiples of 3

Explain to students that the objective of this lesson is to understand what **multiples** are and to find patterns of multiples in a 100s chart. Tell students that multiples are numbers in a special pattern. For example, multiples of 3 start with 0 and then continue by adding 3 repeatedly. Have the students practice "skip counting" to say the multiples of 3 (e.g., 0, 3, 6, 9, 12, . . .).

Tell the students that they will be using their crayon to color in multiples of 3 on their blank 100s chart on the *Multiples of Three* Student Page **[SMN page 115]**. Before allowing the students to begin, ask them:

- What are a few numbers you will color in?

Content Note: *The main problem students may experience is they might believe multiples of 3 are numbers that end in 3. Listen carefully to make sure students include numbers that do not end in 3 like 9 and that they do not include numbers like 13. You might even ask students why 13 is not a multiple of 3, even though there is a 3 in the ones place.*

After students have colored in the multiples of 3, ask them:

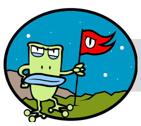
- What patterns do you see?

Remind students of the directional mathematical terms that were introduced in the previous lesson such as vertical, horizontal, diagonal, row, and column. The *100s Chart Pattern Language* Student Page **[SMN page 117]** is a helpful reminder of the language they could use. To encourage a discussion, ask the students:

- When you move diagonally in the chart, do you notice any patterns?
- If you move horizontally, what patterns do you see in the rows?
- If you move vertically, what patterns do you see in the columns?
- What is the 3rd multiple?
- What is the 9th multiple?

Talk Move: *Use Adding On. Ask students to add more to a classmate's response.*

For the more advanced students, you may want to make the connection that the 3rd multiple of 3 is the same as 3 x 3. If that is too much information in this lesson or if students are not ready for this connection, you may want to refer back to this lesson when you introduce multiplication algorithms in Lesson 8.



Explore

2.

Explain the Lab

In this investigation, students will explore patterns of multiples. Have them work in differentiated groups based on the results of two questions on the unit pretest. Allow flexibility in meeting the needs of your particular students. If needed, you may want to put them in smaller groups within the larger groups to allow for more student participation.

Groups Formed by Scores on Questions 5 and 6 on the Pretest		
Fibonacci Lab Group	Diophantus Lab Group	Kovalevsky Lab Group
Scored 0 or 1 point on items combined	Scored 2 points on items combined	Scored 3 to 4 points on items combined
Student Names	Student Names	Student Names
Student Pages for Lessons		
Searching for Patterns—Fibonacci Student Page **[SMN pages 119-121]**	*Searching for Patterns—Diophantus* Student Page **[SMN pages 123-127]**	*Searching for Patterns—Kovalevsky* Student Page **[SMN pages 129-131]**

In general, the complexity of the patterns increases from the Fibonacci Lab Group to the Kovalevsky Lab Group. Instruct all groups to first look at their special 100s chart. Each group has different patterns on its 100s chart represented by different symbols such as circles or triangles. Encourage students to talk about the chart with other members in their group. Then, tell the students there are several questions about their 100s chart. Have them work together to answer these questions.

Once the groups have completed the assigned questions, engage the students in a Jigsaw, re-forming the groups to ensure that at least one person from each Lab Group is in the newly formed group. Students should share the patterns they examined. They can discuss the following questions from the beginning of the lesson with their new charts:

- When you move diagonally in the chart, do you notice any patterns?
- If you move horizontally, what patterns do you see in the rows?
- If you move vertically, what patterns do you see in the columns?

Encourage students to use the 100s chart language from the last lesson (see **[SMN 117]**). They can also discuss when they have the same numbers marked on their papers (e.g., 6, 12, 18).

Share and Discuss

For the discussion, put a laminated 100s chart and the following table on the board:

Multiples in 100s Charts	
Vertical Patterns	Diagonal Patterns

Have students examine their 100s chart. Because each group has different multiples, each group will have something unique to contribute to this discussion. Ask students to see if any of the multiples on their charts have vertical patterns. Place the number in the table, and then color the pattern on the laminated chart. The only multiples that are not represented in any of the 100s charts are 5 and 10, and those are easy to illustrate. Ask:

- What different patterns do you notice in the vertical patterns? Sample answer: *For the multiples of 3, every third number in a column is highlighted and the difference between the highlighted numbers in that column is 3.*
- Does this pattern explain other numbers?
- Do you see any diagonal patterns?

Add those to the chart and highlight them on the laminated chart. Again, ask the students to explore the difference between each number in a diagonal.

Finally, discuss what it means when a number has more than one symbol on it (e.g., circle, triangle, sun, cloud). It means that a number is a multiple of different numbers. For example, 6 is a multiple of both 2 and 3, so both symbols would be on 6. Ask students to explain different examples (see Differentiate Further for some questions).

Collaborate and Communicate

The class will consolidate their mathematical thinking by addressing a number on the 100s chart that all groups identified as being a multiple on their respective pages: 48. Specifically, have small groups consider the question, “The number 48 is a multiple of what numbers? Why might it be a multiple of so many numbers?” Since groups will have identified different patterns, consider forming small groups with students from the Fibonacci, Diophantus, and Kovalevsky groups so they can exchange these ideas.

Small groups should record their explanation on a whiteboard or large chart paper. Focus questions on their work by asking them to clarify and expand on their ideas. Mathematicians often look to generalize their ideas beyond one case, so encourage students to do the same. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group noticed that most of the numbers they worked with were even, and they identified four as another factor of 48.*	**B.** [Possible response] *This group realized 2, 3, and 6 are factors of 48 and conjectured that since 2 x 3 = 6, that's why 2 and 3 must be factors of 48.*	**C.** [Possible response] *This group pointed out that 24 also is a multiple of 2, 3, 6, and 8, just like 48.*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**

Generalizing, or realizing the underlying mathematical structure, is a manner in which mathematicians think. Students at this age should look for repetition across situations to make their thinking more efficient, such as noticing number properties like evenness and oddness.

Share and Discuss

This sample dialogue highlights how to help students generalize properties of even numbers. The teacher builds off students' ideas to help them understand the importance of generalization.

Teacher: Liam, can you explain how your group found factors of 48?

Liam: We realized that a lot of the numbers we were looking at were even numbers so we noticed that 4 is another factor of 48 (see Response A).

Teacher:	Ok, so you applied the pattern you noticed to figure out another factor. Did any other group do something similar to Liam's group?
Anahita:	No, we did something different. We realized that 2, 3, and 6 are factors of 48 because we tried doing 2 multiplied by 3 and found 6 so we figured that 2 and 3 must also be factors of 48.
Teacher:	Interesting, so Liam's group found that 4 is a factor of 48 because it is even, and your group found 2, 3, and 6 were factors of 48, correct? *(Revoicing talk move)*
Anahita:	Yes.
Teacher:	Does anyone realize any similarities between these two answers?
Anahita:	No, we found different numbers that were factors of 48.
Teacher:	True, these two groups found different numbers for their answers, but what about the numbers they found? Is there a rule or pattern that was common to both groups?
Liam:	Oh! We both found that some factors of 48 that are even numbers!

Students were soon able to see that although they had different answers, both groups realized they could generalize their overall findings to state that their answers had a rule in common (evenness). Provide feedback and positive reinforcement to students and encourage using generalization throughout the rest of the unit. Inform students that when mathematicians can generalize information, they are better able to learn new information. Define generalization if students have difficulty understanding the term.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. An *Extra 100s Chart* **[SMN page 133]** is provided for students in case they need another chart. If students are identifying the multiples of a number as having a specific number in the ones place (e.g., 13 and 23 are multiples of 3 because a 3 is in the ones place), they may use a counter to help them keep track of the moves (e.g., use the counter as a finger pointer tool to emphasize counting by the factor and marking those numbers on the chart—3, count 1, 2, 3, mark 6). It could also be fun to create a 100s chart on the playground to allow for students to play with the patterns.

Some students may benefit from further challenge of the lesson concepts, if students complete their assignments early, they may search for patterns using the *500s Chart* **[SMN page 135]** or work on the *Super Challenges* Student Page **[SMN page 137]**.

You can also ask students:

- Which numbers tend to have the most shapes on them—even or odd numbers? Why?
- Do all even numbers have multiple shapes on them?

Review

Summarize what multiples are and remind the students of the patterns they saw in the 100s chart. Conclude with the ideas the students developed in the discussion.

Debrief and Look Ahead

4.

Debrief Content and Skills

Ensure that students have a solid understanding of multiples. Ask students:

- What important patterns did you notice in the 100s chart?
- How would mathematicians describe multiples?
- How might multiples be helpful while solving problems that go beyond the 100s chart?

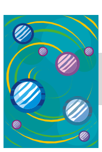
Talk Move: *Use Wait Time. Give students extra time to think about their answers.*
Talk Move: *Use Adding On. Ask students to add more to a classmate's response.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on how mathematicians work together. Review some of the ideas students shared at the beginning of class and offer examples of how students worked together like mathematicians during class. Give students time to create artifacts for the time capsule that demonstrate how they continued working together like mathematicians.

Look Ahead

In the next lesson, students will have an opportunity to work more with multiples as they begin to focus on multiplication.



Assess

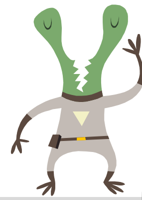
5.

Practice Searching for Patterns

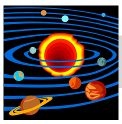
If you need a separate assessment, use the *Searching for Patterns—Homework* Student Page **[SMN pages 139-141]**. You may choose to send it home, use for morning work, or give the students time in class to complete the assessment.

Multiplication Madness—

Getting to Know Planet Nine Aliens

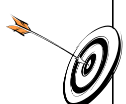
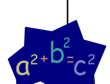
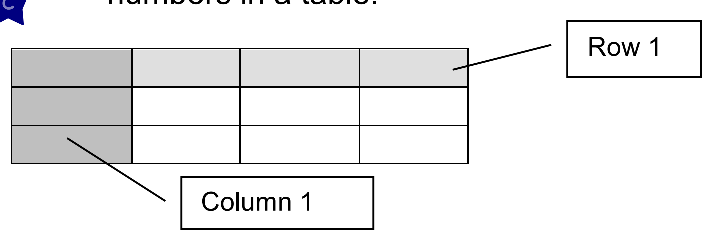
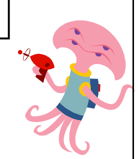


LESSON 7



Big Ideas

Problems that can be solved using multiplication can also be solved using counting strategies or repeated addition strategies. A focus on problem solving should allow students to use different methods and develop personal preferences. In this lesson, students will continue to build their conceptual understanding of the relationship between repeated addition and multiplication.

Lesson Objectives 	• Students will be able to solve multiplication problems using repeated addition. • Students will be able to organize information using a table. • Students will be able to repeatedly add 10 (count by 10s) starting with any number.
Materials 	• Student Page—*Getting to Know Planet Nine Aliens* [SMN pages 143-149] • Student Page—*Getting to Know Planet Nine Aliens—Homework* [SMN page 151] Remind students that manipulatives are available to use strategically and appropriately.
Mathematical Terms 	• **Column:** A vertical arrangement of items or numbers in a table. • **Row:** A horizontal arrangement of items or numbers in a table. 

Selected Mathematical Practices	• MP2: Reason abstractly and quantitatively • MP4: Model with mathematics

Differentiation	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• tiered activities
	• formative assessment
	Process
	• 4Cs (21st Century Skills)
	  ○ critical thinking
	  ○ creative thinking
	  ○ communication
	Product
	• oral, visual, and written opportunities
	• multiple ways to demonstrate knowledge, understanding, and skills
	Learning Environment
	• whole group/small group/individual instruction

Lesson Preview

In this lesson, students explore multiplication by examining Planet Nine aliens with features that differ from those of humans.

Launch

1. **Thinking Like Mathematicians**

As a whole class, discuss strategies that students can use when they have difficulty with a problem or become confused. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask students:

- What do mathematicians do when they are trying to figure out a difficult problem?
- What strategies could be used when you are confused with a math problem?

If students struggle to generate ideas, give students examples of how other students have solved difficult problems.

Example: Johnny, a student in another third-grade class, would take a break and look at each important part of the equation. Then, he would list each step to solve the problem. He realized that even though the question was set up as a multiplication question, he could solve the problem when he thought about it as a repeated addition problem.

Toes by Tens

To begin this lesson, gather the students as a whole group and ask them:

- How many toes are on all of the humans in the classroom?

Next, ask students to share the strategies they used to determine the total.

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

You might consider recording these strategies on a poster or interactive white board to demonstrate to students that there are multiple ways to solve a word problem. A variety of solution strategies may include:

- Count by 10s
- Count by 1s
- Count by 5s
- Other innovative ways students use to come up with solutions.

As you listen to students' responses, try to make the following connections:

- Connection between counting by 10s and addition 10 + 10 + 10 + 10. . . .
- Connection between the answer and the number of students. For example, if there are 23 students, there will be 230 toes (23 with a 0 in the ones place).

As an extension, construct a table relating the number of students in the class to the number of toes. Here is an example:

Number of Students	Number of Toes
10	100
11	110
12	120
13	130
14	140

As you construct the table, use the terms **row** and **column** to explain to students how to label and fill in the table. Start and end the table wherever you choose, including the actual number of students as one of the values in the table. Ask:

- What is the relationship between the number of students and the number of toes?

Sample answer: *The number of toes is the number of students with a zero in the ones place.*

- Why does the number of toes always have a zero in the ones place?
  Sample answer: *When you count by 10s starting at a multiple of 10, each subsequent number has a zero in the ones place.*

Talk Move: *Use Revoicing. Restate students' responses to clarify their answers.*
Talk Move: *Use Repeat/Rephrase. Ask students to restate what a classmate shared.*

As a challenge, ask students:

- How would this problem change if they were calculating fingers and toes?

Have them add a column for "Number of Fingers" and another for "Total Fingers and Toes." Students might notice that the number of fingers and toes is a sum of the two quantities or that it is two times the number of toes (or fingers).

Next, have students determine the number of fingers, number of toes, and the total fingers and toes for 4 people. Use this example to illustrate several equivalent mathematical expressions. Each boxed-in expression below represents one possible way of completing the task to find either the number of fingers or the number of toes.

Content Note: *Students do not need to come up with multiplication strategies if they have not yet been formally introduced to the operation.*

$$\boxed{10 + 10 + 10 + 10} = \boxed{4 \times 10} = \boxed{5 + 5 + 5 + 5 + 5 + 5 + 5 + 5} = \boxed{5 \times 8} = \boxed{40}$$

Emphasize how important it is to use an equal sign only when two or more quantities are equal. In this case, each of the boxed expressions has a total value of 40. If students do not come up with each of the methods boxed in above, show them another method and ask if they can explain why it works. A sample discussion illustrates the case of adding multiple 5s:

Teacher: I was thinking about the problem in this way. (*Writes 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 on the board.)* Can anybody explain why I might have thought this way and whether or not it would work?
***Benjamin*:** Well, you probably thought about each hand.
Teacher: There are only 4 people and I have eight 5s. Can you add on to your thought?
Benjamin: If there are 4 people that means there are 8 hands. And each hand has 5 fingers. So, you need to add the 5 eight

times or add 8 five times. But it doesn't matter because you still get 40 fingers at the end.

Teacher: Does somebody want to agree or disagree with Benjamin's explanation and explain why? *(Reasoning talk move)*

Nina: I agree with Benjamin. I also noticed that instead of writing all those 5s, you could just multiply 5 and 8.

Teacher: So, Nina thinks that adding eight 5s is the same as multiplying 8 and 5. Thumbs up if you agree.

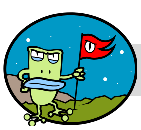
Talk Move: *Use Reasoning. Ask students if they agree or disagree with the other student's strategy and more importantly why they disagree or agree.*
Talk Move: *Use Adding On. Ask students to add more to a classmate's response.*

A similar discussion should guide the case where students examine the total number of fingers and toes among 4 people.

As an optional challenge for students, try reversing the problem using a scenario like the following:

> *It is a beautiful, sunny day and several kids and their parents are at the playground. Altogether, there are 410 fingers at the playground. How many people are at the playground? Explain your thinking.*

Reversing the problem is an effective strategy for demonstrating the inverse relationship between multiplication and division even if students have not yet been formally introduced to either concept. They can use their knowledge of repeated addition or repeated subtraction to answer the question. If 410 is an overwhelming number for some students to work with, consider giving groups of students different numbers of fingers for this problem and having them explain to the rest of the class how they found their answers.



Explore

2. **The Planet Nine Aliens Arrive**

Have students open to *Getting to Know Planet Nine Aliens* Student Page **[SMN pages 143-149]**. Read the scenario at the top of the page and ensure students' understanding of the problem. Using the information provided, fill in the first few rows with students together. Encourage students to use different strategies to help them find their answer. Ask students what strategies they might use. Model how students can create a picture in their minds, do a think-aloud, draw a picture, use repeated addition, skip-count, or write number sentences.

Before students begin working, discuss the difference between columns and rows as they need this information for the first question. Next, have students complete *Getting to Know Planet Nine Aliens* Student Pages **[SMN pages 143-149]**.

Collaborate and Communicate

The focus of the class discussion will be on the question, "If another group of Planet Nine aliens has 12 toes each, how many would be in a group with 60 toes? How do you know?" While students will have worked on this question on their worksheets, they should present their thinking as a small group on a whiteboard or large chart paper. Focus on students' ideas and help them present them logically for others to be able to follow their train of thought, similar to what mathematicians do. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group rounded 12 to 10 and estimated that there would be a little less than 6 Planet Nine aliens.*	**B.** [Possible response] *This group decided to subtract 12 from 60 until they had no toes left.*	**C.** [Possible response] *This group represented 12 using one big toe (10s) and 2 pinkies (ones) and repeatedly added them until they got to 60.*

Examine and Elaborate

3. Highlight Students' Mathematical Thinking

There are multiple ways to solve math problems, so mathematicians have to present their ideas, including unique ones, in a logical way so others can follow their way of thinking. Students should eventually realize that the various ways groups validly solved this problem result in the same answer: five 12-toed Planet Nine aliens have a total of 60 toes.

Share and Discuss

The following sample dialogue showcases how to ensure students present their ideas logically. They naturally will get better at this over time, and mathematical communication becomes easier when they have a solid grasp of the underlying mathematical concepts in addition to an understanding or over-focus on just the process of figuring out a possible solution.

Teacher: Alright, who wants to share with the class how their group figured out an answer?
Luca: I do! (see Response C).
Teacher: Ok, could you explain your answer for the class using a method that you think will help your classmates understand what your group did?
Luca: I'll draw it on the board. First, because these Planet Nine aliens have 12 toes, we made 1 big toe equal to 10 and 2 pinkie toes each equal to 1.
Elizabeth: So, what did you do next to find out how many Planet Nine aliens there were?
Luca: We just kept adding 1 big toe and 2 pinkie toes over and over until we got to 60. We had to add them together 5 times to get 60 toes so that's how we knew there were 5 Planet Nine aliens.
Teacher: Thank you for sharing Luca. Could someone comment on how you understood what Luca's group did?
Sia: Luca explained the steps they used when they were finding the number of Planet Nine aliens so we could understand what they were thinking.
Teacher: Did anyone else notice something Luca did to make it easier to understand?
Taijah: He wrote it on the board and used pictures of toes to show what they did.

Throughout this dialogue, the emphasis should be on students explaining their process to each other so that their peers may understand how they found their answers. Within this example, Elizabeth outwardly spoke up to clarify Luca's process. If this does not happen on its own, guide students to present their answers logically and accurately so everyone can understand and learn from their examples.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. Consider having the students draw a picture of the Planet Nine alien as they complete the *Planet Nine Alien Visit* Student Page **[SMN page 149]**. The concrete drawing may help them if they are using repeated addition to complete the page.

Some students may benefit from further challenge of the lesson concepts. As students answer questions during the lab, encourage students who are comfortably noting repeated addition to include the multiplication sentences in their answers.

Students can also be asked to consider counting by 10s, but with a different starting point to emphasize how increases in 10 function in a base 10 system. For example:

- Ask students to count by 10s starting with 2.

2, 12, 22, 32, 42, . . .

- Ask students to count by 10s starting with 27.

27, 37, 47, 57, . . .

- Differentiate for students who grasp the idea quickly by asking them to start counting by 5s or 10s starting with ½.

By 5s: ½, 5 ½, 10 ½, 15 ½, . . .
By 10s: ½, 10 ½, 20 ½, 30 ½, . . .

For all students, this lesson offers opportunities for students to create their own species of Planet Nine aliens. As they work on creating their Planet Nine alien challenge them to consider why the Planet Nine alien might need a different number of eyes, fingers, or toes. Next, ask students if anyone encountered a challenging problem while creating their own species. Find out if other students can do the calculations related to this species.

Content Note: *Students may enjoy having pictures of their own alien species displayed in the class.*

Debrief and Look Ahead

4.

Debrief Content and Skills

Ensure that students have a solid understanding of the relationship between repeated addition and multiplication. Ask the students:

- What strategies did you find helpful while working on today's problems?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on strategies that mathematicians use when they become confused or stuck on a difficult problem. Review some of the ideas students brainstormed at the beginning of class and offer examples of how students acted like mathematicians while persevering through difficult problems. Give students time at the end of the lesson to create an artifact for the time capsule that demonstrates how they persevered through difficult problems.

Look Ahead

Students continue to explore multiplication through an examination of the Planet Nine alien species. The next lesson goes beyond calculations, asking students to use a multiplication algorithm to represent problems.

Assess

5. **Practice Comparing Strategies**

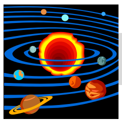
Use questioning techniques to determine which strategies students used to determine totals for their tables. This information can be transferred to the process of solving multiplication problems.

Getting to Know Planet Nine Aliens—Homework Student Page **[SMN page 151]** may also be used to assess students' progress.

Multiplication Madness—

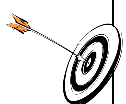
Seeing Stars

LESSON 8



Big Ideas

Multiplication allows us to write repeated addition problems more concisely. It can also be represented using pictures, symbols, or words.

Lesson Objectives 	• Students will be able to write a multiplication algorithm from a pictorial representation of a problem. • Students will be able to create a pictorial representation of a multiplication problem to solve it. • Students will apply the commutative property of multiplication when solving problems.
Materials 	• Student Page—*Planet Nine Alien Manipulatives* [SMN page 153] • Student Page—*Planet Nine Alien Multiplication Tables* [SMN pages 155-161] • Student Page—*Eye Love Multiplication!* (**OPTIONAL**) [SMN page 163] • Student Page—*Multiplication Chart* [SMN page 165] • Student Page—*Picture This!* [SMN page 167] • Student Page—*Picture This! Your Turn!* [SMN page 169] • Student Page—*Planet Nine Alien Manipulatives for Picture This!* [SMN page 171] • Student Page—*Multiplication Problems—Homework* [SMN pages 173-175] • Scissors • Glue • Unifix cubes or circle tiles Remind students that manipulatives are available to use strategically and appropriately.

Mathematical Terms	• **Commutative Property of Multiplication:** The order that factors are multiplied does not change the product. • **Factors:** The numbers in a multiplication problem that are multiplied together to arrive at the product. • **Multiplication:** A mathematical operation in which a number is added to itself a certain number of times. • **Product:** The answer to a multiplication problem.
Selected Mathematical Practices	• MP1: Make sense of problems and persevere in solving them • MP4: Model with mathematics

Differentiation	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• tiered activities
	• formative assessment
	Process
	• 4Cs (21st Century Skills)
	  ○ creativity
	  ○ critical thinking
	  ○ communication
	Product
	• oral, visual, and written opportunities
	• multiple ways to demonstrate knowledge, understanding, and skills
	Learning Environment
	• whole group/small group/individual instruction

Lesson Preview

Students begin exploring multiplication tables by considering Planet Nine aliens with different numbers of eyes. Manipulatives are provided to aid visual learners in constructing multiplication representations. Students also create their own story problems that can be represented and solved with multiplication.

Launch

1. **Thinking Like Mathematicians**

As a whole group, discuss how mathematicians react when they disagree about a solution. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask students:

- How do mathematicians react when they disagree about a solution?
- How might discussing disagreements help mathematicians?
- When the Planet Nine aliens arrive, how might they know we are mathematicians by the way we react when we disagree about solutions?

If students struggle to generate ideas, explain that when mathematicians disagree, they listen respectfully to others' perspectives and think about why their answers are different.

Representing Multiplication

Tell students that their Planet Nine alien friends love to count things and they are very curious to know the answer to the following question. However, the Planet Nine aliens need their human friends' mathematical assistance. Ask:

- In a group of 7 Planet Nine aliens each with 2 eyes, how many eyes are there all together?

Give students time to respond to the question without referring to the strategies discussed in Lesson 7. Encourage students to find multiple solution methods, including: repeated addition, drawing a picture of the situation, counting by 2s, or other solutions students may offer.

Next, introduce the multiplication notation of the algorithm 7 x 2 = 14 to solve this problem. Teach students explicitly the concept of **multiplication** and how it can be solved using the following methods. When presenting each method, frame examples as the work of other third-grade students or classes. Multiplication is an operation in which a number is added to itself a certain number of times.

1. **Using Pictures:** Show students pictures of 2-eyed Planet Nine aliens grouped as shown below using a copy of the *Planet Nine Alien Manipulatives* **[SMN page 153]**.

2. **Using Repeated Addition:** 2 + 2 + 2 + 2 + 2 + 2 + 2 = 14

   The quantity 2 added 7 times

3. **Using the Multiplication Algorithm:** 7 x 2 = 14 *(Read: 7 times 2 equals 14)*. Teach students that the numbers 7 and 2 are called **factors**, or the numbers that are multiplied together in a multiplication problem and the number 14 is called the **product** or the answer to a multiplication problem.
4. **Verbal Model:** Demonstrate how to say the multiplication algorithm within the context of the word problem. Say to students, "The number of Planet Nine aliens times the number of eyes on each Planet Nine alien equals the total number of eyes."

Understanding the Commutative Property of Multiplication

The Planet Nine aliens are at it again with their curious questions about counting objects in the universe! To help students understand the commutative property of multiplication, read the following problem to the students:

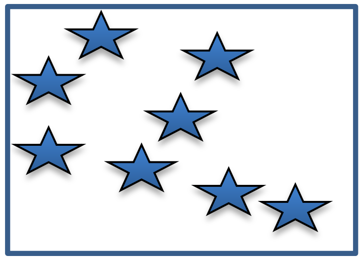
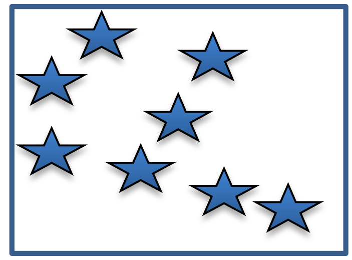
> *The Planet Nine aliens were stargazing and spotted 2 constellations. Each was made up of 8 stars. How many stars were in the constellations?*

Give students time to think of multiple ways to solve this problem and share their ideas as a whole group.

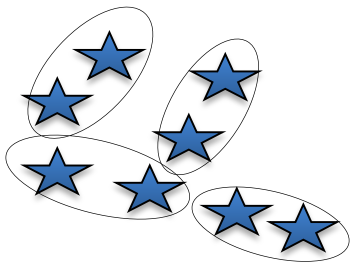
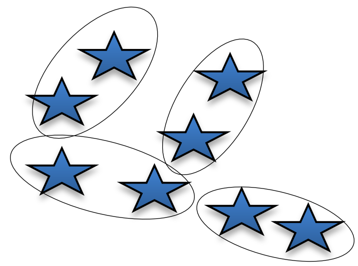
Talk Move: *Use Wait Time. Give students extra time to think about their answers.*
Talk Move: *Use Reasoning. Ask students if they agree or disagree with the other student's strategy and more importantly why they disagree or agree.*

Discuss the following strategies if the students did not mention them. Present them as strategies used by other third-grade students and emphasize the mathematical language.

1. **Inverse of Multiplication Algorithm:** 2 x 8 = 8 x 2 = 16. Discuss with students the idea that two groups each with eight objects produce the same product as eight groups each with two objects. Explain to students that this demonstrates the **commutative property of multiplication**.
2. **Using Pictures to Help Students Understand the Commutative Property:** Display 16 stars in two groups for the students to see. Show students how to use pictures to represent 2 x 8 = 16, as in the following:

Next, show students how to use pictures to represent 8 x 2, or eight groups of two, as in the following picture.

Explain that either grouping strategy will still produce the same product of 16.

3. **Verbal Model:** Elaborate on the description of the commutative property of multiplication by discussing the inverse of a multiplication number sentence within the context of a word problem. For example, model this to students by saying: "The total number of constellations times the number of stars in each constellation equals the total number of stars; and, the number of stars in each constellation times the number of constellations equals the total number of stars." Allow students an opportunity to practice saying a similar statement to a partner.

Explore

2. Organizing Multiplication Facts in a Table

In this Explore activity, students will be developing a conceptual understanding of multiplication using manipulatives to create multiplication number sentences. Ask students to turn to the *Planet Nine Alien Manipulatives* Student Page **[SMN page 153]**. Provide students with scissors and instruct them to cut out each Planet Nine alien on the manipulatives page or they could use Unifix cubes or circle tiles to represent the aliens.

Next, students complete the *Planet Nine Alien Multiplication Tables* Student Pages **[SMN pages 155-161]**. Students are to group the cut-out pictures of the Planet Nine aliens to determine how to write the multiplication number sentence for the total number of eyes for each group of Planet Nine aliens. Some of the rows have already been filled in. Depending on the level of your students, you may need to demonstrate how to use the cut-out pictures to write the multiplication number sentence.

Before they begin, model how to fill in the first few rows of the table on **SMN page 155**. Include the following in your discussion:

Discuss the concept of zero by asking:

- Why does it make sense that the product of each multiplication problem containing zero is zero?
  Sample answer: *Since there are no Planet Nine aliens, the total number of eyes has to be zero. It doesn't matter that each Planet Nine alien has 2 eyes if there are no Planet Nine aliens in the group.*
- Do you think a number times zero always equals zero?
  Look for students to defend their answers by providing additional cases such as the following: If the Planet Nine aliens had 5 eyes and there were no Planet Nine aliens, what would the total number of eyes be? What if they had 100 eyes each? How many eyes are in a group of zero Planet Nine aliens?

For more complex questions, see the Differentiate Further Section.

Collaborate and Communicate

Have the class discuss an extension that addresses multiples of five: "Can you get exactly 52 eyes in a group of 5-eyed Planet Nine aliens? Why or why not?" Small groups should record their thinking on a whiteboard or large chart paper. Focus on students' ideas and have them try to represent them in different ways to ensure others understand them. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group used pictures of eyes and added them repeatedly and "skipped over" 52.*	**B.** [Possible response] *This group thinks that you can get 52 eyes since 5 is a digit in 52.*	**C.** [Possible response] *This group listed the equations representing the 5s multiplication table in order starting with 5 x 1 and ending with 5 x 11.*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**

Mathematicians regularly think about whether their ideas are valid as they attempt to solve problems. Students should realize that there are multiple solutions to problems and they should consider if the ones that are presented mathematically hold true.

Share and Discuss

The following sample dialogue highlights how to respectfully disagree with others' contributions and how to build off of others' ideas. The teacher positions students to determine the soundness of others' ideas rather than correcting the students herself.

Teacher: Alright class, let's see what everyone came up with. Blake, what did your group come up with (see Response B)?

Blake: We thought because there was a 5 in 52, that we could get 52 eyes.

Teacher: Ok, so it sounds like you noticed that 5 is a digit in 52 and concluded that 5 would go into 52. Did anyone solve this problem differently? *(Revoicing talk move)*

Dallas: Yes, we drew out pictures of eyes and counted them.

Teacher: Ok, so you have a different answer then the other group?

Dallas: Yes, I think we did it the right way.

Teacher: Well just because the process of finding the answer was different, doesn't mean it was wrong. What did your group get as an answer, Rory (see Response C)?
Rory: We wrote out multiplication equations starting with 5 x 1 and 5 x 2, etc.
Teacher: Ok, how about if someone from each group comes up and shows their work. [Students share their work on the board]. So, it seems that everyone did it differently. What do we think?
Dallas: I think Blake's group is wrong.
Teacher: Let's try saying it a different way. Do you disagree with how they arrived at the solution? The process is a very important component of the math problem, more so than just the right or wrong answer.
Dallas: Yes.
Teacher: Why? *(Reasoning talk move)*
Dallas: Well, just because 52 has a 5 in it does not mean that there are 52 eyes.
Teacher: Ok, so how can this problem be figured out with the 100s chart?
Rory: Oh, I know! The 100s chart shows that at the end of the numbers, they have a 5 or a 0, but not at the beginning of the number. . . .
Teacher: Can you be more specific?
Rory: Multiples of 5 would be like 10, 15, 20 [points to the chart].

Students should be encouraged to disagree with the process of how someone arrived with the solution, rather than simply disagreeing with their classmates. This dialogue demonstrates how a teacher can have students be more explicit when explaining their reasoning and why they disagree with another solution. Furthermore, it shows how students can disagree appropriately with peers by using evidence to support their claims.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. For this lesson, emphasize that students should use concrete models (e.g., pictures), if needed (item number 8 on the unit pretest can be used to see if students focus on concrete models or are able to represent the information numerically). Additionally, when students begin to create their own word problems, they may need additional support. First, you may want groups of students or the class to brainstorm games the Planet Nine aliens might play, clothes they might wear, and food they might eat. The students can use that information to create the contexts for their word problems. You have ___ Planet Nine aliens with ___ eyes each.

They need _______ for each of their eyes. How many _______ do they need?

Some students may benefit from further challenge of the lesson concepts. As they create their own problems, encourage them to use more complex numbers such as 7, 8, and 9 in their word problems.

You can also increase the complexity of the questions:

- If there are 4 Planet Nine aliens, each with zero eyes, how many eyes would there be all together?
- If zero Planet Nine aliens with 4 eyes each are represented by 0 x 4, how would you represent 4 Planet Nine aliens with no eyes?
- How are 0 x 4 and 4 x 0 different with respect to this problem? How are they similar?

To provide an optional challenge, have students test their skills using the *Eye Love Multiplication!* Student Page **[SMN page 163]**. This gives students the opportunity to write in new multiplication problems.

Multiplication Chart (OPTIONAL)

Introduce students to using the *Multiplication Chart* Student Page **[SMN page 165]**. If available, use an overhead projector or other technology so students can follow along as you explore together.

Known Factors to Find Product

Ask students:

- How might you use the chart to help you find the product of two different factors?

Show students how the multiplication chart can help them find the product for two factors. For example, if they want to know the answer for 5 x 8, they would begin by locating the 5 in the top row and the 8 in the column to the far left. Then they would find the number where the column and row meets in the chart, which is 40. Ask students to use what they know about the commutative property to find the inverse of this multiplication problem (8 x 5 = 40). Students will be happy to know that due to the commutative property, students will only have to memorize half of the facts on the multiplication chart.

Known Product to Find Factors

Next, ask students:

- How might you use the multiplication chart to help you find the factors for a specific number?

Show students how they can use the multiplication chart to help them find two factors for a particular number. For example, if they want to know what two factors make 40, they would begin by finding a 40 in the chart. Then they would find the number at the top of the column that the 40 is in, and the number at the far left of the row that 40 is in.

Take some time to explore the multiplication chart. Ask students to find patterns.

- What do you notice about the numbers going down in each column?
- What do you notice about the numbers going across in each row?

Give each student a laminated copy of the multiplication chart to help them develop the automaticity of their multiplication facts (although a more important goal is to develop conceptual understanding). Encourage students to work on understanding multiplication in contextual situations. Provide additional word problems for practice and ask students to write their own.

To further challenge students, assign the *Picture This! Your Turn!* Student Page **[SMN page 169]** for seatwork, homework, or an assessment.

Debrief and Look Ahead

4.

Debrief Content and Skills

Ensure that students understand how to write a multiplication algorithm given a pictorial representation. Ask students:

- How would a mathematician explain multiplication?
- What strategies do mathematicians use to solve multiplication algorithms?
- Can multiplication only be represented by numbers? How else could a multiplication problem be presented?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on how mathematicians react when they disagree. Review some of the ideas students brainstormed at the beginning of class and offer examples of how students reacted like mathematicians when they disagreed during class. Give students time to create artifacts for the time capsule that demonstrate how they reacted like mathematicians when they disagreed.

Look Ahead

This lesson reinforced the commutative property as students gave different representations for the same problem. In the next lesson, students will explore multiplication arrays and how they demonstrate the commutative property of multiplication.

Assess

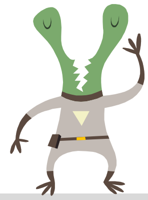
5. **Practice Multiplication With Pictures**

Have students complete the *Picture This!* Student Page **[SMN page 167]**. An additional copy of the *Planet Nine Alien Manipulatives* Student Page **[SMN page 171]** has been provided for the homework. Read the directions on **SMN page 167**. Ask students how they can use the pictures on **SMN page 171** to represent the problem. In the column for 2-eyed Planet Nine aliens, they need to cut out and glue as many Planet Nine aliens as required to get 24 eyes. It is important for students to recognize that they do not need to use all of the pictures to represent each problem. At the bottom of the page, have them write the corresponding multiplication problem (12 x 2 = 24).

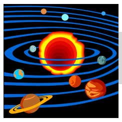
The *Multiplication Problems—Homework* Student Page **[SMN pages 173-175]** may be used as a homework assignment.

Arrays—

Arranging Planet Nine Aliens

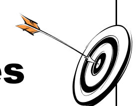


LESSON 9



Big Ideas

Multiplication can be represented using rows and columns in an array. Exploring ways to organize information into arrays will help students further their understanding of the relationship between multiplication and grouping. The ability to recognize these problems aids students' ability to apply multiplication strategies to real-world situations such as finding the area of a room to determine how many square feet of carpet to buy.

Lesson Objectives 	• Students will be able to explain how multiplication arrays are used to represent real-world scenarios. • Students will demonstrate the commutative property through visual models.
Materials 	• Student Page—*Acting Planet Nine Aliens—Arrangement A* [SMN page 177] • Student Page—*Acting Planet Nine Aliens—Arrangement B* [SMN page 179] • Student Page—*Chair Manipulatives* [SMN page 181] • Student Page—*Marching Band Planet Nine Aliens—Fibonacci* [SMN page 183] • Student Page—*Marching Band Planet Nine Aliens—Diophantus* [SMN pages 185] • Student Page—*Marching Band Planet Nine Aliens—Kovalevsky* [SMN page 187] • Student Page—*Collectibles—Fibonacci* [SMN pages 189-191] • Student Page—*Collectibles—Diophantus* [SMN pages 193-195] • Student Page—*Collectibles—Kovalevsky* [SMN pages 197-199] • Pattern blocks • Unifix cubes or circle tiles Remind students that manipulatives are available to use strategically and appropriately.

Mathematical Terms	• **Array:** A way to organize information in rows and columns.
Selected Mathematical Practices	• MP3: Construct viable arguments and critique the reasoning of others • MP8: Look for and express regularity in repeated reasoning

Differentiation	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• tiered activities
	• varied levels of challenge
	• know (information, facts, vocabulary), understand (concepts, big ideas, connections), apply (skills, processes)
	• real-world application
	Process
	• questioning strategies
	• 4Cs (21st Century Skills)
	  ○ creativity
	  ○ critical thinking
	  ○ collaboration
	  ○ communication
	Product
	• oral, visual, and written opportunities
	• multiple ways to demonstrate knowledge, understanding, and skills
	• meaningful and respectful tasks
	Learning Environment
	• flexible grouping
	• whole group/small group/individual instruction

Lesson Preview

In this lesson, students explore multiplication arrays and how they can demonstrate the commutative property. They will begin by exploring how arrays are arranged and then they will explore the commutative property further.

Launch

1. **Thinking Like Mathematicians**

In small groups or pairs, have students write down their ideas on a whiteboard or chart paper. Ask the students:

- How do mathematicians communicate their ideas?
- How do mathematicians react when they disagree about a solution?
- If Planet Nine aliens were to come to our classroom, how would they know that we communicated like mathematicians?

If students are struggling to generate ideas, explain to students that mathematicians share their ideas and solutions respectfully to other mathematicians. Mathematicians are also flexible and open to other mathematicians' ideas so they can learn from one another. After students share their ideas, remind students to refer back to these ideas throughout the lesson.

Classroom Arrays

Invite the students to consider the following problem.

Planet Nine aliens have decided to put on a play in the classroom. They want to fit as many people in the classroom as possible, and they want everyone to have his or her own chair. How many students can they invite to their play?

Display a large floor diagram of a classroom to the whole group on an overhead or electronic board. You can use the chairs provided on **SMN page 181**. Hand each student a picture of one chair. Next, have the students come up to the board and place their chair wherever they would like their guest to sit.

When they are done, ask students:

- Can you describe what you notice about how the chairs were placed on this diagram?

Talk Move*: Use Wait Time. Give students extra time to think about their answers.*

If the students place the chairs all over the room, prompt the students to think about a more organized system. Note how Ms. A. Ray uses Talk Moves in the following discussion about improving the arrangement of the chairs:

Ms. A. Ray: Can anyone think of ways to better organize the chairs?
Sydney: You could line the chairs up.
Ms. A. Ray: Does anyone agree or disagree? Why? *(Reasoning talk move)*

Daevon: Yeah, you could have them in straight lines across the class.
Ms. A. Ray: Who can show us what Sydney and Daevon have just explained? Thank you, Sam. [Sam draws or moves chairs across and down]. Who can explain what Sam just drew?
Samira: He drew chairs going across and down the room with spaces between each row. It looks like there are more chairs in the room now.
Ms. A. Ray: What happened when we put the chairs all over the room?
Lance: It looked like it would be hard to move around.
Hunter: They were spread out too far; now you can fit more.
Ms. A Ray: Let's see what happens.
Kelse: This kind of looks like a 100s chart. It has rows and columns.
Ms. A. Ray: Kelse, that's an interesting connection.

Changing Chairs

Now that the students have explored the idea of organizing the chairs in a systematic manner, students will examine the commutative property of multiplication by seeing that the same amount of chairs fits in a room with the same measurements, but may be oriented two different ways (e.g., 5 rows with 3 columns allows the same number of seats as 3 rows with 5 columns).

Give the students the diagram *Acting Planet Nine Aliens—Arrangement A* Student Page **[SMN page 177]** and the *Chair Manipulatives* Student Page **[SMN page 181]**, and Unifix cubes or circle tiles. Allow students to cut out and arrange the chairs on the diagram of the room.

Have students work with a partner to answer the following question using the diagram of room *Arrangement A*:

- If this is the classroom and these are the chairs, how many friends could the Planet Nine aliens invite to the play?

After students have taken some time to arrange the chairs, have students write the algorithm that represents what they did to determine the total number of friends that could be invited to the play. Next, discuss the students' strategies to arrange the chairs and their algorithms. Be sure to offer one explanation that shows the multiplication **array**. Explain to students that an array is a way to organize information in rows and columns. Here is an example of a possible discussion with the students:

Ms. A. Ray: How did you figure out how many chairs fit in the room?
Drake: I moved the chairs around and figured out that I could fit 6 chairs across and 4 chairs down. I think the room will fit 24 chairs. I added 6 + 6 + 6 + 6 to get 24.
Ms. A. Ray: Do you agree or disagree, Myrionne? *(Reasoning talk move)*

Myrionne: I agree with his total, but I multiplied 4 x 6 to get 24 because there are 4 rows and 6 columns of chairs.

Repeat the process with *Acting Planet Nine Aliens—Arrangement B* Student Page **[SMN page 179]**. Have students write the algorithm that explains what they did. Then ask the students:

- What do you notice about the total number of friends the Planet Nine aliens can invite using Arrangement B?
  Sample answer: *It does not matter whether there are six rows and four columns or 4 rows and 6 columns; the total number of chairs will be the same.*

To follow up on this type of response, remind students that this is an example of the **commutative property**. To support discussions about this property, it is helpful to name the array consistently. For example, the figure below could be called a “3 by 4” or “4 by 3” array. It depends on whether you reference the columns or rows first. Tell the students:

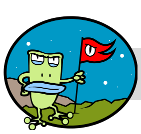
- Traditionally, mathematicians state the row first, then the column. What does the “4” in “4 x 3” represent—is it the row or column?

Combining Rooms

Next, display the following word problem for the students to solve:

> *Planet Nine aliens decided they want to invite more friends to the play. In their school, they have removable walls. If each classroom is the same size, and the Planet Nine aliens open up 3 rooms, how many friends can they invite now? (72 friends)*

Allow students to explore different ways of solving this problem. Students can work in pairs or small groups. Some may just add 24 three times. Others may look at the number of rows and columns. Some students may even add in 2 extra rows because they don’t think you need the aisles in between the classrooms. It is more important to make sure that students are using good reasoning than finding one answer for this question.



Explore

2.

Marching Band Planet Nine Aliens

In this investigation, students will be exploring arrays by completing the *Marching Band Planet Nine Aliens* Student Pages **[SMN pages 183-187]** in their differentiated groups based on the results of their comfort with multiplication (see student work in Lesson 8 to make decisions using the chart that follows).

Next, students will be exploring multiplication and arrays further. Students will be divided into small groups based on their comfort with multiplication. They will complete the *Marching Band Planet Nine Aliens* Student Pages **[SMN pages 183-187]**. Instruct students to carefully consider all the possibilities for arranging the Planet Nine aliens.

Groups Formed by Readiness With Multiplication		
Fibonacci Lab Group	Diophantus Lab Group	Kovalevsky Lab Group
Students who are most comfortable with repeated addition or who need support with addition.	Students who are feeling more comfortable with multiplication, but occasionally rely on repeated addition.	Students who have transitioned to explaining problems with multiplication instead of repeated addition.
Student Names	Student Names	Student Names
Student Pages for Lessons		
Marching Band Planet Nine Aliens—Fibonacci Student Page **[SMN page 183]**	*Marching Band Planet Nine Aliens—Diophantus* Student Page **[SMN page 185]**	*Marching Band Planet Nine Aliens—Kovalevsky* Student Page **[SMN page 187]**

As students work on the investigation, encourage them to use different methods based on their comfort levels. Some groups may prefer to draw the examples, others may want to use counters, some may initially write the addition algorithm, and finally some may be ready to write down the multiplication algorithm (4 x 5, 5 x 4, etc.).

Collaborate and Communicate

Have the class build off their work with arrays to consider the role of the commutative property of multiplication in real-world contexts. Have small groups respond to a new scenario:

The city realized they have a limited budget, so they need to make a smaller stage for the band. There's room for only 12 seats. It's important for the sound to carry across the width of the stage so the whole audience can appreciate the music. The musicians in the back row need to have at least 2 fellow musicians in front of them to be able to harmonize.

Small groups should help the city figure out how they might configure the stage given this scenario. They should record their explanation on a whiteboard or large chart paper. Remind students that mathematicians often use mathematics in real-world contexts that bring up unique constraints for possible solutions. Focus on groups' ideas by asking them questions related to what a city official might inquire about their plans. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group decided that having one row of 12 seats would be the best experience for concert goers.*	**B.** [Possible response] *This group proposed to build a stage that can seat 3 rows of four or 4 rows of three.*	**C.** [Possible response] *This group suggested to construct a stage with 4 seats across and 3 seats deep.*

Examine and Elaborate

3. Highlight Students' Mathematical Thinking

Mathematicians regularly ask themselves if any given answer makes sense. Students similarly should wonder if their solution meets the constraints related to the city budget and performance. Ask students:

- Why should mathematicians check their answers to see if it makes sense?

If students struggle, ask them to consider what would happen at the music performance if they configured the stage incorrectly.

Share and Discuss

The following dialogue presents how mathematical thinking involves regularly checking in about the viability of a solution. The teacher facilitates the interactions among students to help them think in this way.

Teacher: Quinn, would someone from your group like to share what your conclusion was for how the seats should be arranged?

Quinn: Sure, (see Response A) we decided that just having one row of 12 seats would probably be the best for anyone going to the concert.

Teacher: Alright and why is that?

***Quinn*:** Because it is the easiest way to set it up, and they would be seated in a single line, so they wouldn't have anyone sitting in front of them.

Teacher: Ok, thank you for sharing your thinking. Class, what do you think the mayor will say if he comes and sees this arrangement? Why might the mayor agree or disagree? *(Reasoning talk move)*

Payton: I don't know, it might take up a lot of space and our group was thinking that the stage could be built into 3 rows of four, or 4 rows of three.

Teacher: Ok, that is a different solution, and what do you think a mayor will say about that plan?

Payton: Maybe that it will take too much time?

Teacher: Can anyone add to Payton's idea? What are some pros and cons of each idea? *(Adding On talk move)*

The purpose of this dialogue is for everyone to express their group's ideas and analyze the benefits of creating the stage a specific way or the problems they might encounter. Encouraging students to think critically about efficient, realistic solutions helps them approach problems like mathematicians.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. For students who have difficulty organizing the information presented, consider providing students with pattern blocks to organize the arrays.

Some students may benefit from further challenge of the lesson concepts. For students who have been regularly using multiplication, encourage students to describe the array using a multiplication number sentence. If students demonstrate mastery of the Fibonacci or Diophantus pages, and finish their work early, you can provide them with an anchor activity such as completing the Kovalevsky student page.

Ask students to think of other real-world scenarios in which they would need to use arrays to solve problems.

Debrief and Look Ahead

4.

Debrief Content and Skills

Have students reflect on what they have learned today. Ask the following questions to the whole class:

- When would you use arrays in the real world? When are they helpful?
  Sample answer: *They are helpful when the information can be organized into rows and columns. If I were to plant a garden I might have 3 rows of corn and 2 rows of tomatoes. I could draw a diagram to show how many of each plant I have and where each plant is located in my garden.*
- When would organizing information into arrays not be helpful?
  Sample answer: *They are not helpful when you have especially large amounts (at least not to draw). If I wanted to find out how many cookies there are in 50 boxes of cookies an array might take too long to draw.*

Talk Move*: Use Wait Time. Give students extra time to think about their answers.*

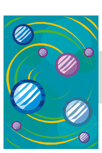
Have students in each group think of at least one real world example and share it aloud.

Debrief Thinking Like Mathematicians

Remind students that today's lesson emphasized how mathematicians communicate with one another, and how they react when they disagree/agree about a solution. Give the students a few minutes at the end of the lesson to create or place an artifact into the time capsule that demonstrates how they communicated with one another, and how they responded to disagreements like mathematicians.

Look Ahead

In the next lesson students will explore the inverse relationship between multiplication and division.



Assess

5.

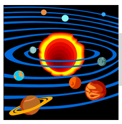
Practice Multiplication

The *Collectibles* Student Pages **[SMN pages 189-199]** may be completed in class as an assessment or for homework. Assign students the most appropriate level for the Student Pages. Students may remain in the same groups as they did for the Explore section or may complete a more challenging level based on their performance in class. The *Collectibles* Student Pages require slightly different thinking for the Fibonacci and Diophantus groups than the *Marching Band Planet Nine Aliens* worksheets.

Multiplication Madness—

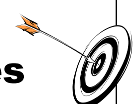
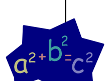
Meet Multiplication's Friend, Division

LESSON 10



Big Ideas

When division and multiplication are taught in isolation, students may struggle to understand the inverse relationship between the two. A thorough understanding of this relationship provides students with additional strategies to use as they confront problems that can be solved using *either* operation.

Lesson Objectives 	• Students will describe the process of division. • Students will write number sentences using both division and multiplication.
Materials 	• Student Page—*Exploring Factors of 12* [SMN pages 201-205] • Student Page—*Finding Factors* [SMN pages 207-209] • 12 pattern blocks for each group of 3 students • Unifix cubes or circle tiles Remind students that manipulatives are available to use strategically and appropriately.
Mathematical Terms 	• **Divisible:** A whole number is *divisible* by another whole number if the remainder after dividing the two numbers is zero. • **Division:** A mathematical operation in which a number is subtracted from itself a certain number of times. • **Sets:** Another term for a group. • **Prime Number:** A whole number greater than one with two factors: 0 and itself.

Selected Mathematical Practices	• MP 4: Model with mathematics • MP7: Look for and make use of structure

Differentiation	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• formative assessment
	• “teaching up” (aim high, provide scaffolding)
	Process
	• 4Cs (21st Century Skills)
	    ○ critical thinking
	    ○ collaboration
	    ○ communication
	Product
	• oral, visual, and written opportunities
	• summative assessment
	Learning Environment
	• flexible grouping

Lesson Preview

Students explore division as the inverse operation of multiplication.

Launch

1. **Thinking Like Mathematicians**
As a whole class, discuss this unit’s vocabulary and how mathematicians would use the mathematical terms. Ask the students the following question:
    - How do mathematicians use mathematical language?
    - Can someone use one of the mathematical terms in a sentence to explain what the term means to the rest of the class?

Remind students that in today’s lesson they will try to use mathematical language when they are communicating their ideas to other students.

Why Are Multiplication and Division Friends?
Tell students that today they will focus on multiplication’s good friend, **division**. Arrange students in pairs or groups of three, and give each group a set of 12 pattern blocks. Ask students to divide the pattern blocks into four equal piles. Ask:

- How many shapes are in each pile?

Explain that division is sometimes used when we have a group of objects and we want to make equal **sets**, or groups, of these objects. In this case, students have divided 12 shapes into 4 equal sets to determine that there are 3 objects in each set or pile.

Show students one way to write the division equation using symbols:

12 shapes ÷ 4 piles = 3 shapes in each pile.

Ask students:

- How could a multiplication problem be used to represent the piles of pattern blocks?

4 piles x 3 shapes in each pile = 12 total shapes.

It is important for students to verbalize what they are doing at each stage. Encourage the use of mathematical statements like the following:

- I have 12 shapes, and I am dividing them into four equal groups. Each group has 3 shapes.
- I have four piles now, each with 3 shapes. So, I have a total of 12 shapes.

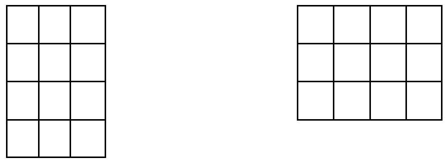
Have students turn to *Exploring Factors of 12* Student Page **[SMN pages 201-205]** and complete #1 based upon this activity. Continue to draw connections between the symbolic and pictorial representations of the problem by emphasizing the words that describe what each number means.

Next, for #2, ask students to create three equal piles with their shapes and write a division and multiplication equation that can be used to represent the situation. As students write their number sentences, prompt them to use statements like those above to describe what they are doing. Ask students:

- How is the grouping of shapes in #2 alike or different from the grouping of shapes in #1?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*
Talk Move: *Use Adding On. Ask students to add more to a classmate's response.*

To make a connection between this lesson and Lesson 9, ask students to take their piles and reform them into arrays. Students should understand and demonstrate that 12 shapes can be made into 3 rows and 4 columns or into 4 rows and 3 columns.



For #3, ask students to put all of their shapes back into a single pile and have them find another way to group their shapes into equal sets that does not use four piles or three piles. Have students write the division and multiplication number sentences. Once students have finished this task, use the following questions to guide a discussion.

- **How many of you made 2 piles? 5 piles? 6 piles? 7 piles? 8 piles? 9 piles? 10 piles? 11 piles? 12 piles?**
  Most students will have either 2 or 6 piles if they followed the instructions given. Some may have 12 piles. Others may have piles that are not equal in number of shapes or that are equal but have a "remainder."
- **What do you think would happen if you tried to make 5 equal piles with your 12 shapes?**
  Students may understand that each pile would have 2 shapes but there would be 2 shapes remaining. (Have a student demonstrate on the whiteboard or large chart paper.) You might also show what happens when you try to organize the 12 objects into 5 rows. Two rows would have to be longer than the rest. Mathematicians would say, "12 is not ***divisible*** *by 5."*
- **Twelve is divisible by which other numbers besides 3 and 4?**
  Twelve is divisible by 2 and 6. Students should examine 2 piles each with 6 shapes and 6 piles each with 2 shapes. Ask students to organize their shapes into an array of equal length rows and columns.

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Finally, have students model one pile of 12 shapes or 12 piles each with one shape. Summarize the information by telling students they have discovered the "factors" of 12. List the factors for students and show how pairs of factors from the list have a product of 12. Students can record this for #6. Explain to students that mathematicians also would say "Twelve is divisible by 1, 2, 3, 4, 6, and 12."

FACTORS OF 12: 1, 2, 3, 4, 6, 12

Explore

2. Finding Factors

Students can be grouped by readiness for the next task. Students with a sophisticated understanding of multiplication should be paired together and given a number with many factors to explore. Students who may have struggled to conceptually grasp multiplication can be paired up and given a number with fewer factors to explore.

Once students have been paired, give each pair a different number to factor using manipulatives. Ask students to use the *Finding Factors* Student Page **[SMN pages 207-209]**. The lists below provide some guidance to help select numbers for students to factor according to readiness groups.

Numbers with fewest factors: 8, 14, 22
Numbers with some factors: 15, 18, 20, 28
Numbers with many factors: 24, 48, 72, 96

Once students have been assigned a number to factor, ask them to count the number of manipulatives equal to their assigned number. Be sure there are no extra manipulatives on students' desks. Then, have students find a way to divide the shapes into equal sets or piles as they did in the Launch section of this lesson.

After students finish finding at least one model for their assigned number, encourage communication across readiness groups by asking students to share their first model verbally. An example for a group with 14 shapes might be, "We have seven piles. Each pile has 2 shapes." The rest of the class can then determine how many shapes the group has in total. Be sure to provide ample wait time for students to figure out some of the larger products.

Have students continue working by finding other ways to create piles with equal numbers of shapes. For students who finish early, increase the level of challenge by having them examine a number with more factors. Examine student pages for missing factors.

Content Note: *Students may often miss the single pile, represented by one times the number. It is more important that students understand the relationship between multiplication and division than it is that they find every factor for the assigned number.*

Collaborate and Communicate

Have the class contemplate the question, "How are multiplication and division related?" Small groups should record their explanation on a whiteboard or large chart paper. Focus on student ideas to help them attend to all concepts of the problem—multiplication and division—which is what mathematicians do. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group built off their understanding of arrays and pointed out how, with division, you could take away one row at a time.*	**B.** [Possible response] *This group focused on symbolic notation and listed several examples of related fact families.*	**C.** [Possible response] *This group noticed that, for multiplication, they passed out equal groups of chips, and, for division, they collected them.*

Examine and Elaborate

3. Highlight Students' Mathematical Thinking

Mathematicians ensure they address all parts of a problem. In this case, students should fully explain the relationship between multiplication and division.

Share and Discuss

The sample dialogue has the teacher facilitate the discussion in a way that builds off students' conceptual understanding before proceeding to the symbolic notation represented by equations.

Teacher: Can anyone demonstrate how multiplication and division are related?

Hayden: I can! Our group gave each person the same amount of chips when we multiplied but when we divided, we collected all of the groups of chips (see Response C).

Teacher: Ok, thanks! Can anyone describe what Hayden just shared in their own words? *(Repeat/Rephrase talk move)*

London: I think I can, so basically they gave out the chips for multiplication, and then collected chips for division, so basically they did the opposite.

Teacher: Does anyone want to add onto London's description? *(Adding On talk move)*

Eden: Basically with multiplication, you are making the number bigger or spreading out the number, while dividing you are making the number smaller or bringing it back together?

Teacher: So, can we say that the **inverse**, or the opposite operation, of multiplication is division?

Hayden: Yes, can division be the inverse of multiplication though?

Teacher: Great question, what does everyone think?

This discussion illustrates how the students can help figure out the inverse of multiplication and division with teacher guidance. If students are having a hard time understanding that both multiplication is the inverse of division and vice versa, try to demonstrate this on the board using symbolic notations, or have a student that seems to understand the concept visually show their classmates what inverse means.

To wrap up the discussion and summarize the learning objectives of the lesson, remind students that, by recognizing division word problems, they can also use multiplication to arrive at a solution. For instance, if students need to solve 12 ÷ 2, they may use their knowledge of multiplication facts to recognize that 2 x 6 = 12, so 12 ÷ 2 = 6. Relate this concept to students' prior knowledge of fact families with addition and subtraction by displaying the following on the board:

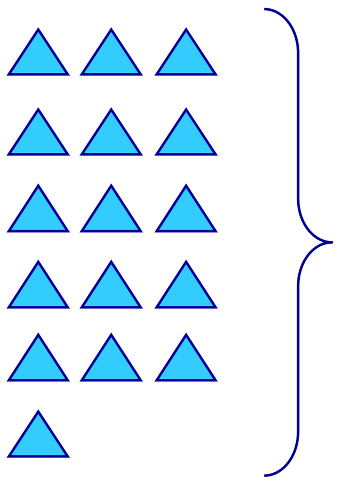
Fact Families for addition and subtraction	Fact families for multiplication and division
2 + 6 = 8 6 + 2 = 8 8 – 2 = 6 8 – 6 = 2	6 x 2 = 12 2 x 6 = 12 12 ÷ 6 = 2 12 ÷ 2 = 6

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. Students who have been using repeated addition might need to see the connections between division and repeated subtraction. For example, they may need to see that 12 minus 6 minus 6 is the same as two equal groups of 6 equal 12.

Some students may benefit from further challenge of the lesson concepts. Ask them to develop a model to show why a particular number *is not* a factor of the number they are factoring. For example, ask a group that is finding factors of 24 to provide a visual model with their manipulatives to show why 7 is not a factor of 24. Stated another way, students would be showing why 24 is not divisible by 7.

As an optional challenge for your advanced math students on the assessment, encourage unique pictorial and symbolic representations like the one pictured below.



(5 x 3) + 1
Five groups of 3 triangles plus 1 extra triangle

It is important to find out which students are able to transition from the use of manipulatives, to the use of pictures, and to the more abstract symbolic representations. If it seems that some students have made this transition easily while others are still struggling to draw connections, you may divide the class up into two groups. The group that has already made the transition can be given the task of finding prime numbers as explained below. The other groups can continue to work on finding the factors for numbers with multiple factors with greater teacher support in highlighting the connections between representations.

Prime Number Task
Explain that a prime number is a number that is divisible only by 1 and itself. In other words, the only factors would be one and the number itself. Tell students that 5 is a prime number. Use manipulatives to demonstrate that a set with 5 items can be made into 5 piles with 1 item each or 1 pile with 5 items. Have students try to make piles with 2 items, 3 items, and 4 items so they can see the remainder each time. Ask students to find three more prime numbers greater than 5, reminding them of the definition.

Debrief and Look Ahead

4. **Debrief Content and Skills**
Have students reflect on what they learned today. Have students discuss the following questions with a partner or in small groups first. Then, discuss these questions as a whole class:

- What are some strategies you can use to solve division problems?
- How can multiplication and division be used to solve the same problem?

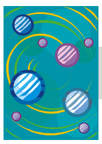
Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians

Remind students that today's lesson emphasized how mathematicians use mathematical terms when communicating ideas and solving problems. Give the students a few minutes at the end of the lesson to create or place an artifact into the time capsule that demonstrates how they used mathematical language when describing division or multiplication.

Look Ahead

In the next lesson, students will have the opportunity to practice building equivalent number sentences using addition, subtraction, multiplication, or division.



Assess

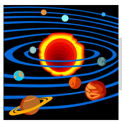
5. Practice Multiplication and Division

Ask students to generate as many multiplication and division equations as they can that include the number 16. Students may use a pile of 16 manipulatives to help them complete this task.

Orbiting Oberon on the Oneida Rocket Ship—

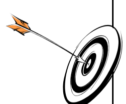
Pre-boarding Task for Ms. Oort's Class (OPTIONAL)

LESSON 11



Big Ideas

Developing flexibility in shifting between different operations is important to building students' mathematical confidence and creativity. Typically, students are given numeric expressions to solve. This lesson offers students the opportunity to receive the answer first instead of the problem, which adds an exciting twist to building skills of adding, subtracting, multiplying, and dividing.

Lesson Objectives 	• Students will understand the concept of equality between multiple number expressions. • Students will create number expressions to arrive at a given solution. • Students will practice their addition, subtraction, multiplication, and division facts.
Materials 	• Student Page—*Number Sentences for 36* [SMN page 211] • Student Page—*Assessment: Creative Mathematicians* [SMN page 213] • Sticker-stars • Unifix cubes or circle tiles Remind students that manipulatives are available to use strategically and appropriately.
Selected Mathematical Practices 	• MP2: Reason abstractly and quantitatively • MP4: Model with mathematics

Differentiation	**Content** • learning objectives • prior knowledge or learner readiness **Process** • 4Cs (21st Century Skills) ○ creativity ○ critical thinking ○ collaboration ○ communication **Product** • oral, visual, and written opportunities **Learning Environment** • whole group/small group/individual instruction

Lesson Preview

In this lesson, students will practice writing numerical expressions to arrive at the number 36. Students will develop their understanding of equality as they flexibly alternate between different operations such as addition, subtraction, multiplication, and division.

Launch

1. **Thinking Like Mathematicians**

As a whole group, have students write their ideas or share their ideas aloud. Ask the students:

- What strategies do mathematicians use when solving a problem?
- How do mathematicians support their answers/solutions?

If students struggle to generate ideas, explain that mathematicians analyze mathematical problems from multiple angles, and believe that math problems can be solved in multiple ways. Mathematicians also show their work when supporting their answers.

How Can We Create the Number 8?

In a whole group format, tell students that their Planet Nine alien friends need their help:

Planet Nine aliens at Oberon Academy are having their annual school holiday party at the Amusement Park. There is a newly-designed Oneida Rocket Ship Ride at the Park. Each grade level has been designated a different math task, which must be completed before boarding the Oneida Rocket Ship. Ms. Oort's grade 3 students have been asked to show that they have been practicing their math skills by creating a number sentence for a secret number that will appear on the *door of the Oneida Rocket Ship before they board. To take a ride on the ship, people have to guess the secret number. Each student could also represent his/her number sentence using sticker-stars. The stickers will create a class mural of constellations on the door of the Oneida Rocket Ship.*

Tell students to imagine that the secret number was 8. The students in Ms. Oort's class would create as many number sentences as possible that show the number 8 and then represent the sentences using pictures. Explain that a third-grade student in another class noticed that possible number sentences are 5 + 3 and 3 + 5 because they total to 8. Ask students:

- Can you think of other number sentences their Planet Nine alien friends could make?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

As students create number sentences, write them with equal signs in between. A sample list is below:

$8 = 9 - 1 = 2 + 2 + 2 + 2 = 4 \times 2 = 24 \div 3 = 2 \times 2 \times 2$

Once a few examples have been listed, ask students:

- Why is it accurate to put an equal sign between each number sentence for 8?

Students should explain that each number sentence has a value of 8. Remind students that an equal sign can be used in between any two or more quantities that have the same value. Have students create 10 more number sentences for the number 8.

Explore

2. **Constellations for 36**

Tell students that Ms. Oort's class now needs their help to create number sentences for the secret number 36. As an optional activity, you may select to have students represent their number sentences using stickers or drawings of stars. These may be used to decorate the door of the classroom.

Students may work independently or with a partner for this activity. Ask students to turn to the *Number Sentences for 36* Student Page **[SMN page 211]**. Students should come up with equations that equal 36. Encourage students to be creative with their responses and try to include all operations. Students should work on incorporating different multiplication and division facts into their expressions, especially facts students struggle with so they have additional practice.

Collaborate and Communicate

Have students answer a question that addresses a computation concept that the class is struggling with or that they can use more practice with. One concept that students might need more practice with is division. If students struggle with division, you could ask the students the following questions:

- What are all the different ways you can divide 36 by two numbers?
- How do you know you've found all combinations?

Small groups should record their explanation on a whiteboard or large chart paper. Mathematicians identify a systematic and organized way to compute, to ensure they have found all true cases. Focus on students' ideas to help them realize this to be the case as they work by emphasizing the second part of the question. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group used the multiplication fact families for 36 to find the related division ones.*	**B.** [Possible response] *This group recorded equations starting by dividing 36 by 1, then 2, and continuing until 6 when they realized they could use the fact families to calculate the answer or quotient.*	**C.** [Possible response] *This group started with a pile of 36 cubes, then asked themselves if they could make one equal group, two equal groups, three equal groups, etc. all the way up to 36.*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**

Mathematicians consider different approaches to problems. Students should realize that however they choose to tackle this problem, doing so systematically and in an organized way will ensure they have identified all possible two-number combinations for dividing 36.

Share and Discuss

This sample dialogue showcases how the teacher has students connect their solutions to those shared by other groups.

Teacher: Can someone share aloud, or demonstrate how their group identified all the possible two-number combinations for dividing 36.

Harlow: Can I go first? Our group used these cubes to help figure out this problem, we counted 36 cubes, and then tried making equal groups (see Response C).

Teacher: Very nice! Justice, would someone from your group describe how you figured this problem out?

Justice: We used the multiplication fact families for 36 to try and find all the numbers that we could use in division too (see Response A).

Teacher: Can anyone tell me how you could combine both of these strategies to come up with the same solution?

Rory: I guess you could use the multiplication fact sheet as a guide to separating blocks?

Teacher: Can anyone add to that? *(Adding On talk move)*

Karter My group wrote down all of the equations, so maybe we could use the cubes to see if it works, and then check the multiplication facts?

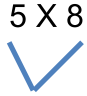
Teacher: Can everyone try all three of these strategies to see if you get the same solution?

Within this dialogue, the teacher is trying to guide the students in making connections between their answers. This helps the students communicate their findings clearly, so that other students may be able to build off of those ideas. Furthermore, it helps students realize that everyone has different strategies and that a student can come to the same solution even if they used another strategy. By sharing process strategies with each other, the whole group can ensure they arrived at all the possible combinations as well.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. As students work on the number sentences for 36, students may need to record the total as they work. They can record all their steps under the line where they record the number sentence (see below).

5 X 8 – 4



40 – 4

Some students may benefit from further challenge of the lesson concepts. For students who need a challenge, encourage the use of mixed operations and parentheses. For example, students might think of 36 as (6 x 7) – 6 or some variation such as (6 x 7) – (3 x 2). Teach students the order of operations for which they must calculate to accurately create these types of number sentences. Any operations within the parentheses should be solved first. Next, division and multiplication should be calculated from left to right. Then addition and subtraction expressions are calculated from left to right.

[OPTIONAL]: Decorate the door of the classroom or a bulletin board with students' visual representations of the number equations for 36.

Debrief and Look Ahead

4. **Debrief Content and Skills**

Ensure that students have a solid understanding that problems can be solved in multiple ways and you can still receive the same answer. Ask students:

- If I gave you the number 10, and asked you to create a problem that equals 10, would everyone create the same problem? Why?
- What operations were used today when solving problems? *Multiplication, division, addition, and subtraction*

Debrief Thinking Like Mathematicians

Remind students that today's lesson emphasized how mathematicians use strategies to solve problems, and how mathematicians support their answers/solutions. Give the students a few minutes at the end of the lesson to create an artifact based on their number sentences. Students could write a quick summary with their group members, on how their group demonstrated mathematical strategies, and supported their solution.

Look Ahead

In the next lesson, students shift from exploring operations to investigating visual repeating patterns.

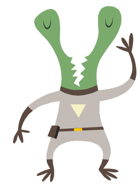
Assess

5. **Practice Writing Multiplication Problems**

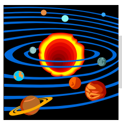
The *Assessment: Creative Mathematicians* Student Page **[SMN page 213]** may be administered as an assessment, seatwork, or homework. In this assessment students are asked to create five multiplication problems and apply the commutative property to each. They will also create two different pictorial representations that could be used to find the answer. As a challenge, encourage students to create a story problem to accompany two of the multiplication problems.

Perplexing Visualizations—

Unlocking the Code

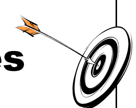


LESSON 12



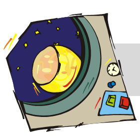
Big Ideas

Mathematicians do not need to look very far to discover a pattern—whether it is in the fabric of one's clothes, in the tiles on the ceiling, on the petals of a flower, or in a list of numbers—patterns are seemingly everywhere! Recognizing and building visual repeating and growing patterns are essential to building a foundation of algebraic understanding.

Lesson Objectives 	• Students will be able to identify, build, and extend repeating and growing patterns. • Students will recognize and describe visual patterns verbally.
Materials 	• Student Page—*Helping Nacci Unlock Her Bag—Fibonacci* [SMN page 215] • Student Page—*Helping Nacci Unlock Her Bag—Diophantus* [SMN page 217] • Student Page—*Helping Nacci Unlock Her Bag—Kovalevsky* [SMN page 219] • Student Page—*Confounding Combinations—Fibonacci* [SMN pages 221-223] • Student Page—*Confounding Combinations—Diophantus* [SMN pages 225-227] • Student Page—*Confounding Combinations—Kovalevsky* [SMN pages 229-231] • Student Page—*Locking Into Creative Combinations—Homework* [SMN page 233] • Unifix cubes or circle tiles Remind students that manipulatives are available to use strategically and appropriately.

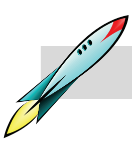
Mathematical Terms	• **Reflection (Flip):** A term for describing the movement of a shape that is flipped to the left, right, up, or down. • **Growing Pattern:** Patterns that increase or decrease in a linear manner. Patterns may “grow” in ascending order such as “2, 4, 6, 8, . . .” or they may “grow” in descending order such as “12, 9, 6, 3, . . .” • **Perfect Squares:** Products that have the same two factors. For example, 81 is a perfect square because 9 x 9 = 81. • **Repeating Pattern:** A repeating arrangement of numbers or objects. • **Rotation (Turn):** A term for describing the movement of a shape in either a clockwise or counterclockwise direction.
Selected Mathematical Practices	• MP3: Construct viable arguments and critique the reasoning of others • MP8: Look for and express regularity in repeated reasoning

Differentiation 	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• tiered activities
	• formative assessment
	• varied levels of challenge
	• real-world application
	Process
	• questioning strategies
	• 4Cs (21st Century Skills)
	  ○ creativity
	  ○ critical thinking
	  ○ collaboration
	  ○ communication
	• hands-on activities/manipulatives
	• connections
	Product
	• oral, visual, and written opportunities
	• multiple ways to demonstrate knowledge, understanding, and skills
	• multiple models and representations
	• meaningful and respectful tasks
	• summative assessment
	Learning Environment
	• flexible grouping
	• whole group/small group/individual instruction
	• growth mindset
	• learning community



Lesson Preview

Discriminating visual patterns is another way to think mathematically. In this lesson, students will explore the nature of growing and repeating patterns visually. Using mathematical terms and having students be precise when explaining their methods will help them make the connection to other patterns they have and will see in this unit.



Launch

1. **Thinking Like Mathematicians**
   Place students into small groups or pairs and ask them to write down or share their ideas with the students next to them. Ask the students the following questions:
   - How do mathematicians communicate their strategies?

- How do mathematicians act when they disagree about a solution?
- When the Planet Nine aliens arrive, how will we show them that we explain our solutions and methods precisely like mathematicians?

Allow at least 4-5 minutes for the students to generate some ideas. Remind students that throughout this lesson, students should try to be as precise as possible so if another student wanted to try their method, they could because it was clearly explained to them.

Exploring Patterns

Tell students that visual patterns appear in surprising places in nature. Ask students:

- What are some examples of visual patterns that you see in the classroom? How about in the real world?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

You may also want to show them a book on the Fibonacci sequence in nature—one option is *Growing Patterns* by Sarah C. Campbell. Reading a book or reviewing a website that discusses the Fibonacci sequence would be a great extension for students to see the connection between math and nature (see Flash Drive for optional resources).

Begin by reviewing the 100s chart from Lessons 5 and 6 where students found patterns in the 100s chart. Ask the students:

- What kinds of patterns did you find?
- How did you find the patterns in the 100s chart?
- What clues did you use?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Tell students that they will be learning how to identify growing and repeating patterns of objects instead of numbers.

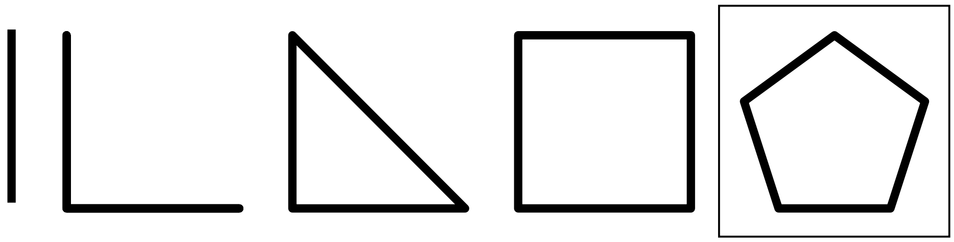
Display the beginning of the two visual patterns for the class below. Do not display the last shape in each pattern yet. Ask students:

- What do you notice?

As students share their observations, guide the discussion to help them understand the difference between a growing pattern and a repeating pattern as described below. Each type of pattern will be presented in a unique way.

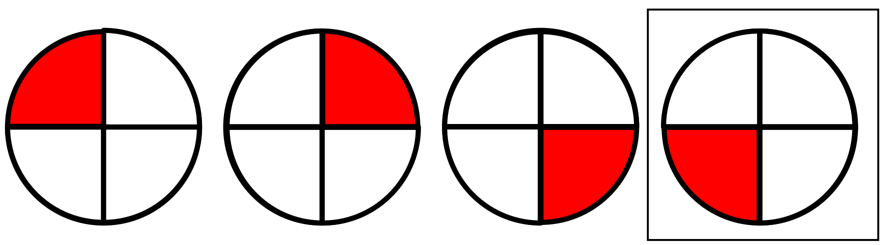
Growing patterns are visual patterns that continue to increase in a linear manner. The basic unit of a growing pattern may need fewer items to identify the pattern than repeating patterns. Growing patterns should be introduced linearly—meaning that to identify the basic unit of the pattern the first shape below should be displayed first, the second shape comes next, and so on. After students have discussed the pattern in the first 4 shapes, ask the students:

- What shape would come next?



For a **repeating pattern**, the items in the basic unit of the pattern need to repeat at least once. As with the growing patterns, ask students:

- What do you notice with this pattern?
- What shape would come next?



Talk Move: *Use Reasoning. Ask students if they agree or disagree with the other student's strategy, and more importantly why they agree or disagree.*

Explore

2. Traveling Planet Nine Alien

In this investigation, students will be recognizing, describing, and extending growing and repeating visual patterns using the SMN pages *Helping Nacci Unlock Her Bag* and *Confounding Combinations* based upon their differentiated groups as informed by students' current level of mathematical communication and comfort with visual patterns.

Have the class discuss the following scenario before dividing them into groups. Tell students that Nacci needs their help:

> *Nacci arrived in the afternoon one day with her luggage in hand. She was quite alarmed when she discovered that her luggage was locked. Poor Nacci forgot the secret combination of symbols that would unlock her luggage! Knowing how absent-minded Nacci can be her mother made sure that all the lock combinations followed a pattern, and she set the first few parts of the lock for her.*

Explain to students that Nacci's luggage has a special type of combination that is set with symbols instead of numbers. Students will work with partners based on their readiness with visualizing flips, slides, and turns in geometry, and their mathematical communication skills. You may also decide to use the Fibonacci Lab Group as a pre-assessment to determine if students are ready for Diophantus or Kovalevsky.

Groups Formed by Readiness With Mathematical Communication and Visual Pattern Recognition		
Fibonacci Lab Group	Diophantus Lab Group	Kovalevsky Lab Group
Mathematical Communication and Visual Pattern Recognition Skills	Mathematical Communication and Visual Pattern Recognition Skills	Mathematical Communication and Visual Pattern Recognition Skills
Student Names	Student Names	Student Names
Student Pages for Lessons		
Helping Nacci Unlock Her Bags—Fibonacci Student Page **[SMN page 215]** *Confounding Combinations—Fibonacci* Student Page **[SMN pages 221-223]**	*Helping Nacci Unlock Her Bags—Diophantus* Student Page **[SMN page 217]** *Confounding Combinations—Diophantus* Student Page **[SMN pages 225-227]**	*Helping Nacci Unlock Her Bags—Kovalevsky* Student Page **[SMN page 219]** *Confounding Combinations—Kovalevsky* Student Page **[SMN pages 229-231]**

Helping Nacci

Next, have students turn to their *Helping Nacci Unlock Her Bag* Student Page **[SMN pages 215-219]**. Review the directions before having students answer the two parts. Once students have finished, elicit responses for the questions. It may be helpful to display the different luggage combinations on the board so that students may refer to them as they explain their answers. Ask students for alternative solution methods to check whether or not some students were able to use different strategies to answer the second question. To further challenge students, ask if the patterns were continued, would they be repeating or growing patterns?

The Fibonacci group works with a pattern of arrows that may be described using the mathematical terminology, **rotation** that also is known as a turn. As the pattern continues, the arrow rotates clockwise by 90 degrees.

During the discussion of Question 2 on the Fibonacci version of *Confounding Combinations* Student Page **[SMN pages 221-223]**, be sure to point out to the whole class that the pattern of dots in the circles is an example of a portion of a special pattern called the **Fibonacci sequence** or **Fibonacci numbers**, as shown below:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, and so on.

Named after Leonardo of Pisa, who was also called Fibonacci, the numbers in the sequence are found by adding the previous two numbers together. So, to determine the next Fibonacci number in the sequence above after 89, add 55 + 89, which equals 144. Interestingly, when these numbers are formed into arrays and grouped together, they form a spiral. This spiral pattern may also be found in nature, such as in the design of a Nautilus shell.

The Diophantus group has a pattern with a filled in portion of the rhombus that reflects from the right side to the left side or vice versa. This **reflection** is also called a flip.

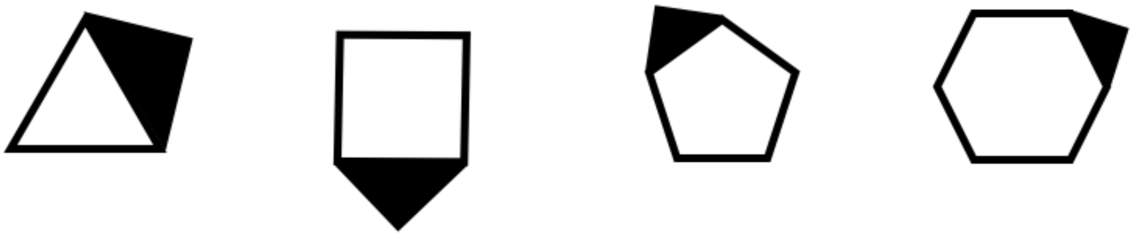
The Kovalevsky group encounters a pattern with symbols decreasing in order of **perfect squares**. Depending upon your students' level of readiness, explain that perfect squares are products that have the same two factors (e.g., 4, 9, 16, 25, 36, and so on). You could also make a connection between perfect squares and the lesson on arrays. For example, explain to students that 81 is a perfect square because if you create an array with 9 rows and 9 columns, it creates a square.

Confounding Combinations

After discussing *Helping Nacci Unlock Her Bag* Student Page **[SMN pages 215-219]**, have students complete the *Confounding Combinations* Student Page **[SMN pages 221-231]** with the same group members. Encourage students to challenge themselves to use some of their new math terminology to explain how they cracked the codes for each pattern and how the patterns are growing or repeating. You may consider posting a word bank for students who need additional support.

Collaborate and Communicate

The class will consider the following new pattern to deepen their knowledge of repeating and growing patterns:



Ask them to:

- Predict the picture for the 8th move. How do you know this will be the picture?

Small groups should record their explanation on a whiteboard or large chart paper. Mathematicians thoroughly check their work. To encourage students to do the same, focus on their initial ideas and ask them to verify that they have uncovered all components of the pattern and how they know this to be true. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group noted a growing pattern where one side keeps getting added to the blank shape, and they counted on to identify a 10-sided shape as the 8th one.*	**B.** [Possible response] *This group discovered that each white shape has the same number of sides as 2 more than its position (e.g., 1 + 2 for the first), and a black triangle on one of its sides (repeating patterns).*	**C.** [Possible response] *This group extended Response B by highlighting how all the sides of each white shape are congruent and rotated so a base is "flat" on the bottom. Also, the black triangle repeatedly is positioned on the top right, then down, then on the top left.*

Examine and Elaborate

3. Highlight Students' Mathematical Thinking

Mathematicians are precise with their work. In this case, students eventually should realize detailed nuances of the pattern.

Share and Discuss

The following sample dialogue showcases how the class realized more detailed components of the pattern. The teacher starts with ideas from groups that were more general and progresses to contributions that are more precise to provide students with the opportunity to experience how to thoroughly investigate a problem.

Teacher: Can anyone tell me what they noticed about this pattern?
Emerson: Our group knew this was a growing pattern, so we knew that one side keeps getting added to the blank shape. We just kept counting until we got to a 10-sided shape. This shape was the 8th shape (see Response A).
Teacher: Great Emerson, your explanation had a great amount of detail, anyone else?
Jesse: Our group realized that each white shape has the same number of sides as 2 more than its position, and a black triangle on one of its sides (see Response B).
Teacher: Ok, that's a great observation, so what was your group's conclusion about this pattern? Can you be more specific?
Jesse: Well we knew it was a repeating pattern.
Teacher: But can someone from your group demonstrate or share with the class what 2 more than its position means?
Jesse: Oh! So for the first pattern we realized it was 1 + 2 then for the second pattern we realized it was 1 + 3 etc.
Hudson: Our group realized the same thing except we also realized that all of the sides of the white shapes are congruent and rotated so that the base is flat on the bottom (see Response C).

Throughout this dialogue the teacher listens to the students' ideas, while also guiding them to be very precise and specific when describing their methods. The teacher emphasizes the amount of detail when students are talking, rather than the correct answer. Students can then build off of each other's ideas when they demonstrate their methods very specifically.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. Students who are struggling to find the pattern can be encouraged to write how the pattern changes from one to the next before trying to determine the overall pattern. For instance, for *Helping Nacci*

Unlock her Bag – Fibonacci, students could note that the arrow points to the left, and then up, and then to the right. Helping the students discriminate all the components of the visual pattern can help them determine how the pattern is changing.

Debrief and Look Ahead

4.

Debrief Content and Skills

Ensure that students have a solid understanding of patterns. Ask students:

- What is the difference between a repeating pattern and a growing pattern?
- How do mathematicians communicate in a precise manner?
- How can mathematicians come to an agreement, even when it seems that they cannot agree on the best solution, and they used different strategies?

Talk Move*: Use Wait Time. Give students extra time to think about their answers.*

Students can answer these questions aloud or make a visual representation to answer the questions. Encourage mathematical language to be used while students present their answers.

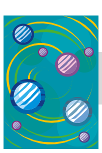
Debrief Thinking Like Mathematicians

Remind students that this lesson focused on how mathematicians are precise when communicating their strategies, and how they settle disagreements when they do not agree on the same strategies and/or solutions. Review some of the ideas students brainstormed at the beginning of class and have students offer examples of how they handled disagreements, and acted like mathematicians throughout the lesson.

Give students a few minutes to create artifacts for the time capsule that demonstrate how they communicated like mathematicians.

Look Ahead

In the next lesson, students will be extending their understanding of visual repeating and growing numerical patterns to numerical repeating and growing patterns.



Assess

5.

Practice Comparing Strategies

The *Confounding Combinations* SMN Student Pages **[SMN pages 221-231]** may be used as an assessment. Ensure that students are able to identify and create repeating and growing patterns.

In addition, assign the *Locking Into Creative Combinations—Homework* Student Page **[SMN page 233]**. Check student work for evidence of understanding the difference between a repeating and a growing pattern.

Repeating Patterns—

Teaching Planet Nine Aliens Mathematical Terms

LESSON 13

Big Ideas

Multiplication serves many purposes in the real world. Students may not realize that multiplication may also be used to quickly make predictions about word patterns. Patterns form in many different real-world scenarios and being able to apply mathematical language to those non-numerical patterns can help students make generalizations.

Lesson Objectives	• Students will be able to identify, build, and extend repeating patterns. • Students will recognize and describe patterns verbally.
Materials	• Student Page—*Teaching Planet Nine Aliens Mathematical Terms* [SMN page 235] • Student Page—*Improving Planet Nine Aliens' Vocabulary—Fibonacci* [SMN pages 237-239] • Student Page—*Improving Planet Nine Aliens' Vocabulary—Diophantus* [SMN pages 241-243] • Student Page—*Teaching Students Planet Nine Alien Words* [SMN page 245] • Unifix cubes or circle tiles Remind students that manipulatives are available to use strategically and appropriately.
Selected Mathematical Practices	• MP2: Reason abstractly and quantitatively • MP8: Look for and express regularity in repeated reasoning

Differentiation

Content

- learning objectives
- prior knowledge or learner readiness
- tiered activities
- formative assessment
- varied levels of challenge

Process

- questioning strategies
- 4Cs (21st Century Skills)
  - creativity
  - critical thinking
  - collaboration
  - communication

Product

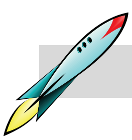
- oral, visual, and written opportunities
- multiple ways to demonstrate knowledge, understanding, and skills

Learning Environment

- flexible grouping
- whole group/small group/individual instruction
- learning community

Lesson Preview

In this lesson, students explore repeating patterns by teaching Planet Nine aliens mathematical terms.



Launch

1. **Thinking Like Mathematicians**

As a whole group, discuss strategies mathematicians use when solving a problem and how they support their answers/solution. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask students:

- What strategies do mathematicians use when solving a problem?
- How do mathematicians support their answers/solutions?
- How may Planet Nine aliens know we are mathematicians by the way that we solve problems?

If students struggle to generate ideas, explain that mathematicians analyze mathematical problems from multiple angles, and believe that math problems can be solved in multiple ways. Mathematicians also show their work when supporting their answers. Refer to ideas students generated during Lesson 12.

Planet Nine Aliens Learn Mathematical Terms

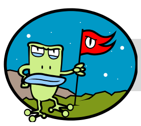
Ask students to look at the *Teaching Planet Nine Aliens Mathematical Terms* Student Page **[SMN page 235]**. Read the first paragraph together. Have students decide which 3 words they should teach the Planet Nine aliens first. Create a list on the board of student suggestions then decide as a class which 3 words will be taught to the Planet Nine aliens. Have students write the 3 words in the same order on the lines provided.

Ask students to work alone or with a partner to complete the two questions on the **SMN Page 235**. Once students have finished, elicit responses for the questions. Ask students:

- Can you think of alternative solution methods?

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*
Talk Move: *Use Revoicing. Restate students' responses to clarify their answers.*

Check whether or not some students were able to use more efficient strategies to answer the second question. This will help guide differentiation choices in the next section.



Explore

2.

Increasing Planet Nine Aliens' Vocabulary

In this section of the lesson, students will be working on one of the levels of the *Improving Planet Nine Aliens' Vocabulary* Student Pages **[SMN pages 237-243]**. Students with strong multiplication skills and students who identified alternative methods in the Launch section should work on the Diophantus version of this task **[SMN pages 241-243]**, which focuses on a repeating pattern with 6 words. Other students should begin with the Fibonacci version **[SMN pages 237-239]**, which focuses on a repeating pattern with 5 words.

As students work alone or in pairs on this task, circulate the room to listen as students describe their thinking. Listen for unique ways that students use to find the word in the pattern. Some questions to guide this listening include the following:

- Are students talking about writing out every word to find the 20th term? The 99th term?
- Are students counting by 5s or 6s and then "adding on" or "taking away"?
- Are students using multiplication?
- Are students drawing pictures to support their thinking?

Examining Students' Responses

Ask a student or pair of students who worked on the Fibonacci version to share the 5 words they chose. List the 5 words on the board. Ask a student from another Fibonacci group to answer the first question using the 5 words on the board. (This will be different than what they have on their paper unless they chose the same word in the fifth position.) Calling on other groups allows students to see that the methods they used can be applied to a different set of words.

Next, ask for a list of 6 words from a student who completed the Diophantus version of this task. Ask *all* students to consider what the 20th word would be for this set of words. Compare and contrast finding the 20th word when there are 5 words versus 6 words on the list. Ask students if one is easier than the other.

Examine the questions related to finding the 99th word using each list of words already on the board. Ask students to share different strategies that could be used to find the 99th word. Highlight as many strategies as possible, asking students to rephrase often to check for understanding of others' methods.

For students who are struggling to identify the 20th word, have them extend the pattern from 5 words (*Improving Planet Nine Aliens' Vocabulary* Student Page **[SMN pages 237-243]**) to the first 10 words. From there, ask students to think about what the 20th word might be.

Collaborate and Communicate

Have the class consider 4 new words: asteroid, comet, meteor, and star. They will work in small groups to answer the following question:

- If you repeat the 4 words over and over again, what will be the 58th word you will say? Explain your thinking.

Have small groups record their explanation on a whiteboard or large chart paper. Since mathematicians sometimes use representations, including symbols and visuals, to communicate their ideas, focus on students' initial ideas and ask if there is a representation that can further clarify their thoughts. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group first wrote the 4 words in lines of 12, then added 12 repeatedly to get to 48, and counted on from there to 58.*	**B.** [Possible response] *This group converted each word to a symbol, then counted off by 10s until they got to 50, and counted by ones to get to 58.*	**C.** [Possible response] *This group worked with the fact that every five times you cycle through the 4 words, you get 20. They repeatedly added 20 to get to 60, then went back 2 words.*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**
One strategy mathematicians use to solve some problems is to find patterns and generalize them. Students should come to realize that there are various strategies to uncover and explain the patterns, which is true not only for this problem, but others as well.

Share and Discuss
This sample dialogue showcases the importance of clearly communicating one's thought process when talking, writing words, and through representations. This is important so that others can determine their validity, which is a key practice for mathematicians engage. The teacher here points out and supports this practice while facilitating the discussion.

Teacher: Alright, can you describe your thinking when you were trying to figure out this problem?
Abynn: I can! I thought the easiest way to do this problem was to make each word a symbol, so that way I could figure out the problem faster (see Response B). After our group made a symbol for each word, we wrote them out 10 times. Then, we kept writing them out 10 times until we got to 50. Then, we counted by ones until we got to 58.
Teacher: Excellent work. What I really liked was the way that you explained each step carefully and logically, so that we were

able to follow along and understand what your group did. Can anyone else describe what your group did with very specific details, so that we could follow along?

Atticus: I can!

Teacher: Alright class, I want you to try and do this on a piece of a paper while Atticus explains his work. Go ahead, Atticus.

Atticus: Ok, our group wrote all of the words out 8 times, and then we realized that every 5 times you cycle through the 4 words, you get 20. Then we kept adding to 20 until we got to 60, and then we just skipped back 2 words (see Response C).

Teacher: Alright everyone, did you write down each step that Atticus said? Were you able to understand each step to solve the problem? *(Wait Time talk move)*

Trinity: Yes, I was able to do it! I like this method!

Teacher: Were there any similarities between Atticus's and Abynn's methods?

Taylor: They both used symbols!

This dialogue emphasizes how students need to communicate clearly and logically for their peers to understand how they solved a problem. The emphasis was placed on how the student explained the method, so that other students could follow along. Once the students were able to follow along, the teacher guided questions to help students realize similarities with how groups solved this problem. Follow-up discussion should focus on the validity of each method, and how students can make it easier for other students to follow along.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. You can encourage students to write out several rounds of the words to see initial patterns, color code the words, use what they know about multiples of the number, work with "easier" multiples (e.g. 5s and 10s), and/or add or subtract to home in on the final element.

Some students may benefit from further challenge of the lesson concepts. You can have students explicitly connect repeating patterns to previous lessons where they identified multiples. For example, for the *Teaching Students Planet Nine Alien Words* Student Page **[SMN page 245]**, students are asked to identify the 54th word. Knowing that 52 is a multiple of 4 can help them determine that bleebee is the 52nd word, so freligo would be the 54th word.

Debrief and Look Ahead

4. **Debrief Content and Skills**
Ensure that students have a solid understanding of how multiplication can be used when making predictions about word patterns. Ask students:
- Can you use multiplication when predicting a pattern? How?
- Are there any repeating patterns within this classroom right now? Where? How is it a pattern? Can you predict what would come next if the pattern continues?
- What strategies were used when communicating the methods that were used when solving the problems?

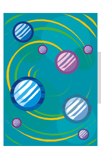
Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians
Remind students that this lesson focused on strategies that mathematicians use to solve problems. Review some of the ideas students brainstormed at the beginning of class and have students offer examples of how they communicated like mathematicians while they worked together during the lesson.

Give students a few minutes to create artifacts for the time capsule that demonstrates how they worked together like mathematicians.

Look Ahead
Students will extend their understanding of patterns to see the similarities and differences between repeating and growing patterns in the next lesson.

⚠ **For Lesson 16, please plan for an individual to come to your classroom, so students have the opportunity to demonstrate their time capsule.**

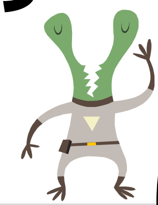


Assess

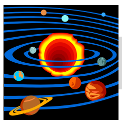
5. **Practice Teaching Mathematical Terms**
Administer the *Teaching Students Planet Nine Alien Words* Student Page **[SMN page 245]** to assess students' comprehension of repeating patterns. This page may be used as a homework assignment or an in-class assessment.

Growing Patterns—

Coasting at the Amusement Park!

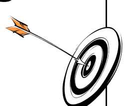


LESSON 14



Big Ideas

Growing patterns are different from repeating patterns in that they show either an increasing or decreasing number of pictures, numbers, or symbols with respect to a function or formula. For example, in the growing pattern 3, 6, 9, 12, 15, the numbers are increasing by multiples of 3. The formula to predict any term (e.g., the 15th term) would be 3 x t or 3t, where t is a variable representing the predicted term. The 15th term in this growing pattern would be 3 x 15 = 45. Determining the formula of a growing pattern allows students to make predictions and generalizations about the predicted term in the sequence.

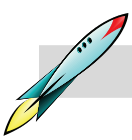
Lesson Objectives 	• Students will identify growing pattern problems that require multiplication to solve. • Students will solve growing pattern problems using visual representations. • Students will explain how to identify and solve growing pattern problems using visual representations. • Students will make predictions with patterns using a variable in a formula. • Students will develop a formula using a variable to describe a growing pattern.

Materials	• Student Page—*Coasting at the Amusement Park! Table* [SMN page 247] • Student Page—*Coasting at the Amusement Park!—Fibonacci* [SMN page 249] • Student Page—*Coasting at the Amusement Park!—Diophantus/Kovalevsky* [SMN pages 251-253] • Student Page—*Rollercoaster Cars Cutouts* [SMN page 255] • Student Page—*Super Challenges* [SMN page 257] • Student Page—*Coasting at the Amusement Park!—Homework* [SMN page 259] • Unifix cubes or circle tiles • Yarn or string • Scrap paper Remind students that manipulatives are available to use strategically and appropriately.
Mathematical Terms	• **Formula:** A rule or function for a pattern of numbers to make a prediction about a specific term. • **Variable:** A symbol or letter that represents a number or amount. For example, *a* is the variable in $4 \times a = 12$ or $4a = 12$.
Selected Mathematical Practices	• MP2: Reason abstractly and quantitatively • MP7: Look for and make use of structure

Differentiation 	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• tiered activities
	• formative assessment
	• varied levels of challenge
	Process
	• questioning strategies
	• 4Cs (21st Century Skills)
	   ○ creativity
	   ○ critical thinking
	   ○ collaboration
	   ○ communication
	Product
	• oral, visual, and written opportunities
	• multiple ways to demonstrate knowledge, understanding, and skills
	Learning Environment
	• flexible grouping
	• whole group/small group/individual instruction
	• learning community

Lesson Preview

Students will be exploring and building growing patterns using concrete visual materials. In addition, students will be introduced to the concept of using variables in a number sentence to predict patterns.



Launch

1. **Thinking Like Mathematicians**
As a whole group, discuss how mathematicians use tools appropriately, and strategically. Consider writing a list of the students' ideas on a whiteboard or chart paper. Ask students:
   - How do mathematicians use tools?
   - Why do mathematicians use tools? What purpose do these tools serve?
   - When our guest comes to see our time capsule, how will they know that we used tools creatively and appropriately?

   If students struggle to generate ideas, explain that mathematicians use tools appropriately. This means that the tools are not to be used as something to play with, but as tools to help figure out the answers to math problems. Tools can help students visualize the math problem they are trying to figure out. An example that was given from another third-grade student could also be mentioned: *Marcus from another third-grade class suggested that yarn can be used to measure the length or width of an object.*

A Rollercoaster Ride at the Amusement Park!

Ask students:

- Can anyone tell me what they remember about repeating patterns?

Talk Move: *Use Adding On. Ask students to add more to a classmate's response.*

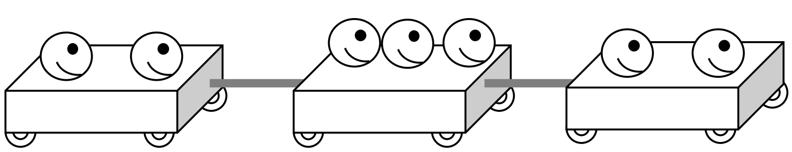
Explain to students that today they will be exploring **growing patterns** in word problems, and how to use multiplication to make predictions in these patterns.

Read the following story to the students:

> *Toxo and his Planet Nine alien friends decided to join their human friends for a trip to the Amusement Park. They all decided to brave the Rocket Rollercoaster together! Toxo had to stretch his neck to peer at the top of the Rocket Rollercoaster because it was so high! As he did, he noticed that the first and last car of the train of connected cars on the rollercoaster could only fit 2 beings. But, the middle car of the rollercoaster could fit 3 beings. Toxo found this observation very interesting. He began to wonder how many beings could fit altogether in the cars of the Rocket Rollercoaster.*

Tell students that they are going to help Toxo figure out how many beings could fit altogether in the 3 cars on the rollercoaster.

Display the following or a similar diagram on the board. Alternatively, you could demonstrate this problem using manipulatives. As a fun option, you could use 2 pieces of yarn or string to form the shape of the rollercoaster and then set the cars on the yarn or string. Tell students that the smiley faces or circular manipulatives represent the beings and the boxes represent the rollercoaster cars (see Flash Drive for the image below to display for the whole group).



Ask the students the following questions:

- How many beings can fit in the first or last car? *(2)*
- How many beings can fit in the middle car? *(3)*
- How many beings can fit altogether in the 3 cars? *(7) Encourage students to explain different strategies for arriving at the total.*

Possible answers might include counting all the smiley faces in the diagram or adding 2 + 3 + 2 = 7 or (2 x 2) + 3 = 7.

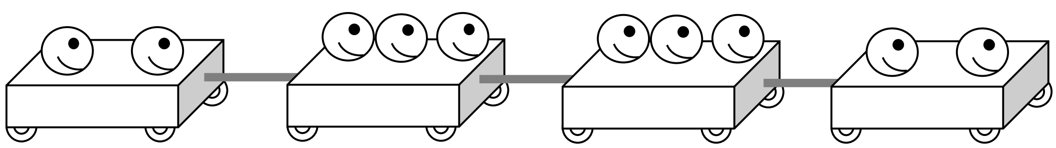
Adding on More Cars

Explain to the students that the Amusement Park decided to add another car to the middle of the Rocket Rollercoaster since it was such a popular ride! Ask students:

- How many beings altogether would now be able to fit in the cars?

Give the students a few minutes to solve this problem with a partner. Students might choose to draw their own diagram on paper, use the manipulatives for this lesson, or use mental math. Encourage students to solve the problem cooperatively with their partners to determine a way to solve the problem.

Have students come to the board to show what the diagram (or model using manipulatives) would look like if one more car was added to the middle of the rollercoaster. The student's diagram should be similar to the following:



Ask the students the following:

- How many beings can fit in all the cars now that an additional car has been added? *(10)*
  Encourage students to explain different strategies for arriving at the total. Possible answers might include counting all the smiley faces in the diagram, adding 2 + 3 + 3 + 2 = 10, or calculating (2 x 2) + (2 x 3) = 10.

Talk Move: *Use Revoicing. Restate students' responses to clarify their answers.*

Next, have students open to the *Coasting at the Amusement Park! Table* Student Page **[SMN page 247]**. Have students record the number of beings that can fit in 3 cars and 4 cars. Ask students:

- What do you notice about the pattern of beings?

Students should be able to explain that the number of beings is increasing by 3 for the middle cars and that this is an example of a growing pattern.

Developing Formulas

Explain to the students that they will create a **formula** to predict the total number of beings that could fit altogether in any given number of cars. Share the definition and example of a formula with the students.

Formula: A rule or function for a pattern of numbers to make a prediction about a specific term (e.g., 4 x A = 12, 4a = 12, 12 = 4 x A, 12 = 4a).

Use the following guiding questions to help students create a formula as a whole class activity.

Teacher: Let's work on creating the formula using a variable to help us predict. What are we trying to figure out with the formula?
Jeff: We are trying to figure out the total number of beings that fit in the cars.
Teacher: Do we have to know how many cars right now?
Chris: You said that we are figuring out a formula for any number of cars.
Ellie: Oh yeah, we are going to use a letter, right?
Teacher: Who can add on to that? *(Adding On talk move)*
My'asia: Can we use the letter C to stand for the number of cars?
Teacher: Yes, we can use C as our variable. What do we know about the number of beings that can fit on the roller coaster?
Scout: Each car can fit 3 beings, but the first and the last car only have 2 people in each car.
Teacher: How can we use multiplication and a variable to show how to figure out how many beings fit in the middle cars?
Jeff: We can multiply the number of cars by 3.
My'asia: We could write 3 X C.
Teacher: Is there another way to say that? *(Repeat/Rephrase talk move)*
Ellie: 3C.
Teacher: So, is C all the cars or just the middle cars?
My'asia: Just the middle cars.
Teacher: Do you agree or disagree with My'asia's reasoning? *(Reasoning talk move)*
Jeff: I agree because the end cars only have 2 people in each.
Teacher: Now that we have shown how many beings will fit in the middle, how can we show how many will sit on the ends of the roller coaster?
Jeff: You should add 2.
Chris: Yeah, but we need to add 2 to each end so that would be 4.
Teacher: How can we write the formula to show the middle part and the two ends?
Scout: 2 + 3C + 2 = total number of beings.
Chris: Or we could say 4 + 3C = total number of beings.

Depending on the ability level of the students, instead of having the guided group discussion, you may want to challenge the class or a smaller group of students to figure out how to create the formula on their own.

Be sure to explore at least one other formula. Here is another way to solve this problem. Using the **variable *n*** to represent the total number of cars, including the middle and end cars, the formula would be:

$3n - 2$ = Total number of beings

Since each car can fit 3 beings, except for the first and last car, you would multiply 3 by the total number of cars. (*Be sure that students understand that 3*n *means the same as 3 x* n*).* Then you would subtract 2 to account for the difference in the number of beings in the first and last car. In the example above in Step 2, to find the total number of beings in 4 cars, you would complete the formula as follows:

$n = 4$ cars
$3(4) - 2 = 12 - 2 = 10$ beings

If necessary, have students test this formula using a different number of cars before proceeding to Explore.

Explore

2. **Growing Patterns**

In this investigation, students will be extending their knowledge of growing patterns based on the results of the unit pretest and their differentiated groups.

- **Fibonacci Group:** Encourage students to use the rollercoaster car cutouts and circular manipulatives to extend repeating patterns to work out the problems. Also, the *Coasting at the Amusement Park! Table* Student Page **[SMN page 247]** can be used by the students to organize their thinking.
- **Diophantus/Kovalevsky Group:** Encourage and challenge students to use mental math or to use the formula ($3n - 2$) to find the total number of beings.

Groups Formed by Scores on Questions 10, 11, and 12 on the Pretest	
Fibonacci Lab Group	Diophantus/Kovalevsky Lab Group
Scored 0 to 1 point(s) on items combined.	Scored 2 to 3 points on items combined.
Student Names	Student Names
Student Pages for Lessons	
Coasting at the Amusement Park—Fibonacci Student Page **[SMN page 249]**	*Coasting at the Amusement Park—Diophantus/Kovalevsky* Student Page **[SMN pages 251-253]**

Tell students that they now will have a chance to solve growing pattern problems with Toxo the friendly Planet Nine alien as he spends his day having fun at the Amusement Park. Encourage students to be creative in determining strategies to solve the problems. Remind students that there can be more than one way to solve a problem.

Students may work with a partner or in a small group to complete the *Coasting at the Amusement Park!* Student Pages **[SMN pages 249-253]**. Provide students with copies of the *Rollercoaster Car Cutouts* **[SMN page 255]**. To save time, you might consider having the cars cut out prior to the lesson.

Share and Discuss

After the students have completed the SMN pages, bring the class together for a group discussion. Ask the students the following questions:

- How were these rollercoaster problems examples of growing patterns?
  Sample answer: *As we added each middle car, the pattern increased by groups of 3.*
- What did you notice that happened when a new rollercoaster car was added?
  Sample answer: *Three more beings could ride the rollercoaster when a new car was added.*

Discuss the answers to the problems on the *Coasting at the Amusement Park!* Student Pages. Encourage students to share any strategies that they used to find the solutions to the problems. Use Talk Moves to encourage discourse between the students in the class. For example, you might use the Reasoning Talk Move and ask students "Do you agree or disagree? Why?"

Encourage students in the Diophantus/Kovalevsky group to share any formulas that they discovered that helped them to make predictions about the growing patterns problems.

Collaborate and Communicate

The class will work on and discuss an extension of the original problems. "Imagine that there were 40 cars on the Rocket Rollercoaster! How many beings could ride altogether? Remember that the first and last car can only seat 2 beings." Small groups should record their explanation on a whiteboard or large chart paper. Mathematicians at times break down problems to smaller ones to come up with possible solutions. Focus on students' ideas by asking them to clarify how they might have done this. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group built on their experience working with 20 cars and realized that they could add more cars to the middle by multiplying by 3.*	**B.** [Possible response] *This group multiplied the number of cars by 3, then subtracted 1 two times to account for the first and last car.*	**C.** [Possible response] *This group multiplied 40 cars by 2, then added 38 for the extra being sitting in each of the middle cars.*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**

Mathematicians use visual representations to help them uncover patterns. These jottings may not be clear to others at first when they are working out a solution, and they redraw them when presenting them to others so they can understand their thinking. The presence of an audience is something that students should come to understand when sharing their work.

Share and Discuss

This sample dialogue showcases the varied ways math problems can be solved. The teacher highlights the strategies the groups used to make their thinking clear on paper to others.

Teacher: Can someone tell or demonstrate how their group figured out this problem?

Scout: I can! We all went back to the example that we did earlier that used 20 cars, and just realized that we could add more in the middle if we just multiplied by 3! [see Response A.]

Teacher: Hmmm. . . this sounds like a great idea, but I wonder if a growing or a repeating pattern could be found within your solution or method. Could you please share your group's thinking on the whiteboard or large chart paper?

Scout: Ok. [Student goes to the board.]

Teacher: Now that everyone can see their work, do you have any questions? *(Wait Time talk move).*

Tristan: I still do not understand.

Teacher: Scout, can someone from your group start the problem again from the beginning and display it on the white board?

Vick: I can explain it. We started off with 20 cars in the last example. [He draws 20 squares to represent the 20 cars.] Then, we realized since the car at the beginning and the car at the end can only seat 2 beings, we kept those 2 cars the same and realized we could add more cars in the middle that have more seats, so we just multiplied 20 by 3 and added that on.

Tiana: Where did the other 20 come from? I don't see that on your sheet either. Why not 40? Or maybe 38?

Vick: Oh, yeah. We started with what we knew about the first 20 cars. If you have 40, you would just need to add on another 20. The 20 new cars can be put in the middle, like this. [Vick draws a representation to help explain his thinking.]

Teacher: This is an interesting strategy. Let's hear from a few other groups so that we then can look back at them to see if the methods used uncovered a repeating or growing pattern.

This dialogue serves two purposes, to have students be able to clearly communicate their methods and solutions visually, and to have students further understand the difference between a growing pattern and a repeating pattern. If students need further explanation, summarize the definitions of growing patterns, variables, and formulas. Remind students that recognizing a growing pattern will allow them to be able to make predictions about the pattern. Conclude the lesson with what the students discovered about solving growing pattern problems.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. Continue to encourage students who have a hard time keeping track of the pattern mentally to use the concrete manipulatives.

Some students may benefit from further challenge of the lesson concepts. Encourage students to use mental math or to use a formula such as, $3n - 2$, to find the total number of beings.

Students who finish early may work on the four *Super Challenge* cards **[SMN page 257]**.

Debrief and Look Ahead

4.

Debrief Content and Skills

Ensure that students have a solid understanding of the difference between growing patterns and repeating patterns. Ask the students:

- How is a growing pattern different than a repeating pattern? Students answers will vary, but they should be able to discuss the main points of a growing pattern: *A growing pattern shows either an increasing or decreasing number of pictures, numbers, or symbols. Determining the formula for a growing pattern allows for students to make predictions and generalizations about the predicted term in the sequence.*
- How can you predict what will come next in the pattern? *By using a variable in a formula.*
- Did you use tools when explaining your method/solutions?

Talk Move*: Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians

Remind students that this lesson focused on how mathematicians use tools appropriately, and strategically. Review some of the ideas students brainstormed at the beginning of class and have students offer examples of how they acted like mathematicians while using tools during today's lesson.

Give students a few minutes to create artifacts for the time capsule that demonstrate how they used tools like mathematicians. Remind students that they will need to clearly communicate what they learned, using the artifacts, to the guest in Lesson 16.

Look Ahead

In Lesson 15, students will continue to explore the concept of growing patterns by solving problems using a table. Students will also have an opportunity to learn more about formulating a rule or formula to make predictions about terms in a growing pattern.

For Lesson 16, please plan for an individual to come to your classroom, so students have the opportunity to demonstrate their time capsule.

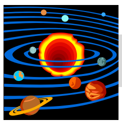
Assess

5. **Practice Growing Patterns**

The *Coasting at the Amusement Park!—Homework* Student Page **[SMN page 259]** homework could be used as a homework assignment or as morning work to be completed in class. If you choose to send this home as homework, include copies of the rollercoaster car cutouts for students who would like to use them.

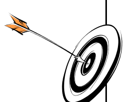
Growing Patterns—
Cookies That Are out of This World!

LESSON 15



Big Ideas

This lesson is a continuation of exploring the concept of growing patterns. The information from growing pattern problems can be used to make predictions or generalizations about a pattern. Mathematicians determine formulas for growing patterns that allow them to make predictions and generalizations about the next number in a sequence. Predicting terms in a pattern may be especially useful for budding chefs in the kitchen when doubling or even quadrupling a recipe!

Lesson Objectives 	• Students will identify, build, and extend a growing pattern to solve a problem. • Students will explain how to make predictions about a term in a pattern. • Students will solve growing pattern problems using a table.
Materials 	• Student Page—*Toxo's Out of This World Ideas!—Increasing and Decreasing Patterns* [SMN pages 261-263] • Student Page—*Toxo's Out of This World Cookie Recipe!—Fibonacci* [SMN pages 265-267] • Student Page—*Toxo's Out of This World Cookie Recipe!—Diophantus* [SMN pages 269-271] • Student Page—*Toxo's Out of This World Cookie Recipe!—Kovalevsky* [SMN pages 273-275] • Student Page—*Toxo's Out of This World Ideas!—Homework* [SMN pages 277-279] • Student Page—*Super Challenges* [SMN pages 281-283] • Unifix cubes or circle tiles Remind students that manipulatives are available to use strategically and appropriately.

Mathematical Terms	• **Formula**: a rule or function for a pattern of numbers to make a prediction about a specific term. • **Terms:** Numbers in a pattern or sequence. • **Variables:** A symbol or letter that represents a number or amount. For example, *a* is the variable in $4 \times a = 12$ or $4a = 12$.
Selected Mathematical Practices	• MP3: Construct viable arguments and critique the reasoning of others • MP8: Look for and express regularity in repeated reasoning

Differentiation	**Content**
	• learning objectives
	• prior knowledge or learner readiness
	• tiered activities
	• formative assessment
	• varied levels of challenge
	Process
	• questioning strategies
	• 4Cs (21st Century Skills)
	  ○ creativity
	  ○ critical thinking
	  ○ collaboration
	  ○ communication
	Product
	• oral, visual, and written opportunities
	• multiple ways to demonstrate knowledge, understanding, and skills
	Learning Environment
	• flexible grouping
	• whole group/small group/individual instruction
	• learning community

Lesson Preview

Students use symbols, tables, mental math, or formulas to identify, extend, and build growing problems within the context of story problems.

Launch

1.

Thinking Like Mathematicians

In small groups, have students review how mathematicians communicate their solutions and/or methods, and how mathematicians persevere through difficult problems. Consider having them record their ideas or share them aloud with the whole class. Ask students:

- How do mathematicians communicate?
- How do mathematicians persevere through difficult problems?
- How have we been communicating like mathematicians?

If students struggle to generate ideas, remind students that mathematicians communicate their ideas by (a) explaining their thinking to others, (b) listening actively (e.g., make eye contact) to others explain their thinking, and (c) asking questions.

Review

Ask students to share what they remember learning from the previous lesson about growing patterns. Explain to the students that today they will be extending their knowledge about growing patterns using formulas and tables. For students who discovered how to develop formulas in Lesson 14, they will have a chance to extend and advance this skill as well.

A Real World Connection

Make a real world connection by asking students if they have ever made cookies using a recipe with an adult. Then ask:

- Have you ever “doubled a recipe?”
- What do you think it means to “double a recipe?”
  Sample answer: *When a baker doubles a cookie recipe, he or she is baking twice as many cookies. Another way to think of this is by multiplying the amount of each ingredient by 2.*
- What happens if you “triple a recipe?”
  Sample answer: *When a baker triples a cookie recipe, he or she triples the amount of cookies made, or the baker multiplies the amount of each ingredient by 3.*
- What do you think it is called when you increase a recipe 4 times? *(quadruple a recipe)*

Talk Move: *Use Repeat/Rephrase. Ask a student to restate what a classmate shared.*

Tell students the following story:

> *Toxo is going on a picnic this afternoon after the trip to the Amusement Park with his Planet Nine alien and human friends. Toxo has a recipe for Planet Nine alien cookies that are out of this world! But, Toxo has a lot of friends. He would like to quadruple his recipe of Planet Nine alien*

cookies so that he can bake 4 batches of cookies. Unfortunately, he is out of one special ingredient, Zorg flour! Toxo needs your help to figure out how many cups of Zorg flour he needs to purchase at the store to bake his 4 batches of cookies.

Give students a few minutes to explore the following questions with a partner.

- What do you need to know to solve this problem? *(# of cups of flour for one batch)*
- How might you plan to solve this problem?
  Encourage students to brainstorm a variety of ideas on how to solve the problem.

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Bring the class back together as a whole group and ask students to share their thinking.

Explain to students that a third-grade student in another class found that one way to solve this problem is to draw a table to organize the **terms (*t*)**, or numbers that are a part of a pattern or sequence, and the missing numbers in the pattern. Draw the following table on the board to guide students in building a pattern to solve this problem. The "*n*" in the table below represents the number of cups of Zorg flour, or the missing variable that the students need to solve. Explain how *t* and *n* are **variables** that can be represented by letters. Remind students that variables are letters or symbols that represent a number or amount. Tell students that one batch of cookies needs 3 cups of Zorg flour.

Number of batches (*t*)	1	2	3	4
Cups of Zorg flour (*n*)	3			

Explain to students that if they know one batch of cookies calls for 3 cups of Zorg flour they can determine how many cups will be needed for two batches. Remind students that when you make two batches, you double your recipe ingredients. Ask for students to come up to the board to fill in the missing terms, or numbers in the growing pattern, in the table for two, three, and four batches of cookies. The completed table should look as follows:

Number of batches (*t*)	1	2	3	4
Cups of Zorg flour (*n*)	3	6	9	12

Have students explain how they arrived at discovering the missing terms to complete the table. Ask students to share any different strategies they used.

Talk Move: *Use Reasoning. Ask students if they agree or disagree with the other student's strategy, and more importantly why they agree or disagree.*

Developing Formulas for Growing Patterns

Next, extend the table so it looks like the following:

Number of batches (*t*)	1	2	3	4	5	6	7
Cups of Zorg flour (*n*)	3	6	9	12			

Ask students the following questions:

- What is the rule for the pattern for the cups of Zorg flour? *(+3)*
- How do you know that +3 is the rule for this pattern?
  Sample answer: *The difference between the first and second term for cups of flour is 3. The difference between the second and third term is also 3, and so on. So, the rule for the pattern is +3.*
- Is the pattern in the table a repeating or growing pattern? Is this pattern increasing or decreasing?
  Students should be able to explain that the numbers in this pattern are increasing so that makes it a growing pattern, whereas in a repeating pattern the numbers would simply repeat.

Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Explain to students that since they know the rule for the growing patterns, they can use that rule to determine the 7th term for each ingredient. Ask students:

- What is the 7th term in this pattern? *(21 cups of Zorg flour)*

Students might arrive at this answer by filling in the rest of the table by counting up by 3s (i.e., 12 + 3 = 15 + 3 = 18 + 3 = 21). Have students come up to the board to complete the table and explain how they found their answers. Use Talk Moves to encourage student communication and thinking. For example, ask the rest of the class if they agree or disagree. Depending on students' ability levels and mastery of multiplication facts, students may notice that you can multiply 7 x 3 to get 21 since there are 7 groups of 3 cups of flour in 7 batches of Planet Nine alien cookies.

Talk Move: *Use Reasoning. Ask students if they agree or disagree with the other student's strategy, and more importantly why they disagree or agree.*

Explain to students that if they know the rule for a pattern, they can make a prediction about any term in a pattern. For example, the rule for the growing pattern of Zorg flour is +3. To find the amount of Zorg flour for 7 batches, you could either add +3 seven times 3 + 3 + 3 + 3 + 3 + 3 + 3 = 21 or you could use a **formula**.

Remind students that a formula is a rule or function for a pattern of numbers to make a prediction about a specific term. Formulas include variables represented by letters for missing terms.

Students should be able to recognize that the repeated addition equation above can also be expressed as 7 x 3 = 21. The formula to find the amount of Zorg flour for any number of batches therefore is 3*t*, where *t* represents the unknown term in the pattern. In this case, we are trying to determine the 7th term in the pattern for the amount of Zorg flour. So, by substituting 7 for the variable ***t*****,** the equation would be 3(7) = 21 or 3 x 7 = 21.

To check for understanding, ask for a volunteer to explain how a mathematician might describe what a formula is, and then have students turn to a partner to explain this concept. Ask students if they have any questions and clarify any misunderstandings before proceeding.

Increasing and Decreasing Growing Patterns

Tell students to open to *Toxo's Out of This World Ideas! Increasing and Decreasing Patterns* Student Page **[SMN pages 261-263]**. Model how to solve questions 1 and 2 with the class. The rest may be completed as a whole group, with a partner, or independently.

If you choose to guide students through the rest of the SMN page, discuss the following:

Other growing patterns exist besides the ones discussed in steps 1 and 2. Present the following table to the students on the board:

Term (*t*)	1	2	3	4	5	6	7
Number (*n*)	50	54	58				

Ask the following questions to foster a discussion:

- What is the rule for the increasing growing pattern in this table? *(+4)*
- What will the 7th term be? *(74)*
- How did you figure out the 7th term?
  Sample answer: *I saw that we needed to add 4 to 58 four more times. I know that 4 x 4 = 16, so I added 58 plus 16 to get 74, which is the 7th term.*
- Could someone share an idea for a story problem to go along with this table? *(Answers will vary.)*
- How was this problem different from the one in steps 1 and 2?
  Sample answer: *This pattern is different because the first term is not the same value as the number for the rule.*

***Talk Move**: Use Wait Time. Give students extra time to think about their answers.*

Draw the following table on the board:

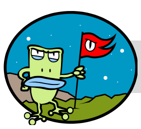
Term (*t*)	1	2	3	4	5	6	7
Number (*n*)	50	42	34				

Ask:

- Are the numbers for *n* increasing or decreasing? *(decreasing)*
- What is the 7th term? *(2)*
- How did you determine the 7th term?
  Sample answer: *I saw that we needed to subtract 8 from 34 four more times. I know that 4 x 8 = 32, so I subtracted 32 from 34 to get 2 which is the 7th term.*
- Could someone share an idea for a story problem to go along with this table? *(Answers will vary.)*

***Talk Move**: Use Wait Time. Give students extra time to think about their answers.*

Discuss how these patterns are different from the patterns in step 1 and 2. When students read word problems that are not accompanied by a table with a pattern already started, they need to pay close attention to the mathematical language clues that will signal whether the pattern is increasing or decreasing.



Explore

2. **Finding a Term in a Growing Pattern**

In this investigation, students will be solving growing patterns based upon their differentiated groups based on the results of the unit pretest.

Have students work with a partner or in a small group to complete the *Toxo's Out of This World Cookie Recipe!* Student Pages **[SMN pages 265-275]**. Students should be grouped based on the table below for this investigation.

Groups Formed by Scores on Questions 10, 11, and 12 on the Pretest		
Fibonacci Lab Group	Diophantus Lab Group	Kovalevsky Lab Group
Scored 0 to 1 point(s) on items combined.	Scored 2 points on items combined.	Scored 3 points on items combined.
Student Names	Student Names	Student Names
Student Pages for Lessons		
Toxo's Out of This World Cookie Recipe!—Fibonacci Student Page **[SMN pages 265-267]**	*Toxo's Out of This World Cookie Recipe!—Diophantus* Student Page **[SMN pages 269-271]**	*Toxo's Out of This World Cookie Recipe!—Kovalevsky* Student Page **[SMN pages 273-275]**

Collaborate and Communicate

All groups have worked on a table with decreasing elements, and they will consider a new one as a class.

Toxo wants some punch. He found a recipe for 15 cups but only needs to make 1 cup for himself. How many teaspoons of Zoogur sweetener does he need? How do you know?

Number of cups of punch	15	14	13	12	11	10	. . .	1
Teaspoons of Zoogur	90	84	78			60		

Small groups should record their explanation on a whiteboard or large chart paper. Mathematicians persevere when trying to solve problems. Focus on students' initial ideas for solving the problem by asking them what the other elements might tell them about the pattern. Here are some possible student responses, and you can record additional ones you observed in your own class in the blank boxes:

A. [Possible response] *This group added a row to the table and discovered that the difference between the number of teaspoons of Zoogur and cups of punch decreases by a multiple of 5 each time.*	**B.** [Possible response] *This group noticed that the number of teaspoons needed decreases by 6 each time and added additional columns to count down to 1 cup of punch.*	**C.** [Possible response] *This group subtracted the number of teaspoons of Zoogur needed for 10 cups of punch from the amount needed to make 15 cups to figure out how many teaspoons were needed for 5 cups of punch, or 30 teaspoons. They then divided this amount into 5 cups (6 teaspoons).*

Examine and Elaborate

3. **Highlight Students' Mathematical Thinking**
Mathematicians persevere in trying to solve problems, even when a possible solution is not immediately clear. Students therefore should try to look at different possible entry points when solving this problem.

Share and Discuss
The following sample dialogue showcases how students uniquely approached this problem. The teacher acknowledges how each solution is valid even though they are distinct ways of solving the problem.

Teacher: Alright class, this problem was a little more challenging. Can someone explain how your group figured out the answer?
Jules: I can*!* Ok, so our group at first was really confused because we could not figure out a pattern, so we wrote out a table starting from 15 all the way down to 1. This made it easier for us because now we weren't missing as many pieces. Then we realized when the cups of punch decreased, the

teaspoons of Zoogur also decreased by 6 teaspoons (see Response B).

Teacher: Awesome, can you or anyone else in the class share some of the strategies that were used when figuring out this problem?

Jesse: Well one strategy we used was writing out the whole problem so we filled in some of the missing information piece by piece which made solving the problem a lot easier.

Mason: They also looked at each row separately to figure out what was going on, and then looked at them together.

Teacher: Mason, I noticed that they looked at the numbers of cups of punch row first, and figured that out, then went down to the teaspoons of Zoogur, wrote that out, then they figured it out. Is that what you were saying? *(Revoicing talk move)*

Mason: Yes! Then they looked at the rows together to figure it out!

Throughout this dialogue, students are encouraged to explain the strategies that they used when they persevere through a difficult problem. The dialogue can be repeated with the other groups, and the class could also come up with a list communally that lists all of the strategies they used to persevere through this difficult problem.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. To support students as they work on their growing patterns, you can prompt students to make a note of the changes to the pattern under the table. Here is an example from *Toxo's Out of This World Cookie Recipe!—Fibonacci:*

Number of batches	1	2	3	4
Cups of Zorg flour	4	8	12	

Students may also need to model the pattern with concrete objects such as the circle tiles, unifix cubes, or they could draw representations of the cups of flour to help them visualize how many cups of flour are being added each time.

Some students may benefit from further challenge of the lesson concepts. For students who are comfortable with multiplication facts or have demonstrated a developing understanding of creating formulas to describe the nth term, encourage them to use multiplication and/or create formulas as they complete the Student Pages **[SMN pages 261-279]**. Students who finish early may work on any of the *Super Challenges* Student Pages **[SMN pages 281-283]**.

You can also encourage students to explore other growing patterns such as the Fibonacci sequence.

Debrief and Look Ahead

4. **Debrief Content and Skills**

Ensure that students have a solid understanding of increasing and decreasing growing patterns. Ask students:

- How are increasing and decreasing growing patterns alike? How are they different?
- How would a mathematicians explain what a formula is?
- Why are formulas helpful when finding terms in growing patterns?

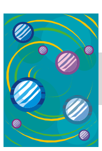
Talk Move: *Use Wait Time. Give students extra time to think about their answers.*

Debrief Thinking Like Mathematicians

Remind students that the mathematical practice for this lesson focused on how mathematicians persevere through difficult problems, and communicate strategies they used when solving a difficult problem. Review some of the ideas students brainstormed at the beginning of class and offer examples of how students communicated like mathematicians.

Look Ahead

In the final lesson, students have the opportunity to demonstrate their understanding of the Big Ideas from this unit. Lesson 16 also has an individual from outside the classroom come and visit the classroom so students have the opportunity to present their time capsule. This activity is scheduled for Day 2 of Lesson 16.



Assess

5. **Practice Growing Patterns**

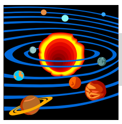
Students should be able to identify the rule for a growing pattern and continue the pattern in a table. Students should also be able to identify a specific term in the growing pattern.

The *Toxo's Out of This World Ideas!—Homework* Student Page **[SMN pages 277-279]** can also be used as a homework assignment.

An Intergalactic Top Secret Mission—

Find the Planet Nine Alien Spaceship Crew!

LESSON 16



Big Ideas

Students gain a deeper understanding of mathematical concepts, skills, and reasoning when applied to a real-world or *out of this world* scenario! In this culminating investigation, students have the opportunity to review and apply the core concepts and skills of this unit. They will also have the opportunity to express and demonstrate to individuals outside of their classroom, how they think like mathematicians. Overall, Lesson 16 is a two-day lesson.

Lesson Objectives	• Students will apply skills and concepts from the unit.

Materials	• Student Page—*An Intergalactic Mission Play* [SMN page 285] • Student Page—*Find Captain Chavir's Crew! Top Secret Clues!—Fibonacci* [SMN page 287] • Student Page—*"Captain Chavir! We Have Found Your Crew!"—Fibonacci* [SMN page 289] • Student Page—*Find Captain Chavir's Crew! Top Secret Clues!—Diophantus* [SMN page 291] • Student Page—*"Captain Chavir! We Have Found Your Crew!"—Diophantus* [SMN page 293] • Student Page—*Find Captain Chavir's Crew! Top Secret Clues!—Kovalevsky* [SMN page 295] • Student Page—*"Captain Chavir! We Have Found Your Crew!"—Kovalevsky* [SMN page 297] • Student Page—*The Planet Nine Alien Amusement Park* [SMN page 299] • Student Page—*Planet Nine Alien Pictures* (**OPTIONAL**) [SMN page 301] • Unifix cubes or circle tiles Remind students that manipulatives are available to use strategically and appropriately.
Selected Mathematical Practices	• MP2: Reason abstractly and quantitatively • MP7: Look for and make use of structure

Differentiation

Content

- learning objectives
- prior knowledge or learner readiness
- tiered activities
- formative assessment
- varied levels of challenge
- “teaching up” (aim high, provide scaffolding)
- know (information, facts, vocabulary), understand (concepts, big ideas, connections), apply (skills, processes)
- real-world application

Process

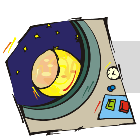
- questioning strategies
- 4Cs (21st Century Skills)
  - creativity
  - critical thinking
  - collaboration
  - communication
- hands-on activities/manipulatives
- connections

Product

- oral, visual, and written opportunities
- multiple ways to demonstrate knowledge, understanding, and skills
- multiple models and representations
- meaningful and respectful tasks
- summative assessment

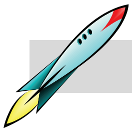
Learning Environment

- flexible grouping
- whole group/small group/individual instruction
- growth mindset
- learning community



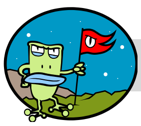
Lesson Preview

Students will review the major concepts and skills from this unit in this two-day culminating lesson. In Day 1, students help the leader of the Planet Nine alien visitors, Captain Chavir, find his spaceship crew at the local amusement park so that they can begin a top-secret mission. Students solve mathematical problems to locate the crew members. In Day 2, to review how they have learned to think like mathematicians, students will present their time capsule artifacts to a guest and explain different ways they thought or acted like mathematicians throughout the unit.



Launch (Day 1)

1. **Strategies Review Poster**
This unit encompasses a variety of thinking and problem-solving strategies. Have students work cooperatively in small groups to brainstorm a list of strategies that they learned for solving math problems throughout this unit. These strategies might include ones that students put into the time capsule. After students have had an opportunity to work together for about 5-10 minutes, have the groups share their progress with the whole group. This will encourage the other groups to expand their lists. After groups have shared, allow more time to continue working on the posters. Conduct a final discussion of the strategies before moving on to the second part of the investigation.



Explore

2. **Locating Planet Nine Alien Crew Members**
In this investigation, students will be using skills from throughout the unit to complete *An Intergalactic Mission Play* **[SMN page 285]**, *Find Captain Chavir's Crew! Top Secret Clues!* **[SMN pages 287, 291, 295]**, and *Captain Chavir! We Have Found Your Crew* **[SMN pages 289, 293, 297]** based upon their understanding and progress throughout the unit.

Before launching into the Explore investigation, have students break up into three groups: Fibonacci, Diophantus, and Kovalevsky. Base your grouping decisions on your knowledge of student progress throughout the unit. Data to guide your decisions may come from the homework or classwork assignments, classroom discussion observations, or observations about student work in small groups.

Groups Formed by Student Progress Throughout the Unit		
Fibonacci Lab Group	Diophantus Lab Group	Kovalevsky Lab Group
Students' conceptual understanding of the unit lessons is still developing.	Students have demonstrated mastery of many of the concepts in the unit but may still be developing an understanding for some concepts.	Students have demonstrated mastery of most of the concepts in the unit.
Student Names	Student Names	Student Names
Student Pages for Final Lesson Option #1		
An Intergalactic Mission Play Student Page **[SMN page 285]** *Find Captain Chavir's Crew! Top Secret Clues!—Fibonacci* Student Page **[SMN page 287]** *"Captain Chavir! We Have Found Your Crew"—Fibonacci* Student Page **[SMN page 289]**	*An Intergalactic Mission Play* Student Page **[SMN page 285]** *Find Captain Chavir's Crew! Top Secret Clues!—Diophantus* Student Page **[SMN page 291]** *"Captain Chavir! We Have Found Your Crew"—Diophantus* Student Page **[SMN page 293]**	*An Intergalactic Mission Play* Student Page **[SMN page 285]** *Find Captain Chavir's Crew! Top Secret Clues!—Kovalevsky* Student Page **[SMN page 295]** *"Captain Chavir! We Have Found Your Crew"—Kovalevsky* Student Page **[SMN page 297]**

An Intergalactic Mission!

Read the *An Intergalactic Mission Play* Student Page **[SMN page 285]** with the students to launch the investigation for this final lesson of the unit. After the play has been shared, discuss the directions for the Explore activity. Each group will be responsible for identifying the locations of the different Planet Nine alien crew members at the Amusement Park. The locations are represented by numbers on the *Planet Nine Alien Amusement Park* Student Page **[SMN page 299]**.

Each group will be using the *Find Captain Chavir's Crew! Top Secret Clues!* Student Page **[SMN pages 287, 291, 295]** to discover the location of Captain Chavir's crew members. Students working at each level might work together as a whole group or cut out the cards and break off into even smaller groups. Students should show their work and record their answers on the *Captain Chavir! We Have Found Your Crew* Student Pages **[SMN pages 289, 293, 297]**.

Encourage students to apply the concepts and skills that they learned throughout this unit and brainstormed for the poster in the Launch section as they solve the world problems.

Note: *Differentiation Tip: If you have observed that students in any of the 3 groups work at a faster rate than the other group members, consider regrouping those students together so that they may work at their own pace. So, there could be multiple Fibonacci, Diophantus, or Kovalevsky groups. For example, if there are 6 students in the Kovalevsky group and 2 in particular are more advanced, have the 2 students work out all of the word problems on their own separately while the other 4 members work together.*

[Optional]: Put a copy of the *Planet Nine Alien Amusement Park* Student Page **[SMN page 299]** on display within easy reach for the students during the investigation. As students determine the secret numbers for the location of the Planet Nine alien crew members, have students come up to the poster and place a labeled picture of the Planet Nine alien on the corresponding number. Students could draw and color a picture of their Planet Nine alien on their own or use one of the Planet Nine Alien pictures **[SMN page 301]** provided.

Collaborate and Communicate

Teachers should identify a question from any of the Student Pages for small groups to solve and record their explanations on a whiteboard or large chart paper. The question should be one that addresses a key idea and the class might need more time to unpack. Record the question, anticipated responses, and observed responses below.

Focus Question:		

Examine and Elaborate

3. Highlight Students' Mathematical Thinking

Students have engaged in and debriefed on mathematical thinking throughout the unit. The manner in which students do so will depend on the question teachers choose to focus on. One option would be to discuss each of the word problems that the students had to solve. Ask students what strategies they used to find the answers.

Share and Discuss

The word problems to find the locations of the Planet Nine aliens are similar for each group except that they increase in complexity and challenge level for each of the groups. Here is a sample dialogue:

Teacher: Would someone please share his/her thinking when you found the number for the location of the Bleep?

Terry: My partner and I thought about our doubles and remembered that 6 + 6 = 12. We then figured out that 5 + 7 = 12. Then 11 + 1 = 12. So, the number of the Ferris wheel car is 3!

Teacher: Does anyone agree or disagree? Why? (*Reasoning talk move*)

Allison: I sort of agree and disagree. There are a few more ways to get to 12. We made a table to get organized so we wouldn't forget any. We found out that 0 + 12, 2 + 10, 3 + 9, and 4 + 8 also make 12! So, that would make the number of the car 6 instead!

Teacher: What do you think about the strategies that group used?

Terry: I like that they made a table to get them all! I'll try that next time.

Differentiate Further as Needed

Some students might need additional support to understand the lesson concepts. Consider having students cut the clues out and solving them one at a time. Decreasing the amount of text on the page can help students focus.

Some students may benefit from further challenge of the lesson concepts. For students who demonstrate understanding quickly, consider having them complete Super Challenges from previous lessons.

Debrief: Thinking Like Mathematicians (Day 2)

4.

Thinking and Acting Like Mathematicians

Tell the students that they will open their time capsule and reflect on all of the different ways they thought or acted like mathematicians during the unit. Explain that a special guest is coming to their classroom and the students will (a) describe how mathematicians think or act, and (b) give evidence to demonstrate how they are mathematicians.

As a whole group, review the time capsule artifacts and divide them into the following three categories (based on the 4Cs and MPs):

- Problem-Solving Skills
  - Lesson 1: How do mathematicians solve problems?
  - Lessons 7 & 15: How do mathematicians persevere through difficult problems?
  - Lessons 11 & 13: What strategies do mathematicians use when solving problems?
- Collaboration and Communication
  - Lessons 2 & 6: How do mathematicians work together?
  - Lessons 3, 9, 15, & 12: How do mathematicians communicate?
  - Lessons 8, 9, & 12: How do mathematicians act when they disagree about a solution?
- Attending to Precision
  - Lessons 4: How do mathematicians support their answers?
  - Lessons 5 & 10: Why do mathematicians use mathematical language?
  - Lesson 14: How/why do mathematicians use tools?

Next, divide students into three groups and assign a category to each group. Students can be divided into groups based on interest, or at teacher's judgment. Ask the students to consider the following questions:

- How will you explain what a mathematician is and how they think and act to our guest?
- Using the artifacts, how will you show our guest ways we are like mathematicians?

If time allows, encourage students to develop creative ways to present their answers and artifacts to the guest.

Invite a guest (e.g., principal, math specialist, other school staff member, community member with a math-related career, etc.) to your classroom and facilitate a time of sharing and discussion as students present the time capsule artifacts to the guest.

If Aliens Taught Algebra Mathematicians' Glossary

Addend: A number that is being added to another number. Example: In 4 + 5 = 9, numbers 4 and 5 are addends.

Algebra: A topic studied as part of mathematics; the study of mathematical symbols (e.g., numbers; equal, addition, or multiplication signs) and the rules for working with these symbols.

Array: A way to organize information in rows and columns.

Associative Property of Addition: The grouping of addends in a number sentence does not change the sum.

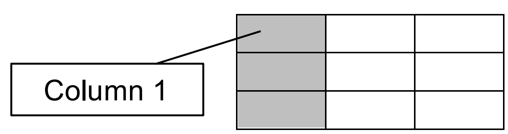
Astronomy: The scientific study of space, stars, planets, and other celestial bodies.

Astronomers: People who study astronomy.

Benchmark: A point of reference.

Column: A vertical arrangement of items or numbers in a list or table.

3
4
5

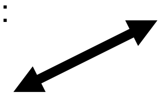


Commutative Property of Addition: The order that addends are added does not change the sum.

Commutative Property of Multiplication: The order that factors are multiplied does not change the product.

Decomposition: Breaking down a number to make a problem mentally easier to calculate.

Diagonal: A line that is on a slant compared to the top and sides of a page. Example:



Divisible: A whole number is *divisible* by another whole number if the remainder after dividing the two numbers is zero.

Division: A mathematical operation in which a number is subtracted from itself a certain number of times.

Equation: A mathematical sentence that contains an equal sign.

Estimate: An educated guess for the answer to an algorithm.

Estimating: Rounding numbers to calculate an answer such as a sum or difference.

Factors: The numbers in a multiplication problem that are multiplied together to arrive at the product.

Flip (Reflection): A term for describing the movement of a shape that is flipped to the left, right, up, or down.

Formula: A rule or function for a pattern of numbers to make a prediction about a specific term.

Function: A rule for calculating sums or differences when using input and output boxes.

Generalization: Stating a conclusion based on a small amount of information, instances, or items.

Growing Patterns: Patterns that increase or decrease in a linear manner. Patterns may "grow" in ascending order such as "2, 4, 6, 8, . . ." or they may "grow" in descending order such as "12, 9, 6, 3, . . ."

Horizontal: Parallel to the horizon. Example:

Inequality Sign: A sign used to represent a number sentence that is not equal ($\neq$).

Mathematics: The study and use of numbers, patterns, and shapes.

Mathematicians: People who study or use mathematics in their work.

Multiple: The product of a whole number and any other whole number. Example: The multiples of 3 are 0, 3, 6, 9, 12, 15, . . .

Multiplication: A mathematical operation in which a number is added to itself a certain number of times.

Number Sentence: A mathematical sentence that contains any sign (equality or inequality)**;** an open number sentence that contains a variable or missing number.

Perfect Squares: Products that have the same two factors. For example, 81 is a perfect square because 9 x 9 = 81.

Prime Number: A whole number greater than one with two factors: 0 and itself.

Product: The answer to a multiplication problem.

Property: A math rule.

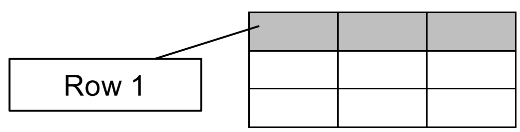
Recompose: Putting numbers back together after decomposing them to make a problem mentally easier to calculate.

Repeating Pattern: A repeating arrangement of numbers or objects.

Rounding: Altering a number so that it is easier to use in calculations.

Row: A horizontal arrangement of items or numbers in a list or table.

3 4 5



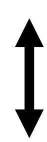
Sets: Another term for a group.

Terms: Numbers in a pattern or sequence.

Turn (Rotation): A term for describing the movement of a shape in either a clockwise or counterclockwise direction.

Variable: A symbol or letter that represents a number or amount. Example: *a* is the variable in 4 x *a* = 12 or 4*a* = 12.

Vertical: At a right angle to the horizon (up and down). Example:



Thinking Like Mathematicians: Challenging All Grade 3 Students

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