If Aliens Taught Algebra

Multiplication and Division Would Be out of This World!

Grade 3

Third Edition

Teacher's Version

Student Mathematician Notebook With Answer Keys

University of Connecticut

June 2019

The National Research Center on the Gifted and Talented (2008-2013)
First Edition

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Thinking Like Mathematicians: Challenging All Grade 3 Students
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Thinking Like Mathematicians: Challenging All Grade 3 Students

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Famous Mathematicians

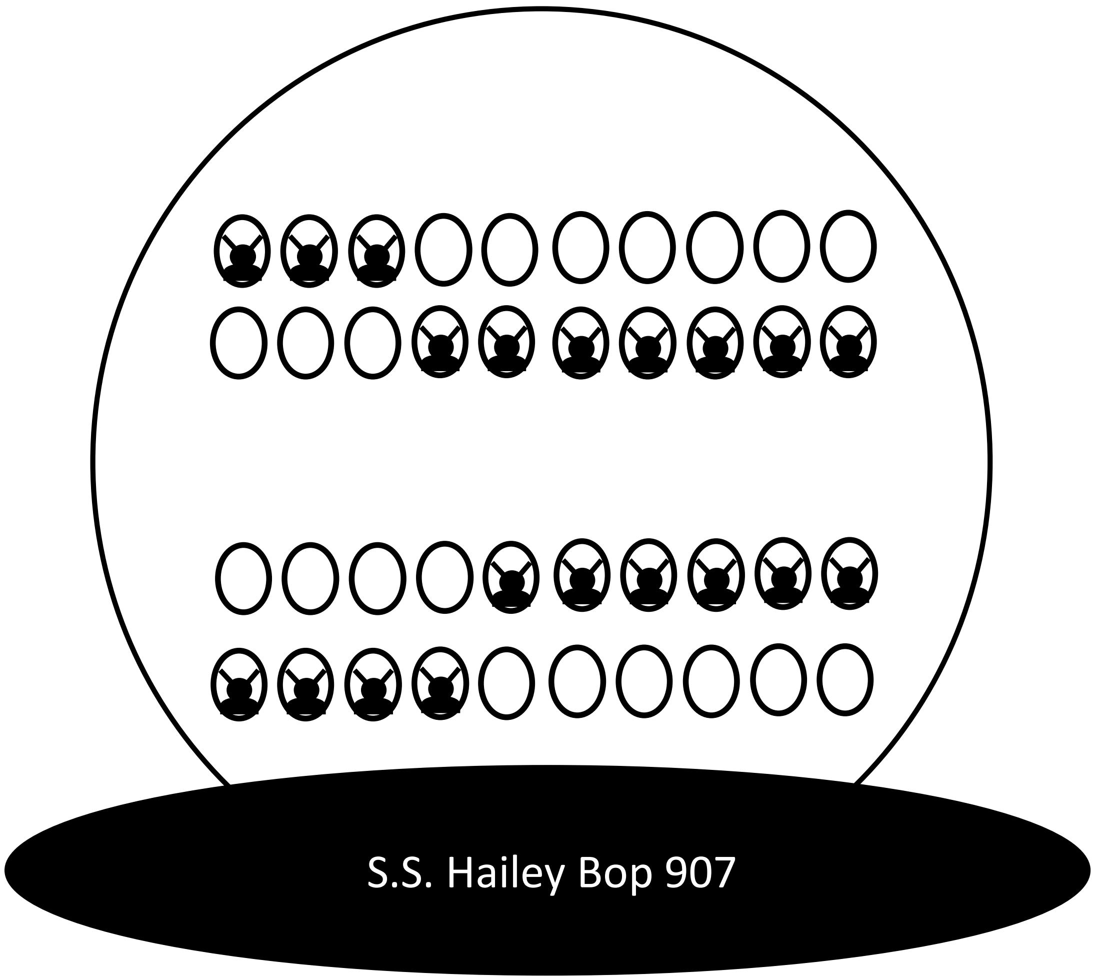
Leonardo Fibonacci (pronounced fee-bow-NAWH-chee) c. 1170 - c. 1250
Leonardo Fibonacci was born in Pisa, Italy, the same place as Galileo Galilei. His father was a merchant who sold goods to North Africa. Leonardo watched the merchants carefully and learned much about money and numbers from them. Leonardo traveled with his father to North Africa. This is where Leonardo learned a new system of numbers that was much easier than the system of numbers he had learned in Europe. He wrote a book about these numbers and introduced them to Europe. Leonardo loved animals. He also studied the patterns on the outside of pineapples. Because of his love of studying patterns, Leonardo discovered a series of numbers that is now called the Fibonacci Sequence. He also studied spiders and dogs and wrote mathematics problems about them. Leonardo wrote 6 books in all and came up with many theories, or ideas, about numbers. Leonardo became so famous that he became friends with the Emperor, and a statue was built of him that still exists to this day. Today there is a magazine about mathematics named after Fibonacci.

Diophantus of Alexandria (pronounced dy-oh-FAN-tuhs)
Diophantus of Alexandria was born in Greece over 1,800 years ago. He lived in Alexandria, Egypt, one of the mathematics centers of the ancient world. Here he studied numbers and came up with many theories about equations. He even invented symbols to represent numbers. He is known as the Father of Algebra. Diophantus wrote 13 books about these theories and inspired many great future mathematicians. One of his books survived over 1,500 years and taught many Europeans about algebra. It is considered the greatest Greek mathematics book in history. A Greek mathematician wrote a book about Diophantus that contained number games and strategy puzzles. One of these riddles is famous and is still difficult to solve, even to this day. Diophantus was married and had a son. Diophantus lived to be 84 years old when most men in his day only lived to be 40 years old.

Sonya Kovalevsky (pronounced koh-vuh-LEHV-skee) 1850 - 1891
Sonya Kovalevsky was born in Moscow, Russia. She is considered one of the brightest female mathematicians since Hypatia. She always loved math. Her father did not believe girls should study math, and he made her leave school at 13 years old. Sonya studied secretly. She borrowed a math book from a neighbor. Sonya showed her neighbor what she learned easily. The neighbor convinced Sonya’s father to let her return to school. Sonya learned geometry and calculus in a few months. She was so good at math that she was one of the first women to earn the highest degree from the local university. Sonya is a true example of an expert mathematician.

Spaceship Seats

The Planet Nine aliens are getting ready for their long trip.
They are boarding and packing their ship.
They need your help to know . . .
How many Planet Nine aliens are ready to go?



1. How many Planet Nine aliens are on board? How do you know?

There are 20 Planet Nine aliens on board.
Answers will vary. *I know because I can visualize the first row of Planet Nine aliens getting into the second row to make 10. The third row Planet Nine aliens can go into the fourth row to make 10. So 10 plus 10 equals 20.*

2. What is another way you could figure out how many Planet Nine aliens are on board?

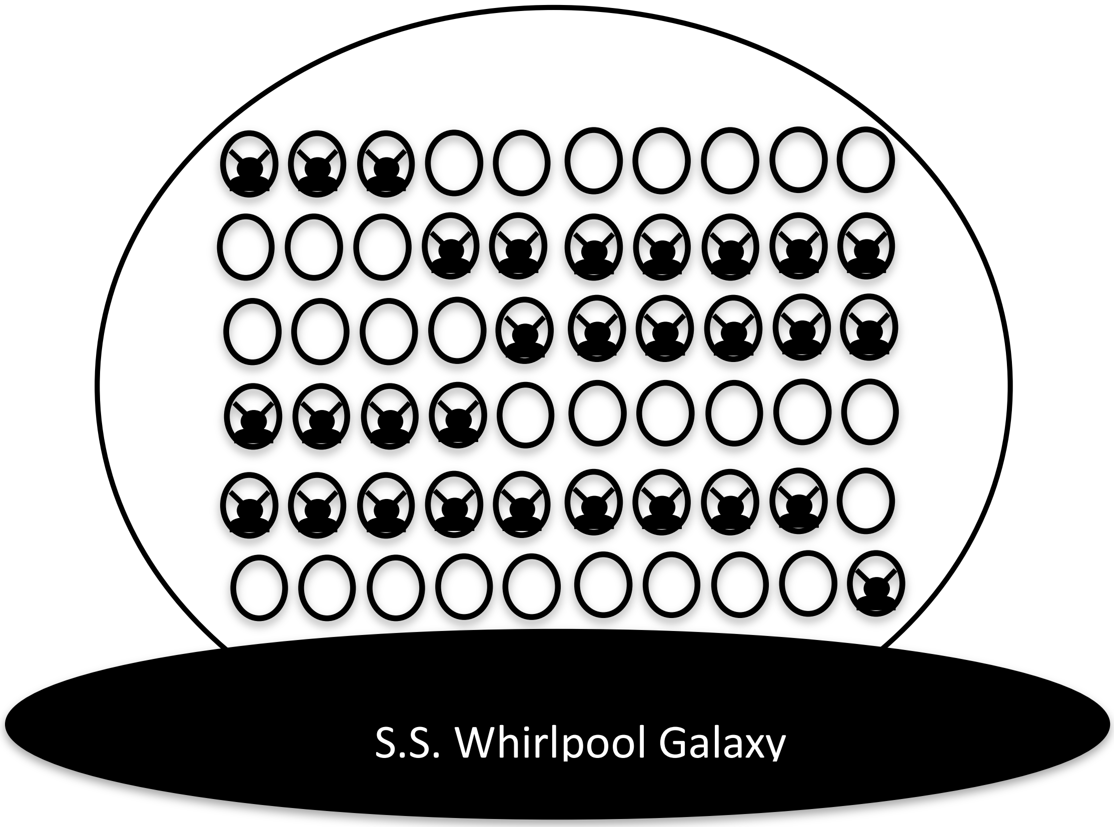
Answers will vary. *I see there are 3 Planet Nine aliens in the first row and 7 in the second. 3 + 7 equals 10. Then in the third row there are 6 Planet Nine aliens and 4 in the last row. So 6 + 4 is 10. Then I know 10 and 10 equals 20 because they are easy to add.*

3. How many more Planet Nine aliens could board?

20 more Planet Nine aliens can board the ship.

Spaceship Seats Challenge

The Planet Nine aliens are getting ready for their long trip.
They are boarding and packing their ship.
They need your help to know . . .
How many Planet Nine aliens are ready to go?



1. How many Planet Nine aliens are on board? How do you know?

There are 30 Planet Nine aliens on board.
Answers will vary. *I know that there are 30 Planet Nine aliens in all because there are 3 Planet Nine aliens in the first row and 7 in the second, which equals 10. In the third row there are 6 and in the fourth there are 4 and that equals another 10. In the fifth row, there are 9 and I added that to the last Planet Nine alien in the sixth row to make another 10. So I added 10 + 10 + 10 to equal 30.*

2. What is another way you could figure out how many Planet Nine aliens are on board?

Answers will vary. *I could visualize the first and second row of Planet Nine aliens to come together to fill one row. If I do that to the other rows, there will be 3 full rows of Planet Nine aliens. I know that each row fits 10 Planet Nine aliens. So that equals 30.*

3. How many more Planet Nine aliens could board?

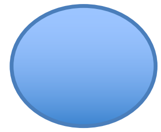
30 more Planet Nine aliens could board.

Packing Party for Earth

The Planet Nine aliens are packing for their trip to Earth. The young Planet Nine aliens have 2 types of toys: Wombles and Snufplets. Their parents are only allowing them to bring a total of 10 toys. What are all the possible combinations they could pack? You may need to use the next page.

Wombles	**+**	**Snufplets**	**= 10**
0	**+**	*10*	**= 10**
10	**+**	*0*	**= 10**
1	**+**	*9*	**= 10**
9	**+**	*1*	**= 10**
2	**+**	*8*	**= 10**
8	**+**	*2*	**= 10**
3	**+**	*7*	**= 10**
7	**+**	*3*	**= 10**
4	**+**	*6*	**= 10**
6	**+**	*4*	**= 10**
5	**+**	*5*	**= 10**

Packing Party for Earth (Continued)

 Wombles	+	 Snufplets	= 10
	+		= 10
	+		= 10
	+		= 10
	+		= 10
	+		= 10
	+		= 10
	+		= 10
	+		= 10
	+		= 10
	+		= 10
	+		= 10

1. How do you know if you have all the possible combinations?

Answers will vary. *I know I have all the possible combinations because I used every number from 0 to 10 for the Wombles and Snufplets. I went in order so I wouldn't skip a number.*

Several of the young Planet Nine aliens decided on their toy combination. You can see their packing list below. Fill in the missing numbers so that each Planet Nine alien is taking 10 toys.

OFFICIAL PACKING LIST				
Planet Nine Alien	**Wombles**	**+**	**Snufplets**	**= 10**
Jupit	**2**	**+**	8	**= 10**
Olive	5	**+**	**5**	**= 10**
BeeBop	**8**	**+**	2	**= 10**
Zulu	1	**+**	**9**	**= 10**
D2L4	**6**	**+**	4	**= 10**

2. How many Wombles in all are being packed? **Hint:** Could you group the Wombles to make 10s? You can use this strategy for any addition problems because it is easier to add 10s.

22 Wombles

3. How many Snufplets in all are being packed?

28 Snufplets

4. If you have some extra time, draw a picture of a Womble and a Snufplet. What would these Planet Nine alien toys be like?

Answers will vary. *Check student's work.*

Super Challenges

Super Challenge 1

Think about the possible combinations of toys if the Planet Nine aliens had a third toy, the Lululo.

Wombles + Lululos + Snufplets = 10

How many different combinations are possible? **(*48 combinations*)** You can't have the same number of Wombles, Lululos, and Snufplets. (e.g., 5 + 4 + 1 = 10)

How do you know you have every possible combination?

I made an organized list.

Does adding a third toy **increase** or **decrease** the number of combinations?

Increase

Super Challenge 2

What numbers would make these number sentences true? **(Hint: There are many possible answers!)**

Answers will vary. *Check student's work.*

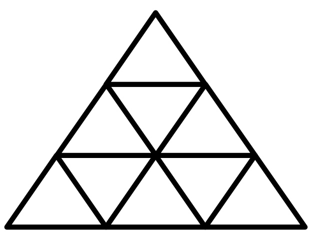
$\triangle$	+	5	=	$\square$
	+	5	=	
	+	5	=	
	+	5	=	
	+	5	=	

Super Challenge 3

(This challenge is from http://nrich.maths.org/192, used with permission.)

Cut out the 9 triangles on page 17 and try to create a triangle that looks like the one below. The touching sides must add up to 10. (**Hint: Grey sides will match with grey sides, white with white, and black with black.**)

Answers will vary. *Check student's work.*



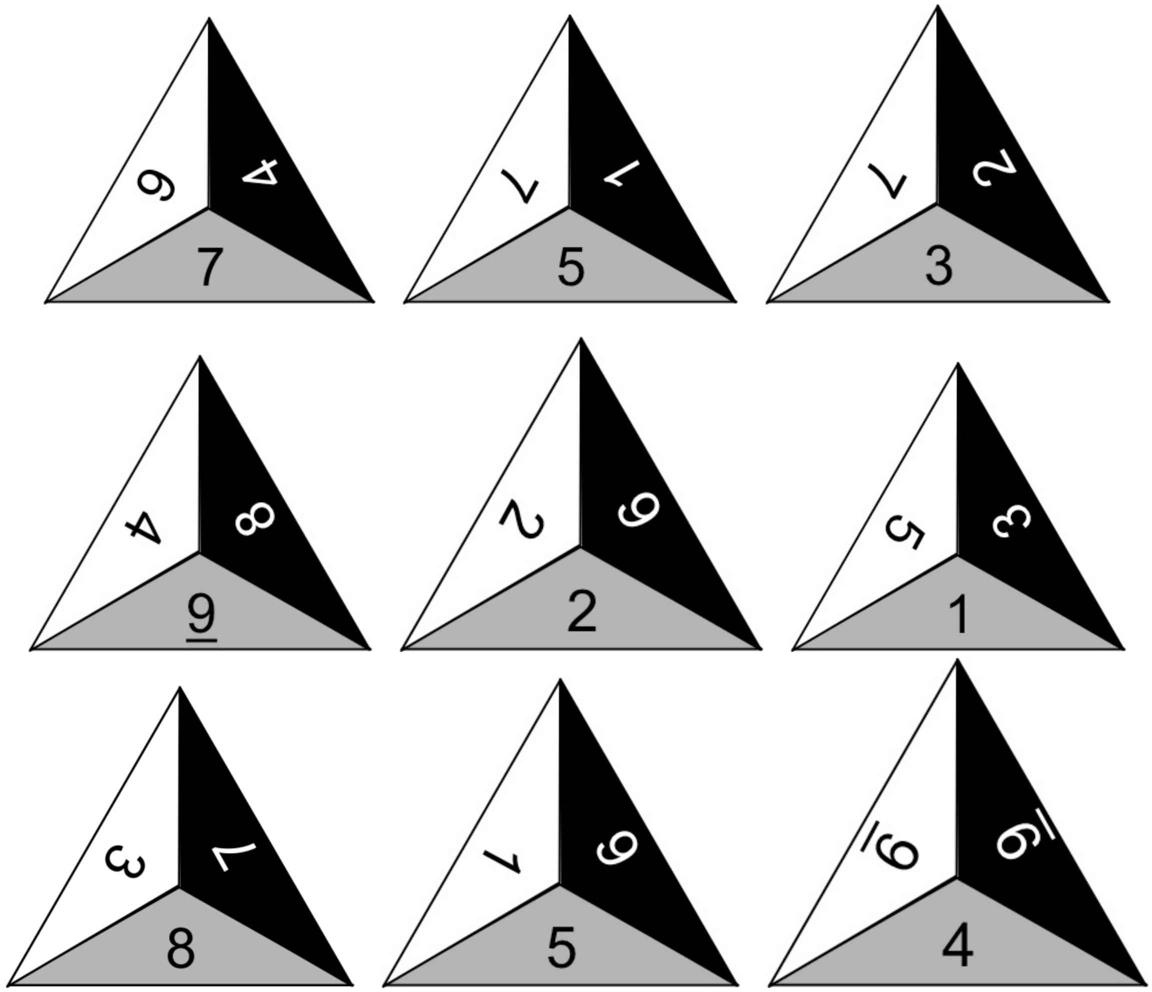
Super Challenge 4

Add **all** of the numbers in the boxes together. Think about making 10s to make your addition easier. (**Hint: Cross off numbers when you use them.**)

1	4	2	9	12
3	6	5	6	7
5	4	6	2	8
2	3	8	7	6
3	5	7	10	5

136 Planet Nine aliens

Triangles for Super Challenge 3



Breaking it Down Practice

Directions: Use what you learned today about the pieces of 10 to add these numbers.

Sample:
8 + 5 =

Start with: *8*
Need _*2*_ to equal 10. **Think: 2 + 8 equals 10.*
Add on remainder: *+3* **Think: 5 – 2 = 3, so 3 + 10 = 13*
Total: *13*

1. 7 + 9 =

   Start with: *9*
   Need __*1*__ to equal 10.
   Add on remainder: *+6*
   Total: *16*

2. 8 + 6 =

   Start with: *8*
   Need __*2*__ to equal 10.
   Add on remainder: *+4*
   Total: *14*

3. 5 + 6 =

   Start with: *5*
   Need __*5*__ to equal 10.
   Add on remainder: *+1*
   Total: *11*

Breaking it Down Practice 2

Directions: Use what you learned today about the pieces of 10 to add these numbers.

Sample:

8 + 5 =

Start with: *8*
Need _*2*_ to equal 10.
Add on remainder: *+3*
Total: *13*

1. 9 + 9 =

   Start with: *9*
   Need __*1*__ to equal 10.
   Add on remainder: *+8*
   Total: *18*

2. 18 + 6 =

   Start with: *18*
   Need __*2*__ to equal 20.
   Add on remainder: *+4*
   Total: *24*

3. 15 + 9 =

   Start with: *15*
   Need __*5*__ to equal 20.
   Add on remainder: *+4*
   Total: *24*

The Magical Number

10

The answer is 10. What is the question?

Think of 10 different ways to equal 10. Try to think of a unique way that no one else in your class will think of!

Examples: 1 + 2 + 4 + 3 = 10 **or** 42 – 32 = 10

Answers will vary. *Check student's work.*

Understanding the ALIEN-R2200—Fibonacci

Directions: Your task is to figure out how the ALIEN-R2200 operates.

IN	OUT
48	50
32	30
19	20
67	70
64	60
23	20
35	40

1. Let's take the first set of numbers. When 48 went in, it came out 50. Color 48 green and 50 red on the number line and draw an arrow from 48 to 50.

40	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**

What could be happening?

Possible response: *The rule might be to add 2.*

2. Now let's take the second set. The number 32 was put through the machine and came out 30. Color 32 green and 30 red on the number line and draw an arrow from 32 to 30.

30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**

What could be happening?

Possible response: *The rule can't be add 2 because now it subtracted 2. So maybe the rule is to round to the nearest 10.*

3. Now let's take the third set. The number 19 was put through the machine and came out 20. Color 19 green and 20 red on the number line and draw an arrow from 19 to 20.

30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**

What could be happening?

Possible response: *The rule is to round the number 19 to the nearest 10 because when the digit in the ones place is a 9, you round up.*

Understanding the ALIEN-R2200—Diophantus

Directions: Your task is to figure out how the ALIEN-R2200 operates.

IN	OUT
48	50
32	30
49	50
67	70
64	60
73	70

1. Let's take the first set of numbers. When 48 went in, it came out 50. Color 48 green and 50 red on the number line and draw an arrow from 48 to 50.

40	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**

   What could be happening?

Possible response: *The rule might be to add 2.*

2. Now let's take the second set. The number 32 was put through the machine and came out 30. Color 32 green and 30 red on the number line and draw an arrow from 32 to 30.

30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**

   What could be happening?

Possible response: *The rule can't be add 2 because now it subtracted 2. So maybe the rule is to round to the nearest 10.*

3. What do you think would happen if you put the number 35 into the ALIEN-R2200?

The number 35 is right in the middle of the number line. I think you should round up to 40.

Understanding the ALIEN-R2200—Kovalevsky

Directions: Your task is to figure out how the ALIEN-R2200 operates.

IN	OUT
48	50
32	30
19	20
67	70
64	60
23	20
35	40

1. What patterns do you notice about how certain numbers come out of the ALIEN-R2200?

I noticed that the numbers with digits less than 5 in the 1s place are rounded down to a 10s number. The numbers that have digits bigger than 5 round up to the next 10s number.

2. What happens when a number with a 5 in the ones place is put into the ALIEN-R2200?

The number 35 has a 5 in the 1s place. When it went in, it came out 40. So the machine rounded it up.

30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**

3. What is the effect of using 5s in that way? What is a possible way to fix the problem when 5s are always rounded up?

The number 35 is right in the middle of 30 and 40. The effect is that you will always be rounding up for the middle number between two 10s. This may be a problem when you are estimating. Maybe you could round down half of the time and round up half of the time so your estimates are closer to the actual answers.

Super Challenges

Super Challenge 1

BeBop said that rounding up seems to be more popular than rounding down. Do you think that is true? Give a few examples of when you would want to round up.

Possible response: *I think rounding up is more popular. For example, if you are planning a party you might round up for the number of cups you need so you don't run out.*

Super Challenge 2

Think about a time when it would be better to round down than to round up. Draw a picture of this below with a caption.

Answers will vary. *Check student's work.*

Super Challenge 3

A **palindrome** is a number that can be read forwards and backwards using the same order of digits. This is a palindrome:

5775

Can you think of three palindromes that would be rounded down to the nearest 10?

Possible response: *3443, 124421, 4994*

Super Challenge 4

One- and two-digit numbers aren't the only numbers that can be rounded. How would you round the following numbers to the nearest 10?

315 *(320)*

2,247 *(2,250)*

$63.42 *($60.00)*

Explain your thinking.

Possible response: *All I did was look at the digit in the 1s place to decide to either round up or down to the nearest 10.*

Understanding Rounding (Extra Practice)

Color each given number on the number line. Draw an arrow from the given number to the 30 and the 40. Decide which path is the shortest. This helps you decide how to round the given number. The first one has been done for you.

1. 34 rounds to _*30*__

30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**

2. 31 rounds to _*30*__

30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**

3. 39 rounds to _*40*__

30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**

4. 37 rounds to _*40*__

30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**

5. 35 rounds to _*40*__

30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**

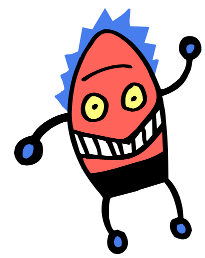
On Your Own

Round each number to the nearest 10.

27 ___*30*___ 11 ___*10*___ 15 ___*20*___ 33 ___*30*___ 79 ___*80*___

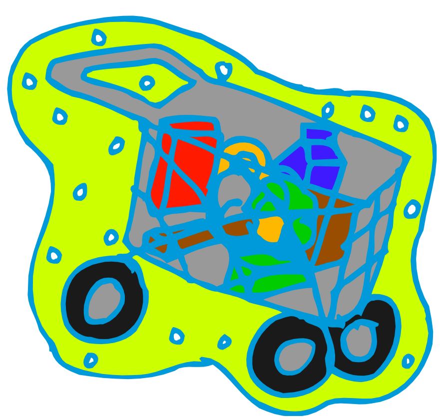
48 ___*50*___ 62 ___*60*___ 54 ___*50*___ 20 ___*20*___ 6 ___*10*___

A Planet Nine Alien Shopping Spree!—Directions

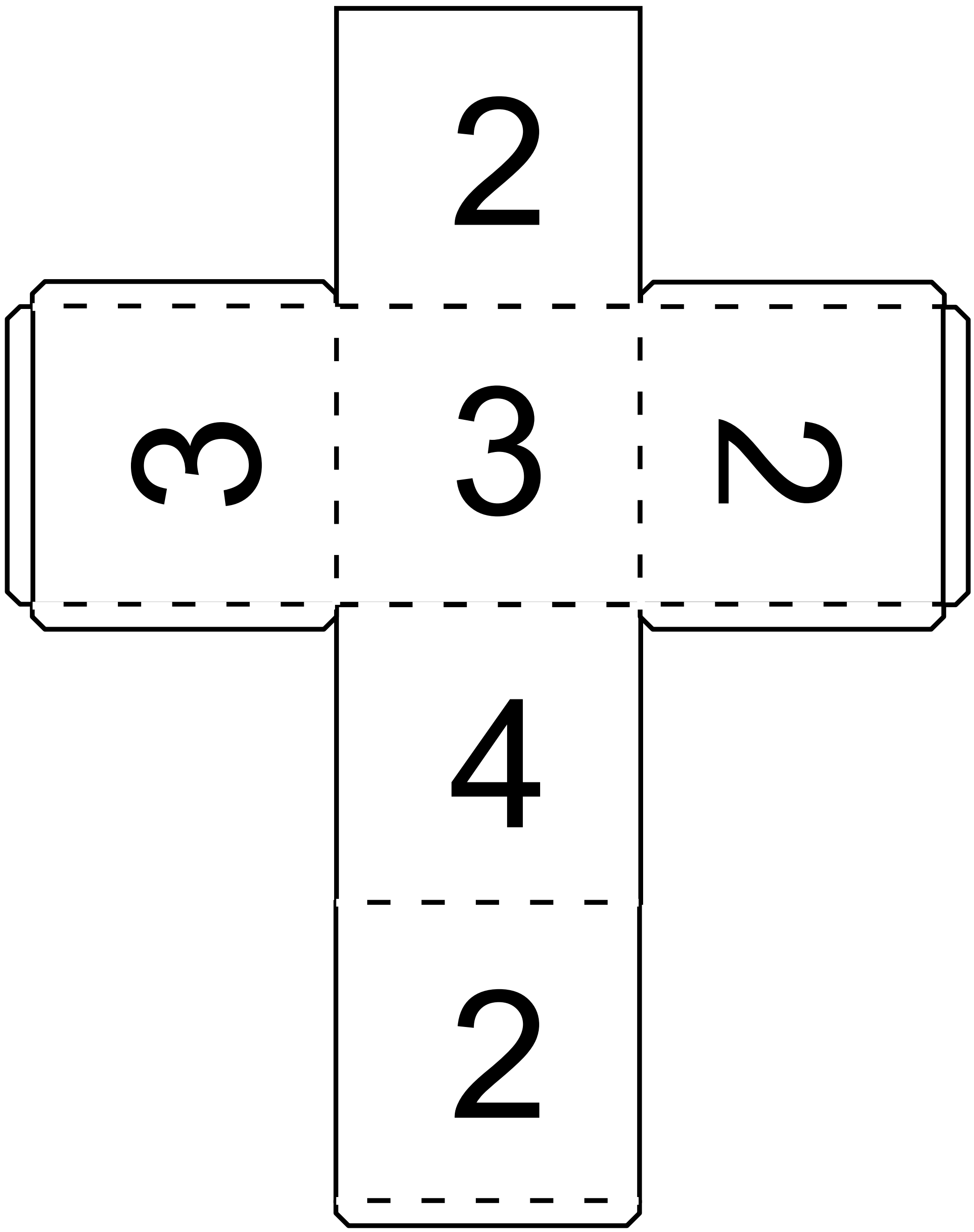


Soland the mathematician would like to go on a shopping spree at *Tommy's Toy Store!* The toy store is located at the Amusement Park, so this is a special kind of toy store. Each customer gets to roll a die to see how many toys he/she can buy at one time. Soland would like to spend less than $100 for each roll. Soland needs help from you and your partner.

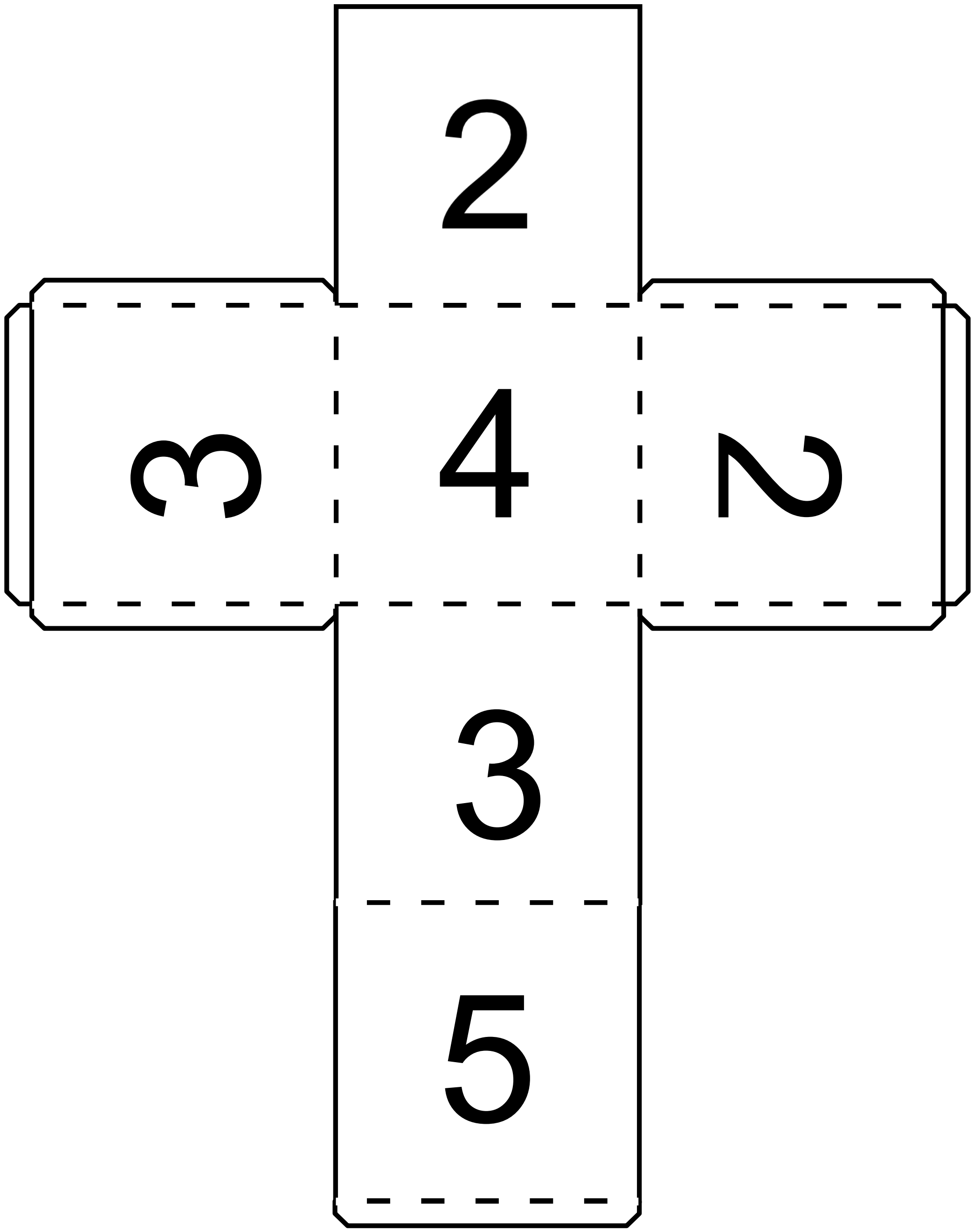
1. Roll the die to see how many toys Soland can buy.
2. Help Soland decide which items she should buy that total **less than $100**.
3. Estimate the cost of the items so Soland does not go over $100 for each roll.
4. Next, use mental math to figure out the exact total for the items.
5. Show your work on the next page.



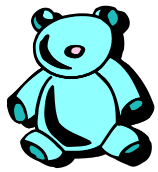
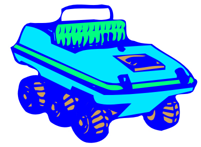
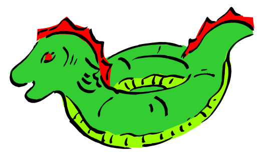
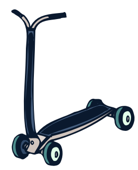
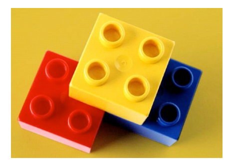
Number Cube—Diophantus



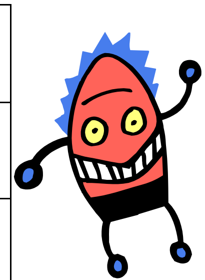
Number Cube—Kovalevsky



Tommy's Toy Store

 $12	 $4	 $23	 $18
 $65	 $26	 $14	 $7
 $28	 $6	 $44	 $3
 $8	 $39	 $5	 $9
 $62	 $2	 $31	 $47

A Planet Nine Alien Shopping Spree!—Diophantus



Items Placed in the Shopping Cart	Cost of the Items
Teddy Bear	*$12*
Ball and Glove	*$8*
Art Easel	*$31*

1. How did you estimate the total cost of all the items? Show your thinking here:

Possible response: *In my head, I rounded $12 to $10, $8 to $10, and $31 to $30. Then I added $10 + $10 + $30 to get $50.*

2. What is the exact cost of all the items? Show your thinking here:

Possible response: *I decomposed $12 into $10 and $2. Then I added $8 to the $2 to make another group of $10. Next, I knew $10 + $10 equals $20. All I had to do was add $20 to $31, which equals $51. I wrote a number sentence to show my thinking:*

$10 + ($8 + $2) + $31 = $51

3. Think of one other way to mentally add the numbers to find the exact cost. Keep in mind the different ways to equal 10.

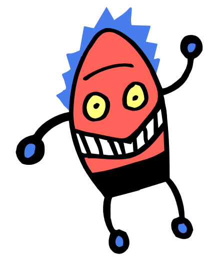
Possible response: *I could also just add $8 to $12 to equal $20. Then add $20 and $31 mentally, which is $51.*

4. How close was your estimate in #1 to the exact cost you found in #2?

Possible response: *My estimate was only $1 less than the exact cost.*

Soland has another **$100** to spend in *Tommy's Toy Store*. The shopping spree continues! This time try to purchase different items. Roll again!

Items Placed in the Shopping Cart	Cost of the Items
Toy Piano	*$65*
Doll	*$7*
Toy Car	*$18*



5. How did you estimate the total cost of all the items? Show your thinking here:

Possible response: *In my head, I rounded $65 to $70, $7 to $10, and $18 to $20. Then I mentally added $10 + $20 which equals $30. Then I added $70 and $30, which is $100.*

6. What is the exact cost of all the items? Use decomposition. Show your thinking here:

Possible response: *I decomposed $7 into $5 and $2. Then I added the $5 to the $65 to equal $70. Next, I add the $2 to the $18 to equal $20. All I had to do next was add $70 to $20, which equals $90. I wrote number sentences to show my thinking:*

($5 + $65) + ($2 + $18) =
$70 + $20 = $90

7. Think of one other way to mentally add the numbers to find the exact cost. Keep in mind the different ways to equal 10.

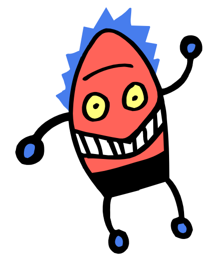
Possible response: *I could also just decompose the $65 into $62 and $3. Next I could add $7 and $3 to equal $10. $62 and $18 is easy to add because $2 and $8 equals another group of $10, so that would be $80. Then add the other $10 to equal $90.*

8. How close was your estimate in #5 to the exact cost you found in #6?

Possible response: *My estimate was only $10 more than the exact cost.*

A Planet Nine Alien Shopping Spree!—Kovalevsky

Items Placed in the Shopping Cart	Cost of the Items
Teddy Bear	*$12*
Ball and Glove	*$8*
Art Easel	*$31*



1. How did you estimate the total cost of all the items? Show your thinking here:

Possible response: *In my head, I rounded $12 to $10, $8 to $10, and $31 to $30. Then I added $10 + $10 + $30 to get $50.*

2. What is the exact cost of all the items? Show your thinking here:

Possible response: *I decomposed $12 into $10 and $2. Then I added $8 to the $2 for another group of $10. Next, I knew $10 + $10 equals $20. All I had to do was add $20 to $31, which equals $51. I wrote my number sentence to show my thinking:*

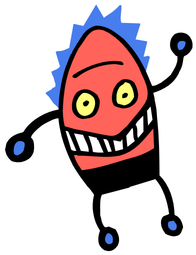
$10 + ($8 + $2) + $31 = $51

3. Think of one other way to mentally add the numbers to find the exact cost. Keep in mind the different ways to equal 10.

Possible response: *I could also just add $8 to $12 to make $20. Then add $20 and $31 mentally, which is $51.*

4. How close was your estimate in #1 to the exact cost you found in #2?

Possible response: *My estimate was only $1 less than the exact cost.*



Soland now has **$150** to spend in *Tommy's Toy Store*. The shopping spree continues! This time try to purchase different items. Roll again!

Items Placed in the Shopping Cart	Cost of the Items
Toy Piano	*$65*
Doll	*$7*
Toy Car	*$18*
Drum	*$23*

5. How did you estimate the total cost of all the items? Show your thinking here:

Possible response: *In my head, I rounded $65 to $70, $7 to $10, $18 to $20, and $23 to $20. Then I mentally added $10 + $20 + $20 which equals $50. Then I added $70 and $50, which is $120.*

6. What is the exact cost of all the items? Use decomposition. Show your thinking here:

Possible response: *I decomposed $7 into $5 and $2. Then I added the $5 to the $65 to equal $70. Next, I add the $2 to the $18 to equal $20. Then I added $70 to $20, which equals $90. Finally, I decomposed $23 into $20 and $3. So $90 and $20 is $110. Then added $3 to equal $113. I wrote number sentences to show my thinking:*

($5 + $65) + ($2 + $18) + $20 + $3 =
($70 + $20) + $20 + $3 = $113

7. Think of one other way to mentally add the numbers to find the exact cost. Keep in mind the different ways to equal 10.

Possible response: *I could also just add $7 to $23 to equal $30. Then decompose $18 into $13 and $5. I could add that $5 to the $65 to equal $70. Next add $70 and $30 mentally, which is $100. Then $100 + $13 is $113.*

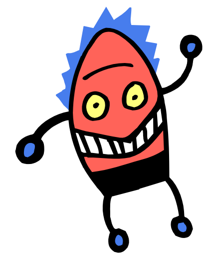
8. How close was your estimate in #5 to the exact cost you found in #6?

Possible response: *My estimate was only $7 more than the exact cost.*

Toy Store Wish List—Homework

Directions: Select any 5 different items from *Tommy's Toy Store* to put onto Soland's wish list and answer the questions below.

Wish List	Cost of the Items
Toy Plane	*$28*
Scooter	*$44*
Legos	*$3*
Checkers	*$9*
Soccer Ball	*$6*



1. Estimate the total cost of all the items. How did you estimate the total cost of all the items? Show your thinking here:

Possible response: *In my head, I rounded $28 to $30, $44 to $40, and $9 to $10. Then I added $30 + $40 + $10 to equal $80. Next, I added $3 and $6 in my head to equal $9. This was easy to add to $80 so the total estimate was $89.*

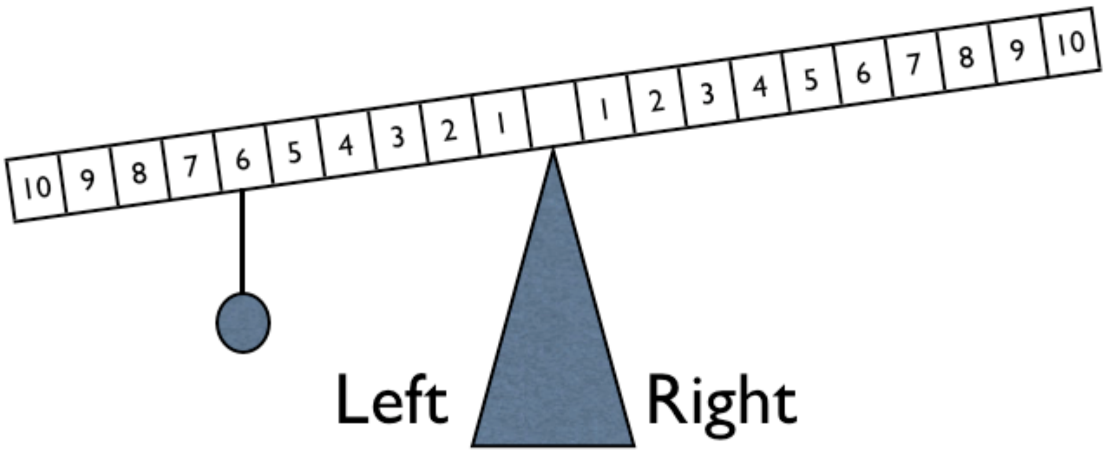
2. What is the exact cost of all the items? Show your thinking here:

Possible response: *First, I added $3 and $9 to get $12. I decomposed $28 into $20 and $8. Then I added $8 to the $12 to make $20. Then I added that to the other $20 to equal $40. Next, I added the $40 to the cost of the scooter to get $84. Then I decomposed $84 into $80 and $4. The cost of the soccer ball was $6, so I added that to the $4 which equals $10. Last, I added the $80 to the $10 to equal a total of $90 for everything on my wish list.*

3. How close was your estimate in #1 to the exact cost you found in #2?

Possible response: *My estimate was only $1 less than the exact answer.*

Introduction to Equal Sides



1. What can you do to balance this scale? ***Remember: The weight on the left may not be moved or taken off the scale*.**

A weight needs to be placed under the number 6 to balance the scale since the weight on the left is under the 6.

2. What if you were given 2 weights that needed to be placed on the right side? Where would you place them to make the scale balance?

   Location: ***Answers will vary.*** *Put a weight under the 4 and 2.*

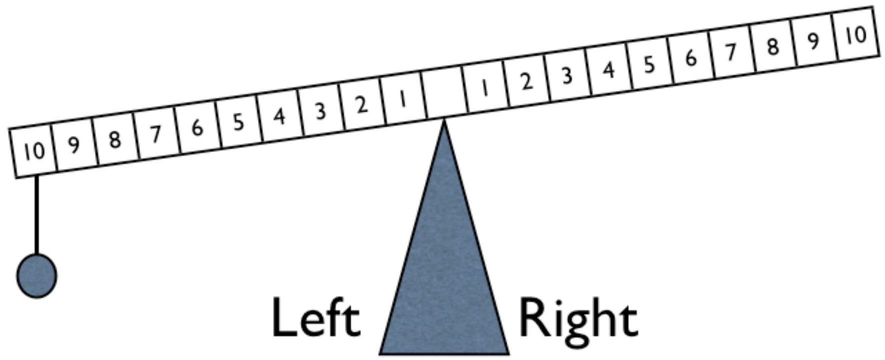
   Number Sentence: *6 = 4 + 2*

3. Where else could you place the two weights on the right side?

   Location: ***Answers will vary.*** *Put a weight under the 5 and 1.*

   Number Sentence: *6 = 5 + 1*

Equal Sides—Fibonacci



1. Your task is to make the balance level. Where would you need to place your weight if you only had 1 weight?

   Location: *Under the 10*

2. What if you had to use 2 weights on the right side?

   Locations: ***Answers will vary.*** *Put a weight under the 4 and 6.*

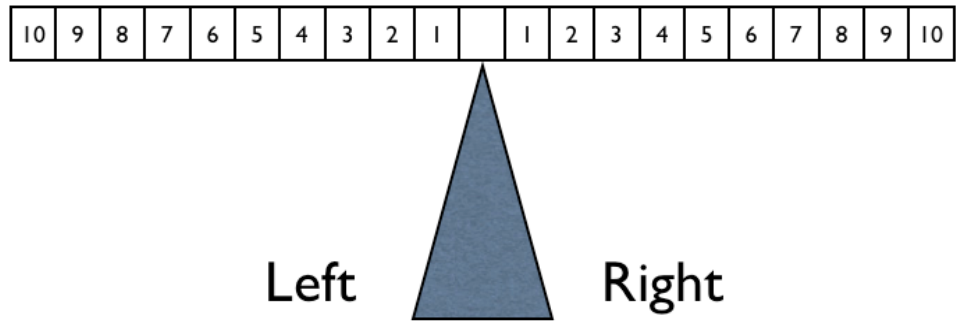
   Number Sentence: *10 = 4 + 6*

3. What if you had to use 3 weights on the right side?

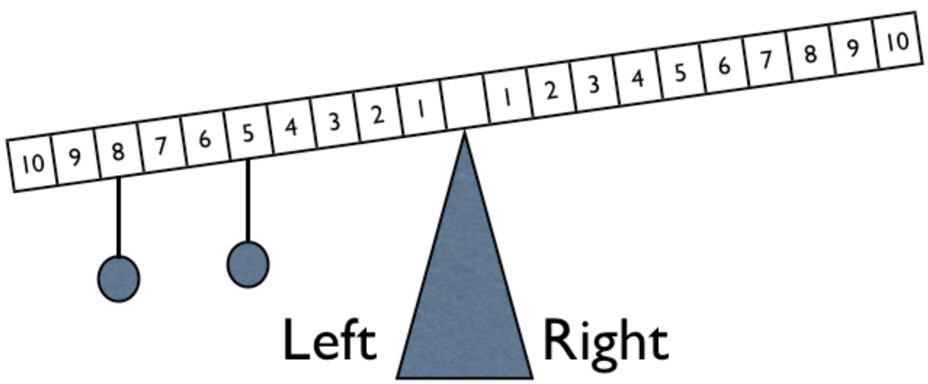
   Locations: ***Answers will vary.*** *Put a weight under the 2, 5, and 3.*

   Number Sentence: *10 = 2 + 5 + 3*

4. Create your own problem using 4 weights on four different numbers to make this scale balance. Write your number sentence below.



Number Sentence: ***Answers will vary.*** *Check student's work.*



5. What do the weights on the left side add up to? *13 pounds*

6. What is one way you could use 2 weights on the right side to balance the scale?

Locations: ***Answers will vary.*** *Put a weight under the 10 and 3.*

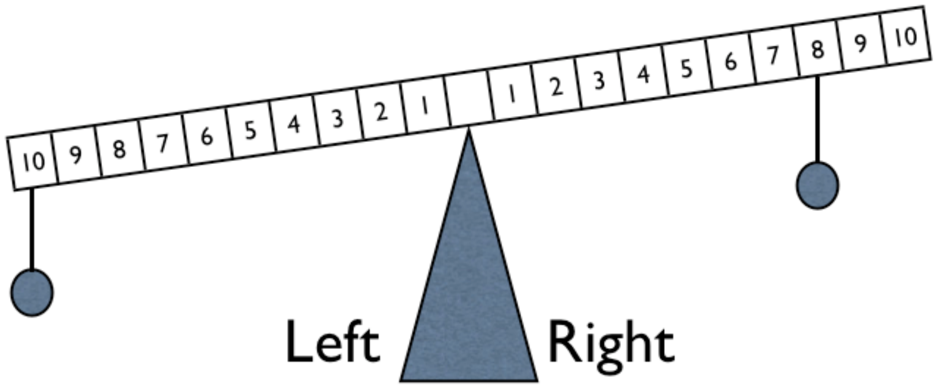
Number Sentence: *13 = 10 + 3*

7. What if you had to use 3 weights on the right side? Where would you place them?

Locations: ***Answers will vary.*** *Put a weight under the 4, 6, and 3.*

Number Sentence: *13 = 4 + 6 + 3*

If you have time before the end of the lesson, think about this problem.



8. What if you could use as many weights as you wanted? How many would you use and where would you place them? Explain where you would place them to make the scale balance.

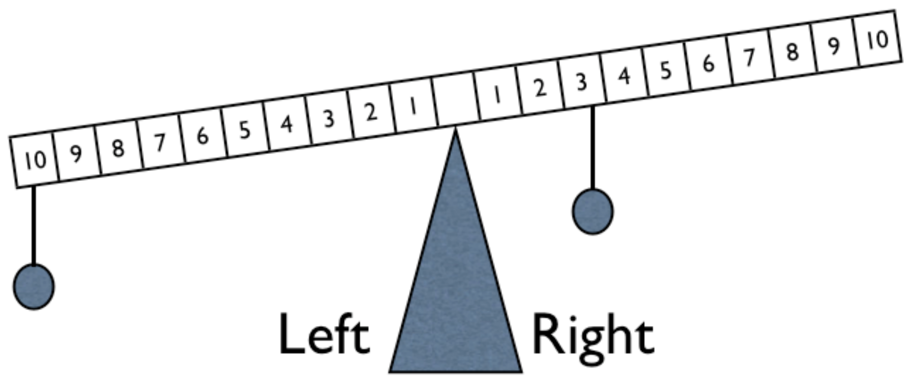
Location: ***Answers will vary.*** *Put 2 weights under the 1.*

Number Sentence: *10 = 8 + 1 + 1*

Explanation:

I know that 8 and 2 equals 10. So I broke up the 2 into two ones because I could use as many weights as I wanted.

Equal Sides—Diophantus



1. Your task is to make the balance level. Where would you need to place a weight if you only had 1 weight?

   Location: *Under the 7*

2. What if you had to use 2 weights on the right side?

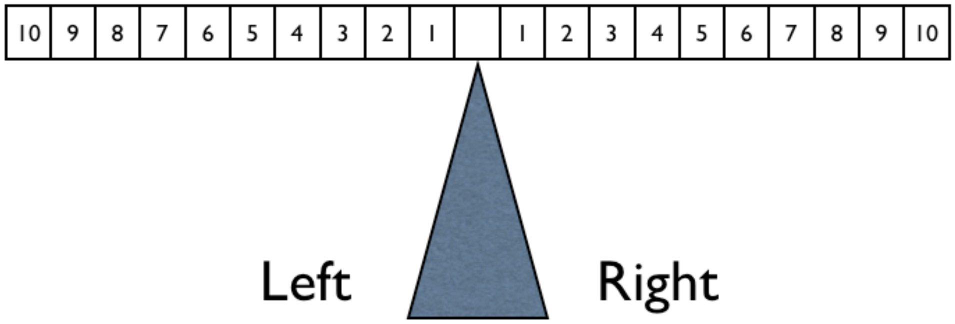
   Locations: ***Answers will vary.*** *Put a weight under the 3 and 4.*

   Number Sentence: *10 = 3 + 3 + 4*

3. What if you had to use 3 weights on the right side?

   Locations: ***Answers will vary.*** *Put a weight under the 3 and 2 weights under the 2.*

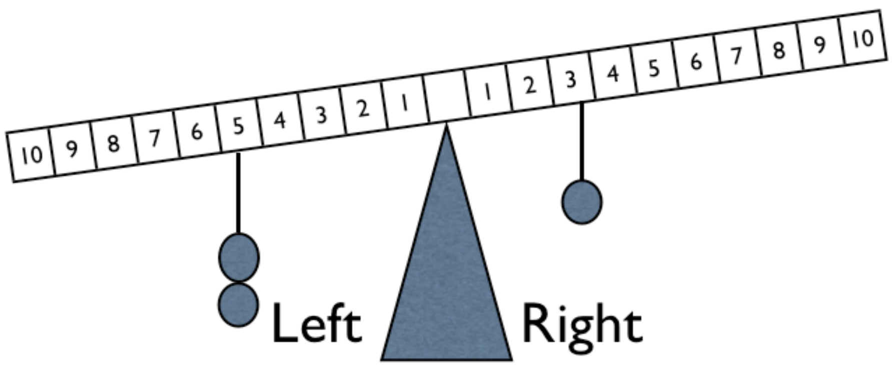
   Number Sentence: *10 = 3 + 3 + 2 + 2*



4. Create your own problem using 4 weights on four different numbers to make this scale balance. Write your number sentence below.

   Number Sentence: ***Answers will vary.*** *Check student's work.*

If you have time before the end of the lesson, think about these problems.



5. On the left side there are two weights under 5, which means there are two 5s or a total of 10. Where could you place 2 weights on the right side to make the scale balance?

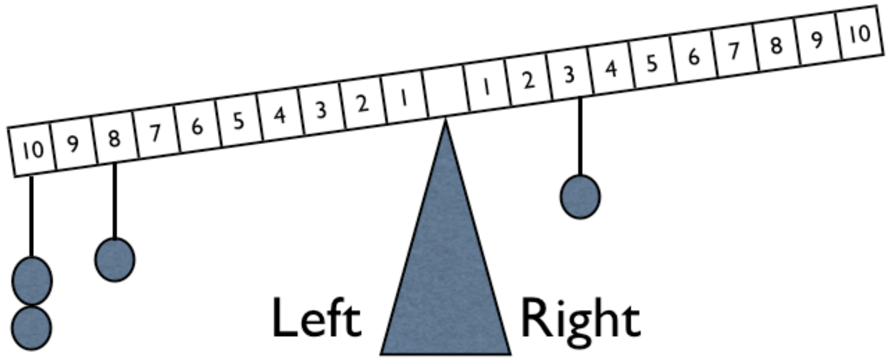
   Locations: ***Answers will vary.*** *Put a weight under the 5 and 2.*

   Number Sentence: *10 = 3 + 5 + 2*

6. What is another way you could place the 2 weights?

   Locations: ***Answers will vary****. Put a weight under the 6 and 1.*

   Number Sentence: *10 = 3 + 6 + 1*



7. What if you could use as many weights as you wanted? Explain your thinking.

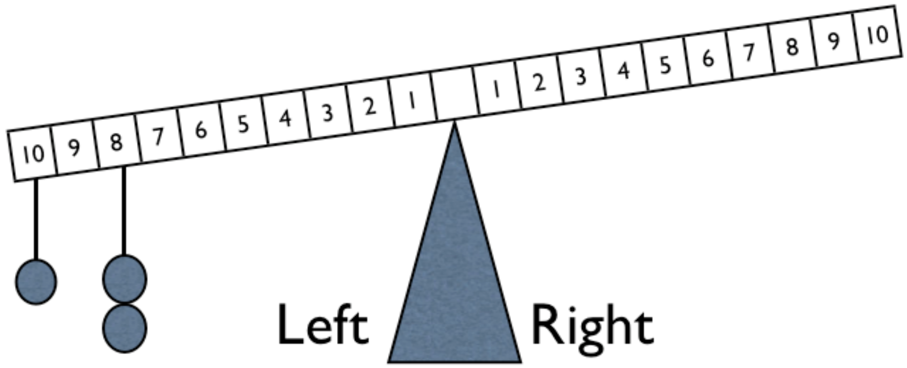
Locations: ***Answers will vary.*** *Put 2 weights under the 10 and 1 weight under the 5.*

Number Sentence: *28 = 3 + 10 + 10 + 5*

Explanation:

I decided to break up the 8 into 3 and 5. Then I just added the two 10s again to balance the scale.

Equal Sides—Kovalevsky



1. This scale has one weight under the 10 to make 10 pounds. Also, there are 2 weights under the 8. That means there are two 8-pound weights on the left side.

   How much weight does the left side have? *26 pounds*

2. Where could you place 4 weights on the right side to make the scale balance?

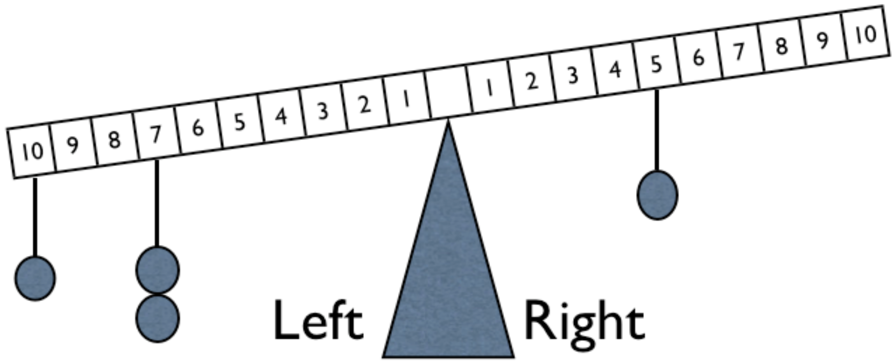
   Locations: ***Answers will vary.*** *Put 2 weights under the 10 and 2 weights under the 3.*

   Number Sentence: *26 = 10 + 10 + 3 + 3 or 26 = (10 + 10) + (3 + 3)*

3. What if you had to use six weights?

   Locations: ***Answers will vary.*** *Put 2 weights under the 7 and 4 weights under the 3.*

   Number Sentence: *26 = 7 + 7 + 3 + 3 + 3 + 3*



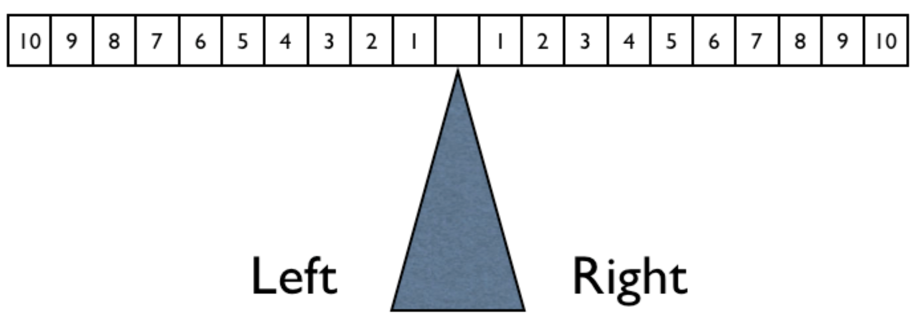
4. How much weight does the left side have now? *24 pounds*

5. Where would you place 4 weights on the right side to make the scale balance?

   Locations: ***Answers will vary.*** *Put 3 weights under the 5 and 1 weight under the 4.*

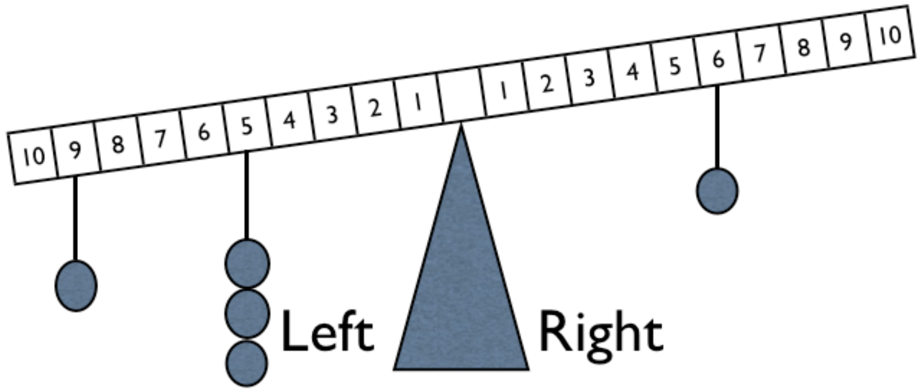
   Number Sentence: *24 = (5 + 5 + 5 + 5) + 4*

6. Create your own problem using 4 weights on four different numbers to make this scale balance. Write your number sentence below.



Number Sentence: ***Answers will vary.*** *3 + 5 + 1 + 4 = 7 + 3 + 2 + 1*

If you have time before the end of the lesson, think about this problem.



7. Where would you place 5 weights on the right side to balance the scale?

   Locations: ***Answers will vary.*** *Put 2 weights under the 4, 1 weight under the 1, 1 weight under the 2, and 1 weight under the 7.*

   Number Sentence: *24 = 6 + 4 + 4 + 1 + 2 + 7*

8. What is another way you could place 5 weights on the right side to balance the scale?

   Locations: ***Answers will vary.*** *Put 1 weight under the 6, 2 weights under the 2, 1 weight under the 3, and 1 weight under the 5.*

   Number Sentence: *24 = 6 + 6 + 2 + 2 + 3 + 5*

9. What if you could use as many weights as you wanted? Explain your thinking.

   Locations: ***Answers will vary.*** *Put 2 weights under the 4 and 2 weights under the 5.*

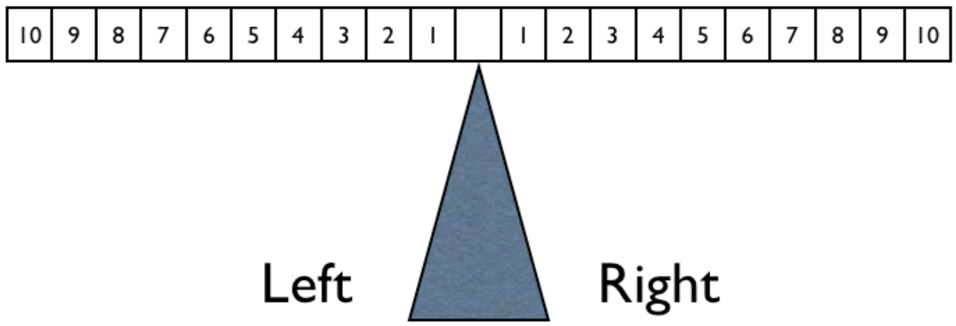
   Number Sentence: *24 = 6 + 4 + 4 + 5 + 5 = 6 + 5 + 5 + 4 + 4*

Explanation:

I decided to pick the same weights to show that it doesn't matter what order you add the addends, you still get the same sum. Even if I group them differently, I get the same answer, too.

Equal Sides—Homework

Directions: Create your own problem using 8 weights on each side to make this scale balance. Write your number sentence below.



Number sentence: ***Answers will vary.*** *3 + 7 + 2 + 4 + 4 + 6 + 3 + 1 = 5 + 4 + 1 + 8 + 2 + 1 + 7 + 2*

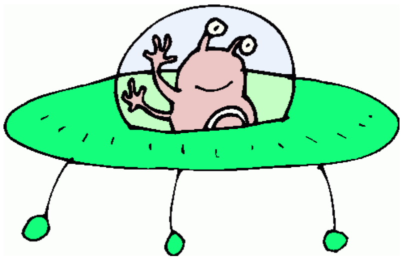
Use the space below if you need it to work out this problem.

Challenge: Make each side total to 30.

Introduction to 100s Charts

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

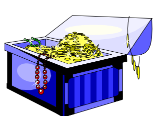
Where Did I Leave My Treasure?



Greetings Earthlings!
I accidentally left my treasure map in your Student Mathematician Notebook. Could you give me directions to find my treasure? I am currently at 19.

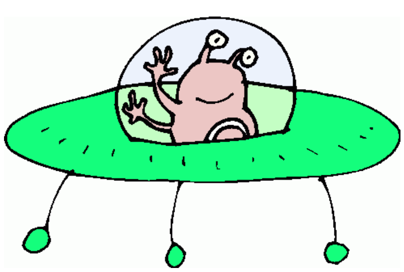
Start Here

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	✖	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



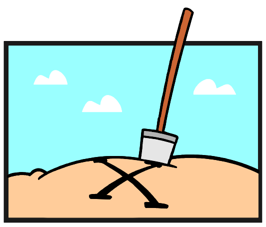
Record your directions here. Can you think of another way to describe how to get to the treasure that no one else will have?

Possible response: *Start at 19. Subtract 7 ones. Add 1 ten. Add 2 ones. Add 4 tens. Finally subtract 1 one. My Treasure is buried there.*

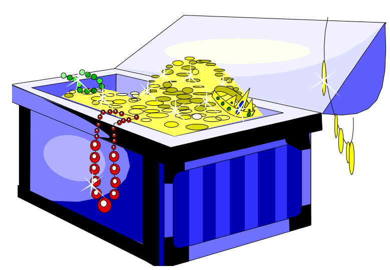


Thank YOU Humans!
I am so grateful for these directions! Soon I will be running my 8 fingers through my long lost treasure!

Treasure Hunt Directions for All Groups

 **Treasure Hider**	If you are the **treasure hider**, you need to place an "X" on your 100s chart where the treasure is hidden. **Do not show your partner!** Make sure your partner can not see where you hid the treasure. After your partner tells you where he/she is starting, you need to give directions to the treasure.	 **Treasure Finder**	If you are the **treasure finder**, circle the number where you started. You can pick any number. Tell the hider where you are starting. Listen carefully to your partner's directions. When he/she is finished giving directions, tell him/her where you believe the treasure is hidden.

Each person in the group should have at least one chance to hide the treasure and one chance to find the treasure. After everyone has a chance to do this, answer the questions. If you have more time, you can use the extra charts to play again. Good luck!



Treasure Hunt—Diophantus (Hider)

*(**Answers will vary.** Check student's work.)*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

I hid the treasure at ________________. My partner is starting at ________________. Here are my directions to the treasure:

Treasure Hunt—Diophantus (Finder)

*(**Answers will vary.** Check student's work.)*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

I am starting at ________________. After following my partner's directions, I believe the treasure is at ________________. Were you right? ________________

Treasure Hunt Questions—Diophantus

1. Did you find this task challenging? Why or why not?

Possible response: *I thought it was easy because I could add 10 to go down and add 1 to go right.*

2. How would you improve your directions?

Possible response: *I would use both an arrow and addition or subtraction.*

3. Tutah just emailed you the following directions: Start at 97, subtract four 10s, then subtract 5, add one 10, add 11, subtract four 10s, and add 2. Where is her treasure buried? Could you give simpler instructions? If so, how?

The treasure is buried at 35. Simpler directions may vary.
Possible response: *Subtract 2. Subtract 6 tens. The treasure is buried there.*

4. Using your knowledge about 100s chart patterns, fill in the missing numbers in the boxes from the 100s chart.

45	*46*	**47**		
		57	*58*	**59**
65	*66*			

Describe how you filled in the missing numbers.

Possible response: *I started at 59 and subtracted one twice to fill in the boxes to the left. I did the same thing with 47 to get 46 and 45. I then added two 10s to 45 and 46 to get 65 and 66.*

Treasure Hunt—Extra Diophantus (Hider)

*(**Answers will vary.** Check student's work.)*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

I hid the treasure at ________________. My partner is starting at ________________. Here are my directions to the treasure:

Treasure Hunt—Extra Diophantus (Finder)

(***Answers will vary.*** *Check student's work.)*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

I am starting at ________________. After following my partner's directions, I believe the treasure is at ________________. Were you right? ________________

Treasure Hunt—Kovalevsky (Hider)

*(**Answers will vary.** Check student's work.)*

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
51	52	53	54	55
56	57	58	59	60
61	62	63	64	65
66	67	68	69	70
71	72	73	74	75
76	77	78	79	80
81	82	83	84	85
86	87	88	89	90
91	92	93	94	95
96	97	98	99	100

I hid the treasure at ________________. My partner is starting at ________________. Here are my directions to the treasure:

Treasure Hunt—Kovalevsky (Finder)

*(**Answers will vary.** Check student's work.)*

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
51	52	53	54	55
56	57	58	59	60
61	62	63	64	65
66	67	68	69	70
71	72	73	74	75
76	77	78	79	80
81	82	83	84	85
86	87	88	89	90
91	92	93	94	95
96	97	98	99	100

I am starting at ________________. After following my partner's directions, I believe the treasure is at ________________. Were you right? ________________

Treasure Hunt Questions—Kovalevsky

1. How is this 100s chart different from the one you were using in the beginning of the lesson? Describe some of the patterns you see.

Possible response: *Instead of having 10 columns, this chart only has five. To go up or down you need to add or subtract 5, but left and right are the same.*

2. What was the most challenging part about this task?

Possible response: *It was difficult to move up and down on the 100s chart because I couldn't just change the 10s place.*

3. How would you improve your directions?

Possible response: *I would remember to say the number of 5s I want my partner to move when I want him or her to move up and down.*

4. Tutah just emailed you the following directions: Start at 97, subtract four 5s, then subtract 2, add one 5, add 2, subtract four 5s, and add 2. Where is her treasure buried? Could you give simpler instructions? If so, how?

The treasure is buried at 64.

Simpler instructions: *Start at 97. Subtract 7 fives. Add 2. The treasure is buried there.*

5. A new 100s chart just arrived that does not look like either of the charts you have been using. When you go down one column, the numbers increase by 7. When you go diagonally down and to the right, the numbers increase by 8. When you travel diagonally down and to the left, the numbers increase by 6. How many numbers are in each row of this chart?

There are seven numbers in each row.

6. Using your knowledge about 100s charts that have 5 numbers in each row like you used for your treasure hunt, fill in the missing numbers in the boxes from the 100s chart.

46	*47*	**48**		
		53	*54*	**55**
56	*57*			

Describe how you filled in the missing numbers.

Possible response: *I started at 55 and subtracted one twice for the 2 boxes to the left. I did the same process for 48 to get 46 and 47. I then added 10 to 46 and 47 to fill the boxes 2 rows below in their columns.*

Treasure Hunt—Extra Kovalevsky (Hider)

*(**Answers will vary.** Check student's work.)*

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
51	52	53	54	55
56	57	58	59	60
61	62	63	64	65
66	67	68	69	70
71	72	73	74	75
76	77	78	79	80
81	82	83	84	85
86	87	88	89	90
91	92	93	94	95
96	97	98	99	100

I hid the treasure at _______________. My partner is starting at _______________. Here are my directions to the treasure:

Treasure Hunt—Extra Kovalevsky (Finder)

*(**Answers will vary.** Check student's work.)*

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
51	52	53	54	55
56	57	58	59	60
61	62	63	64	65
66	67	68	69	70
71	72	73	74	75
76	77	78	79	80
81	82	83	84	85
86	87	88	89	90
91	92	93	94	95
96	97	98	99	100

I am starting at ________________. After following my partner's directions, I believe the treasure is at ________________. Were you right? ________________

Extra 100s Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Super Challenges

Super Challenge 1

Complete the 100s chart. **(Hint: This chart has 5 numbers in each row.)**

81	*82*			
		88	*89*	
				95

Super Challenge 2

Complete the 100s chart. **(Hint: This chart has 5 numbers in each row.)**

26			*29*	*30*
		33		
36			*39*	*40*

Super Challenge 3

Complete the 100s chart. **(Hint: This chart has 10 numbers in each row.)**

16				*20*
	27		*29*	
36		**38**		*40*

Super Challenge 4

Complete the 100s chart. **(Hint: this chart has 7 numbers in each row.)**

		12		
	18		*20*	
24				**28**

Pattern Mysteries—Homework

1. Tutah just texted you the following directions: Start at 29, add three 10s, then subtract 4, add 11, add two 10s, and add 3. Where is her treasure buried? Could you give simpler instructions? If so, how?

The treasure is buried at 89.

Possible response: *To make it simpler, you could just say "Add 6 tens."*

2. It is now your turn to hide the treasure! Decide where the treasure should be buried and write the instructions. Remember to give the starting number.

Answers will vary. *Check student's work.*

3. Using your knowledge about 100s charts that have 10 numbers in each row like you used for your treasure hunt, fill in the missing numbers in the boxes from the 100s chart.

21	*22*	**23**		
		33	*34*	**35**
41	*42*			

Describe how you filled in the missing numbers.

Possible response: *I subtracted 1 to complete the 1st row. I added 10 to 23 and then added 1 to complete the 2nd row. I added 2 tens to 21 and 22 to fill in the third row.*

Multiples of Three

Directions: Color in only the multiples of three.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

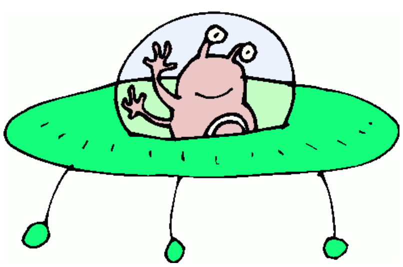
100s Chart Pattern Language

Diagonal

Column
(vertical)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

Row
(horizontal)



Greetings Earthlings!
I love 100s charts! Describe the pattern you see when you color in the multiples of 3.

Possible response: *I noticed that after I colored in all the multiples of 3, they formed 6 diagonal lines on the 100s chart. If I add 9 to any of the multiples, I would get to the next number in the diagonal.*

Searching for Patterns—Fibonacci

1. Focus on the numbers that are circled. What do you notice?

The circled numbers made vertical column patterns.

2. What are these numbers multiples of? (Remember you can pretend to start on 0 and count to the first number. That will help you find the multiples of the numbers.)

They are multiples of 2.

3. Look at the numbers that have triangles in the corners. What do you notice?

The numbers made diagonal patterns on every other row. The first row has one triangle and the next 2 rows have two triangles. This pattern repeats.

4. What are these numbers multiples of?

They are multiples of 6.

5. What is the 5th multiple of 3?

15

6. Look at both the triangles and circles. When do they overlap?

They overlap on the multiples of 2 and 6—some examples are the numbers 12 and 18.

Searching for Patterns—Fibonacci (Continued)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Searching for Patterns—Diophantus

1. Focus on the numbers that have triangles in the lower left corner. What do you notice?

They form diagonals going from the top right to the bottom left.

2. What are these numbers multiples of? (Remember you can pretend to start on 0 and count to the first number. That will help you find the multiples of the numbers.)

They are multiples of 3.

3. Look at the numbers that have suns. What do you notice?

They don't form diagonals.

4. What are these numbers multiples of?

They are multiples of 7.

5. Look at the numbers that have triangles around them. What do you notice?

They form diagonals going from the top right to the bottom left. To get to the next multiple of 9 you could add 10 and then subtract 1.

6. What are these numbers multiples of?

They are multiples of 9.

7. What is the 4th multiple of 9?

The 4th multiple is 36.

8. Look at all of the shapes. When do they overlap? What do you notice?

Every multiple of 9 overlaps with multiples of 3. The only number that is a multiple of 3, 7, and 9 is 63.

Searching for Patterns—Diophantus
(Continued)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Searching for Patterns—Kovalevsky

1. Focus on the numbers that have clouds around them. What do you notice?

If you add 10 and then subtract 2, you will get to the next number with a cloud. They almost form a diagonal pattern on the chart.

2. What are these numbers multiples of? (Remember you can pretend to start on 0 and count to the first number. That will help you find the multiples of the numbers.)

They are multiples of 8.

3. Look at the numbers that have a triangle in the top left corner. What do you notice?

The numbers made diagonal patterns on every other row. The first row has 1 triangle and the next 2 rows have 2 triangles. This pattern repeats as you go down the 100s chart.

4. What are these numbers multiples of?

They are multiples of 6.

5. Look at all of the shapes. When do they overlap? What do you notice?

The shapes overlap on the numbers 24, 48, 72, and 96. I noticed that if I add 24 to these numbers, I will get the next number in the list.

6. If you were to color in the multiples of 3, which shape would be colored in most often? Least often?

The triangles would be colored in the most often because 6 is a multiple of 3. The clouds would be colored in the least often because 8 is not a multiple of 3.

Searching for Patterns—Kovalevsky
(Continued)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Extra 100s Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

500s Chart

501	502	503	504	505	506	507	508	509	510
511	512	513	514	515	516	517	518	519	520
521	522	523	524	525	526	527	528	529	530
531	532	533	534	535	536	537	538	539	540
541	542	543	544	545	546	547	548	549	550
551	552	553	554	555	556	557	558	559	560
561	562	563	564	565	566	567	568	569	570
571	572	573	574	575	576	577	578	579	580
581	582	583	584	585	586	587	588	589	590
591	592	593	594	595	596	597	598	599	600

Super Challenges

Super Challenge 1

Soland does not like when a number has more than one symbol or color on it. Which chart would Ally like more?

- a chart with the multiples of 2 and 3 marked?

  or

- a chart with the multiples of 2 and 4 marked?

Explain your answer.

Possible response: *Ally would like to have a chart with multiples of 2 and 3 because 3 is not a multiple of 2, so there would be fewer overlaps.*

Super Challenge 2

Ask your teacher for a copy of the 500s chart. Compare the 500s chart with the 100s chart. What would happen if you colored in the multiples of 3 on both charts? **(Hint: 600 is a multiple of 3.)** Would the patterns look the same? What if you colored in the multiples of 5? Would the patterns be the same?

Possible response: *If you color in the multiples of 3 on both charts, a diagonal pattern from the top right to the bottom left is made. The difference is that the diagonals are in different places. The patterns for the multiples of 5 are the same and the columns are in the same places.*

Super Challenge 3

Create a 100s chart pattern using a mathematical concept. For example, all the multiples of 3 or all the even numbers could be colored. You could even share it with a partner. Ask him/her if he/she can figure out your pattern.

Answers will vary. *Check student's work.*

Super Challenge 4

What if instead of the rows ending in multiples of 10s, the first row ended with multiples of 5 like this?

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

What do the multiples of 2 look like in this new chart? How is it different from the 100s chart?

Possible response: *The multiples of 2 would form diagonals. On the 100s chart they form columns.*

Searching for Patterns—Homework

1. What multiples are designated with the circle? The triangle?

The circles are on the multiples of 2 and the triangles are on the multiples of 9.

2. What is the 6th multiple of 9?

54

3. Choose a number between 1 and 20. Color in all the multiples of that number. You should not choose a multiple already designated.

Answers will vary. *Check student's work.*

4. Describe the patterns you see.

Answers will vary. *Students may point out if the patterns create columns or diagonals.*
Possible response: *All the even numbers are circled and all the multiples of 9 have triangles on them.*

5. Which multiples share the most numbers?

Answers will vary.
Possible response: *Multiples of 2 and 4 share a lot of numbers.*

Searching for Patterns—Homework
(Continued)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Getting to Know Planet Nine Aliens

A spaceship full of Planet Nine aliens has just landed in your backyard. As the first Planet Nine alien steps out, you notice that it has 1 head, 2 eyes, 6 toes, and 8 fingers. Use the table to keep track of the number of heads, eyes, toes, and fingers as the Planet Nine aliens get out of their spaceship.

Number of Planet Nine Aliens	Number of Heads	Number of Eyes	Number of Toes	Number of Fingers
0	*0*	*0*	*0*	*0*
1	*1*	*2*	*6*	*8*
2	*2*	*4*	*12*	*16*
3	*3*	*6*	*18*	*24*
4	*4*	*8*	*24*	*32*
5	*5*	*10*	*30*	*40*
6	*6*	*12*	*36*	*48*
7	*7*	*14*	*42*	*56*
8	*8*	*16*	*48*	*64*
9	*9*	*18*	*54*	*72*
10	*10*	*20*	*60*	*80*
11	*11*	*22*	*66*	*88*
12	*12*	*24*	*72*	*96*

Name one strategy you used to find your answer.

Answers will vary.
Possible response: *I skip counted by the number of Planet Nine aliens.*

Questions

1. Which column was the easiest to calculate? Explain why.

Possible response: *The column for number of heads was easiest to calculate because it was equal to the number of Planet Nine aliens.*

2. a. How many humans are in a group with 60 toes?

   Possible response: *There are 6 humans in a group with 60 toes.*

   b. How many Planet Nine aliens are in a group with 60 toes?

   Possible response: *There are 10 Planet Nine aliens in a group with 60 toes.*

   c. If another group of Planet Nine aliens has 12 toes each, how many would be in a group with 60 toes? Show your work below.

   Possible response: *There would be 5 Planet Nine aliens in a group with 60 toes. Since you have to add 12 five times to get 60, (12 + 12 + 12 + 12 + 12 = 60), there must be 5 Planet Nine aliens.*

3. Create your own Planet Nine alien species. Work with a partner to create a problem like the one you just did. You must name 3 body parts that the Planet Nine aliens have and tell how many of each body part they have. You MAY NOT use the numbers 1, 2, 6, 8, or 10.

   YOUR STORY:

Stories will vary, *but must include 3 different body parts without using the numbers 1, 2, 6, 8, 10.*

YOUR TABLE TITLE:

Possible response: *Body parts and numbers can vary significantly.*

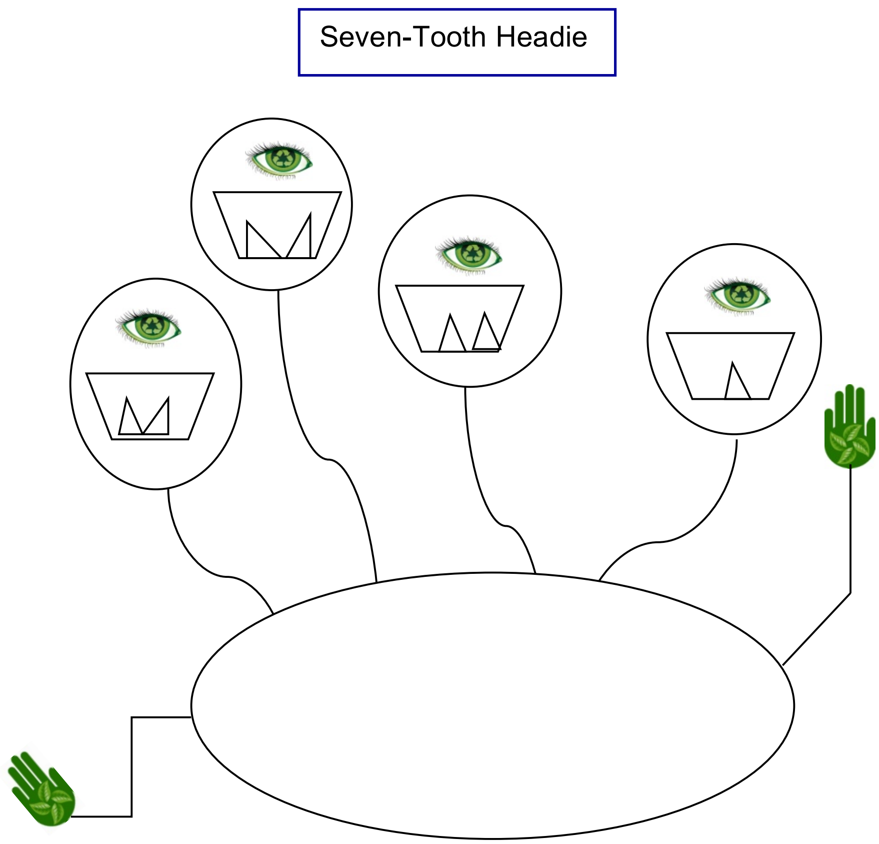
Number of Planet Nine Aliens	Number of <u>*ELBOWS (3)*</u>	Number of <u>*HEADS (4)*</u>	Number of <u>*TEETH (7)*</u>
0	*0*	*0*	*0*
1	*3*	*4*	*7*
2	*6*	*8*	*14*
3	*9*	*12*	*21*
4	*12*	*16*	*28*
5	*15*	*20*	*35*
6	*18*	*24*	*42*
7	*21*	*28*	*49*
8	*24*	*32*	*56*

Name one strategy you used to find your answers.

Answers may vary.
Possible response: *I skip counted by the number of Planet Nine aliens to figure out how many elbows, heads, and teeth they had in all. For example, for 2 Planet Nine aliens, I added 3 + 3 to get to 6 elbows.*

Planet Nine Alien Visit

Draw a picture of a Planet Nine alien from the species you created. Give him or her a name!



Getting to Know Planet Nine Aliens—Homework

Pick any starting number and count up by 2s until you fill in all the lines.

Answers will vary. *Check student's work.*

____ ____ ____ ____ ____ ____ ____ ____ ____ ____ ____

Count up by 10s starting from 13 until you fill in all the lines.

13 23 33 43 53 63 73 83 93 103 113

Fill in the missing boxes in the table.

Number of Planet Nine Aliens	Number of Heads	Number of Eyes	Number of Toes	Number of Fingers
0	**0**	**0**	**0**	**0**
1	**4**	**3**	**7**	**11**
2	*8*	*6*	*14*	*22*
3	*12*	*9*	*21*	*33*
4	*16*	*12*	*28*	*44*

How did you find the number of toes for 4 Planet Nine aliens?

Answers will vary.
Possible response: *I skipped counted by 7 four times to get 28.*

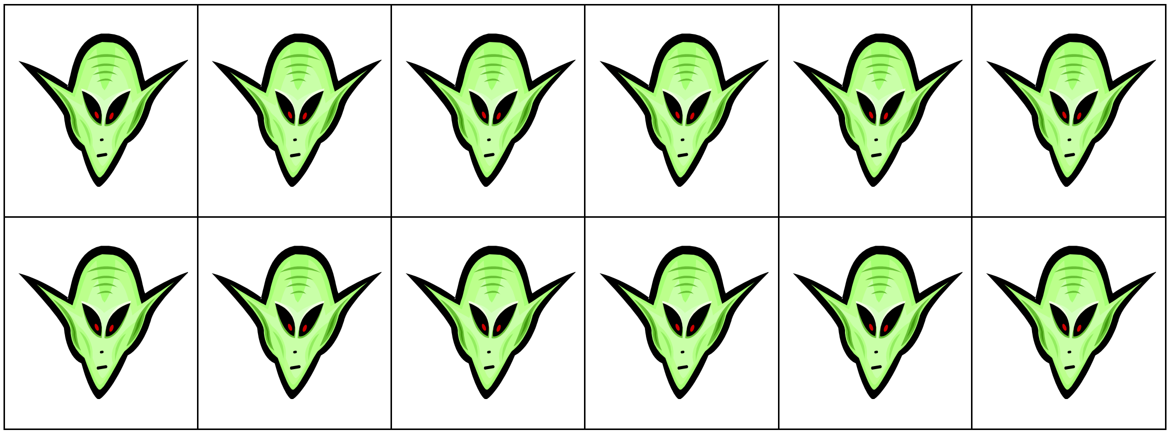
Count up by 6s starting from 0 until you fill in all the lines.

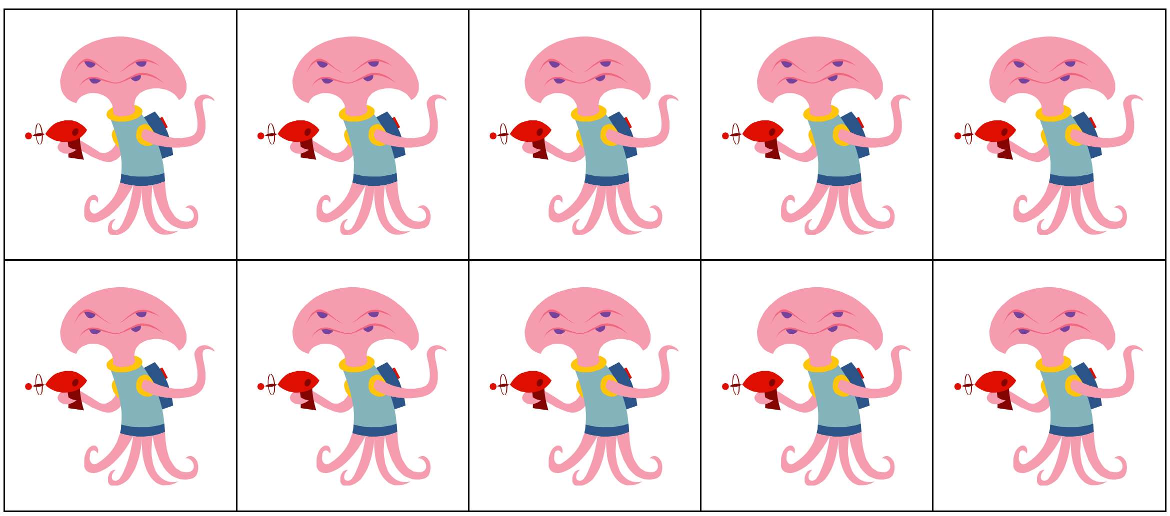
0 6 12 18 24 30 36 42 48 54 60

Count up by 12s starting from 0 until you fill in all the lines.

0 12 24 36 48

Planet Nine Alien Manipulatives





Alien Multiplication Tables

2-Eyed Planet Nine Alien Multiplication Table

Table A

Number of Planet Nine Aliens	Multiplication Problem Used to Find Total	Commutative Property (**Hint:** Flip It!)	Total Number of Eyes
0	0 x 2	2 x 0	0
1	1 x 2	2 x 1	2
2	*2 x 2*	*2 x 2*	*4*
3	*3 x 2*	*2 x 3*	*6*
4	*4 x 2*	*2 x 4*	*8*
5	*5 x 2*	*2 x 5*	*10*
6	*6 x 2*	*2 x 6*	*12*
7	*7 x 2*	*2 x 7*	*14*
8	*8 x 2*	*2 x 8*	*16*
9	*9 x 2*	*2 x 9*	*18*
10	*10 x 2*	*2 x 10*	*20*
11	*11 x 2*	*2 x 11*	*22*
12	*12 x 2*	*2 x 12*	*24*

4-Eyed Planet Nine Alien Multiplication Table

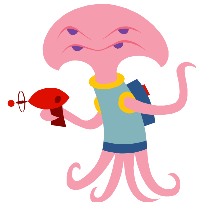


Table B

Number of Planet Nine Aliens	Multiplication Problem Used to Find Total	Commutative Property (**Hint:** Flip It!)	Total Number of Eyes
0	0 x 4	4 x 0	0
1	1 x 4	4 x 1	4
2	*2 x 4*	*4 x 2*	*8*
3	*3 x 4*	*4 x 3*	*12*
4	*4 x 4*	*4 x 4*	*16*
5	*5 x 4*	*4 x 5*	*20*
6	*6 x 4*	*4 x 6*	*24*
7	*7 x 4*	*4 x 7*	*28*
8	*8 x 4*	*4 x 8*	*32*
9	*9 x 4*	*4 x 9*	*36*
10	*10 x 4*	*4 x 10*	*40*
11	*11 x 4*	*4 x 11*	*44*
12	*12 x 4*	*4 x 12*	*48*

Multi-Eyed Planet Nine Alien Multiplication Table

Some students at The Oberon Academy have different numbers of eyes. For example, 2 students at The Academy have 3 eyes. How many eyes do the students have altogether?

Table C

Number of Planet Nine Aliens	Number of Eyes	Multiplication Problem Used to Find Total	Commutative Property (**Hint:** Flip It!)	Total Number of Eyes
2	3	2 x 3	3 x 2	6
5	4	*5 x 4*	*4 x 5*	*20*
3	6	*3 x 6*	*6 x 3*	*18*
8	5	*8 x 5*	*5 x 8*	*40*
4	4	*4 x 4*	*4 x 4*	*16*
6	7	*6 x 7*	*7 x 6*	*42*
3	9	*3 x 9*	*9 x 3*	*27*
11	4	*11 x 4*	*4 x 11*	*44*
10	6	*10 x 6*	*6 x 10*	*60*
7	5	*7 x 5*	*5 x 7*	*35*
9	8	*9 x 8*	*8 x 9*	*72*
12	4	*12 x 4*	*4 x 12*	*48*
6	6	*6 x 6*	*6 x 6*	*36*

Alien Multiplication Tables

(Continued)

(Answers will vary)

1. Why is the first answer the same in Tables A and B?

Possible response: *The first answer is the same because there is a zero in the problem and any time you multiply by zero the product will be zero.*

2. How many 2-eyed Planet Nine aliens are in a group with 12 eyes?

6 Planet Nine aliens

3. How many 3-eyed Planet Nine aliens are in a group with 12 eyes?

4 Planet Nine aliens

4. How many 4-eyed Planet Nine aliens are in a group with 12 eyes?

3 Planet Nine aliens

5. Why is the answer the same for the inverse of each multiplication algorithm?

Possible response: *If the factors in a multiplication number sentence are flipped, the answer will stay the same.*

6. Can you get exactly 12 eyes in a group of 5-eyed Planet Nine aliens? Explain.

Possible response: *No, there could be a total of 5 eyes, 10 eyes, or 15 eyes. You wouldn't have 12 eyes because 12 is not a multiple of 5.*

Eye Love Multiplication!
(OPTIONAL)

Some students at The Oberon Academy have different numbers of eyes. How many eyes do the students have altogether? Create problems for yourself or ask someone to create them for you!

Table D

Number of Planet Nine Aliens	Number of Eyes	Multiplication Problem Used to Find Total	Commutative Property/ Inverse (**Hint:** Flip It!)	Total Number of Eyes
2	3	2 x 3	3 x 2	6

Answers may vary. *Check student's work.*

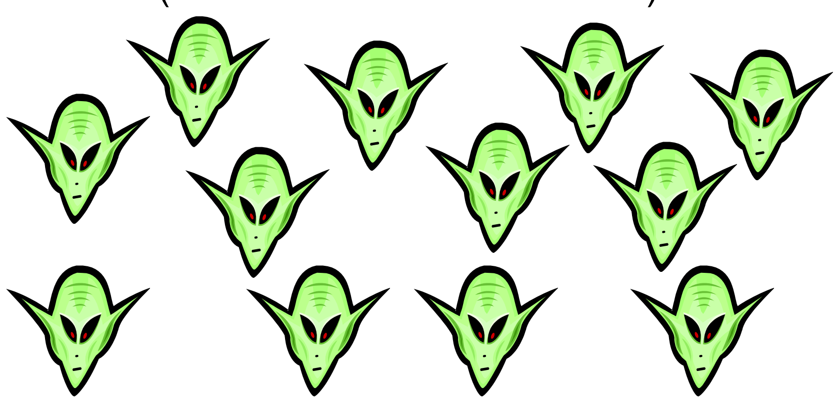
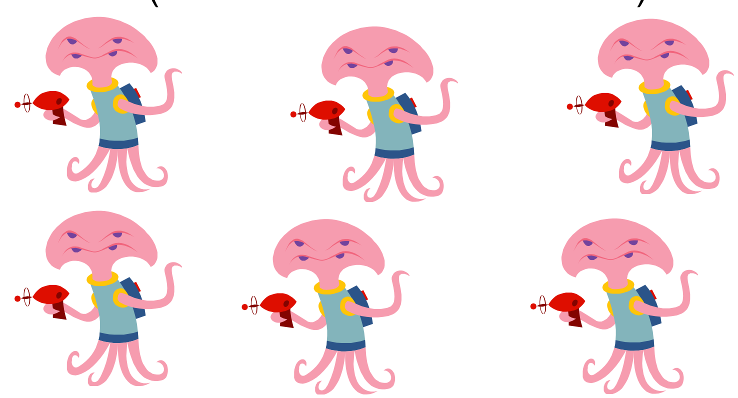
Multiplication Chart

X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Picture This!

(Answers will vary)

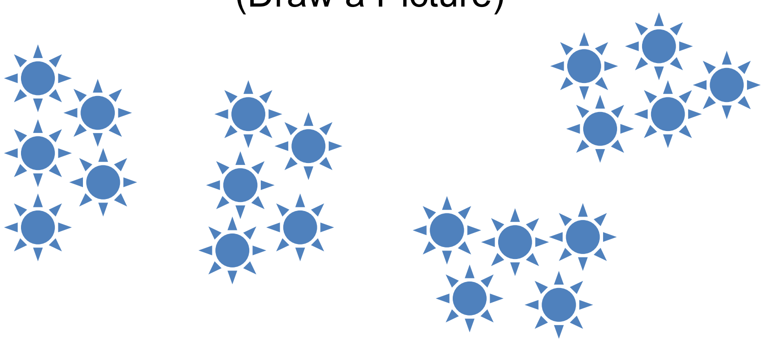
Directions: Glue the number of Planet Nine aliens needed to make 24 eyes for each group. Next, represent the total number of eyes using a multiplication and addition problem. Then, write a word problem.

<table>
<tr><th colspan="2">2-Eyed Planet Nine Aliens</th><th colspan="2">4-Eyed Planet Nine Aliens</th></tr>
<tr><td colspan="2">(Glue Planet Nine aliens here)
</td><td colspan="2">(Glue Planet Nine aliens here)
</td></tr>
<tr><td>Multiplication Problem:
2 x 12 = 24</td><td>Repeated Addition:
2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 24</td><td>Multiplication Problem:
4 x 6 = 24</td><td>Repeated Addition:
4 + 4 + 4 + 4 + 4 + 4 = 24</td></tr>
<tr><td colspan="2">Multiplication Word Problem: Twelve Planet Nine aliens at The Azimuth School for Math and Science have 2 eyes. How many eyes do they have altogether? *The Planet Nine aliens have 24 eyes in total.</td><td colspan="2">Multiplication Word Problem: Six Planet Nine aliens in the grade 3 class at The Pulsar School of the Arts have 4 eyes. How many eyes do they have altogether? *The Planet Nine aliens have 24 eyes altogether.</td></tr>
</table>

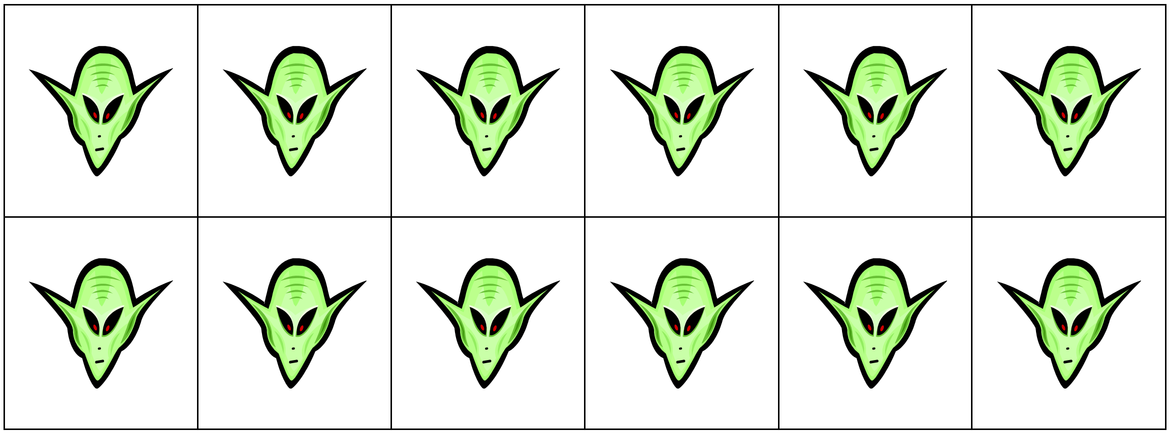
Picture This! Your Turn!

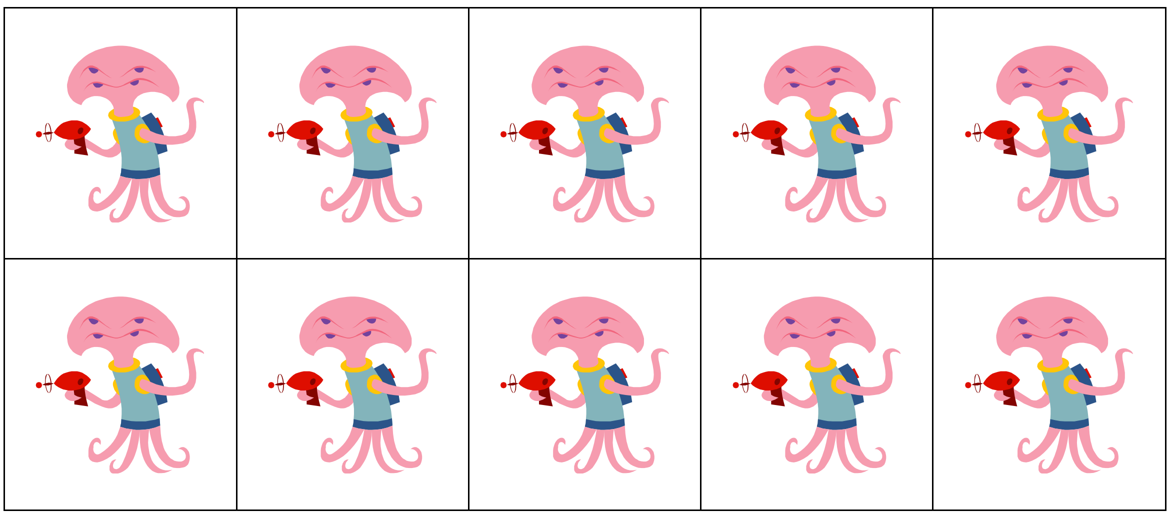
*(**Answers will vary.** Check student's work.)*

Directions: Now it is your turn to create your own multiplication problem. Represent your problem in pictures, numbers, and words.

<table>
<tr><th colspan="2">Gamma-Ray Fruit</th><th colspan="2">Cans of Paint</th></tr>
<tr><td colspan="2">(Draw a Picture)
</td><td colspan="2">(Draw a Picture)
</td></tr>
<tr><td>Multiplication Problem:
5 x 4 = 20</td><td>Repeated Addition:
5 + 5 + 5 + 5 = 20</td><td>Multiplication Problem:
6 x 3 = 24</td><td>Repeated Addition:
6 + 6 + 6 = 18</td></tr>
<tr><td colspan="2">Multiplication Word Problem: Zeefar went to Helios Market to buy gamma-ray fruit. If there are 5 pieces of fruit in 1 package, and Zeefar buys 4 packages, how many gamma-ray fruit does he buy altogether? *Zeefar buys 20 gamma-ray fruits altogether.</td><td colspan="2">Multiplication Word Problem: The art teacher at The Pulsar School of the Arts is giving each kindergarten through grade 5 class 3 cans of paint to use for projects. How many cans of paint does she give to the classes in total? *The art teacher gives the classes 18 cans of paint in total.</td></tr>
</table>

Planet Nine Alien Manipulatives for Picture This!



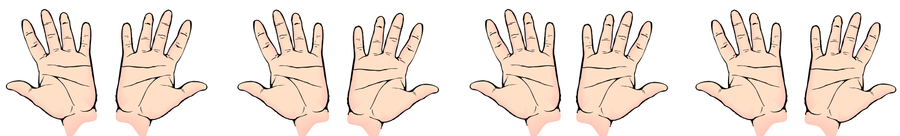


Multiplication Problems—Homework

Directions: You will need to cut out the pictures of hands and six-packs of soda cans on page 175 to complete your homework. Read each problem and solve it by pasting the appropriate number of pictures and writing a multiplication number sentence.

1. There are 4 humans in a room. How many fingers are in the room?

   Paste your pictures here:

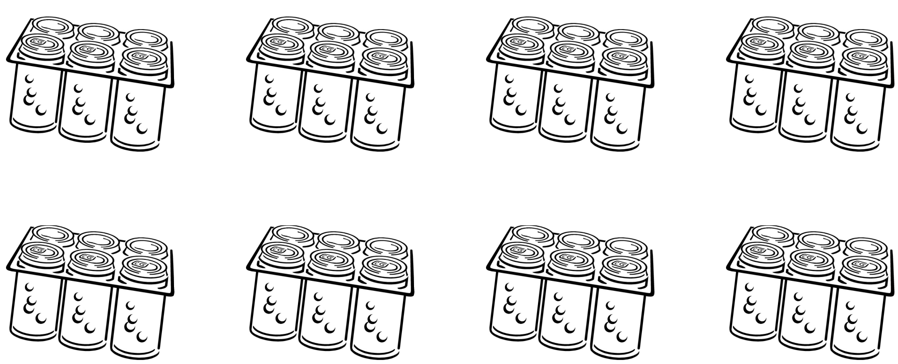


   Multiplication problem with answer:

   Answers will vary. *Check student's work.*
   Possible answer: 8 hands x 5 fingers = 40 fingers

2. Warsu's mom goes to the store and buys 8 six-packs of soda. How many cans of soda does she buy?

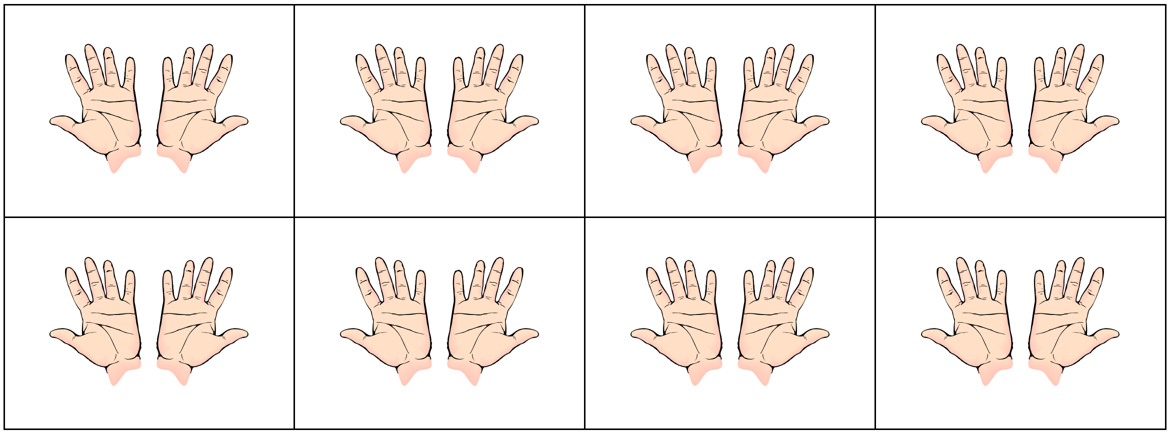
   Paste your pictures here:



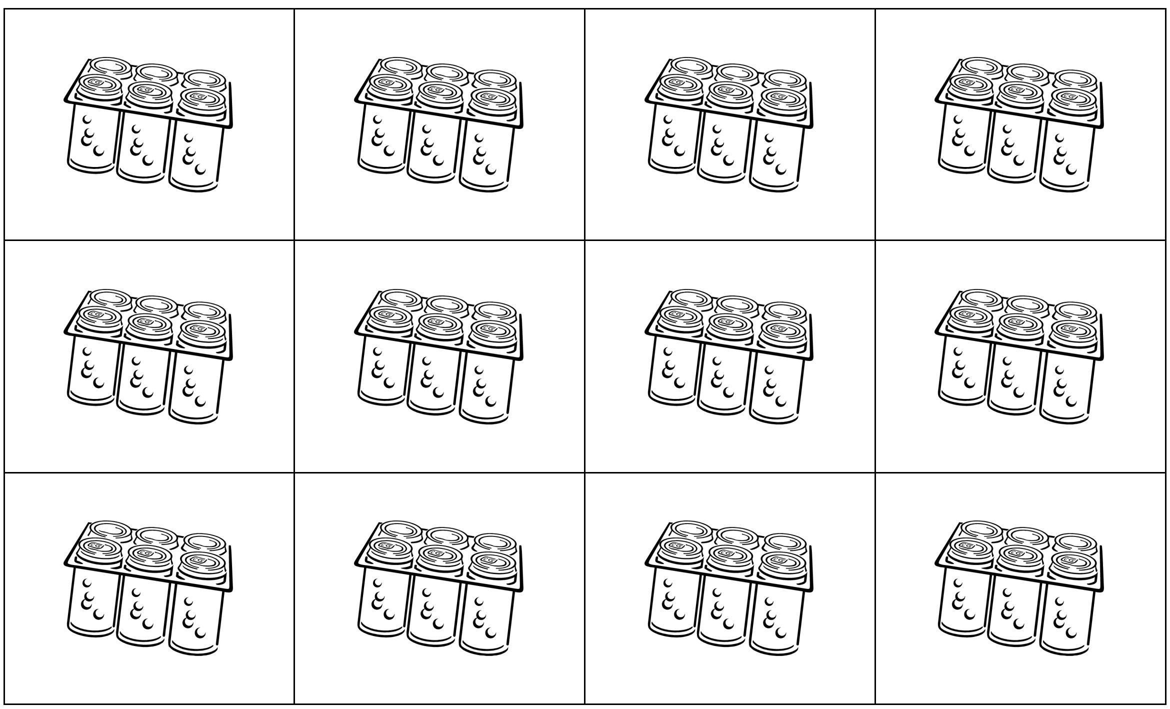
   Multiplication problem with answer:

   Answers will vary. *Check student's work.*
   Possible answer: 8 six-packs x 6 cans of soda = 48 cans of soda

HANDS

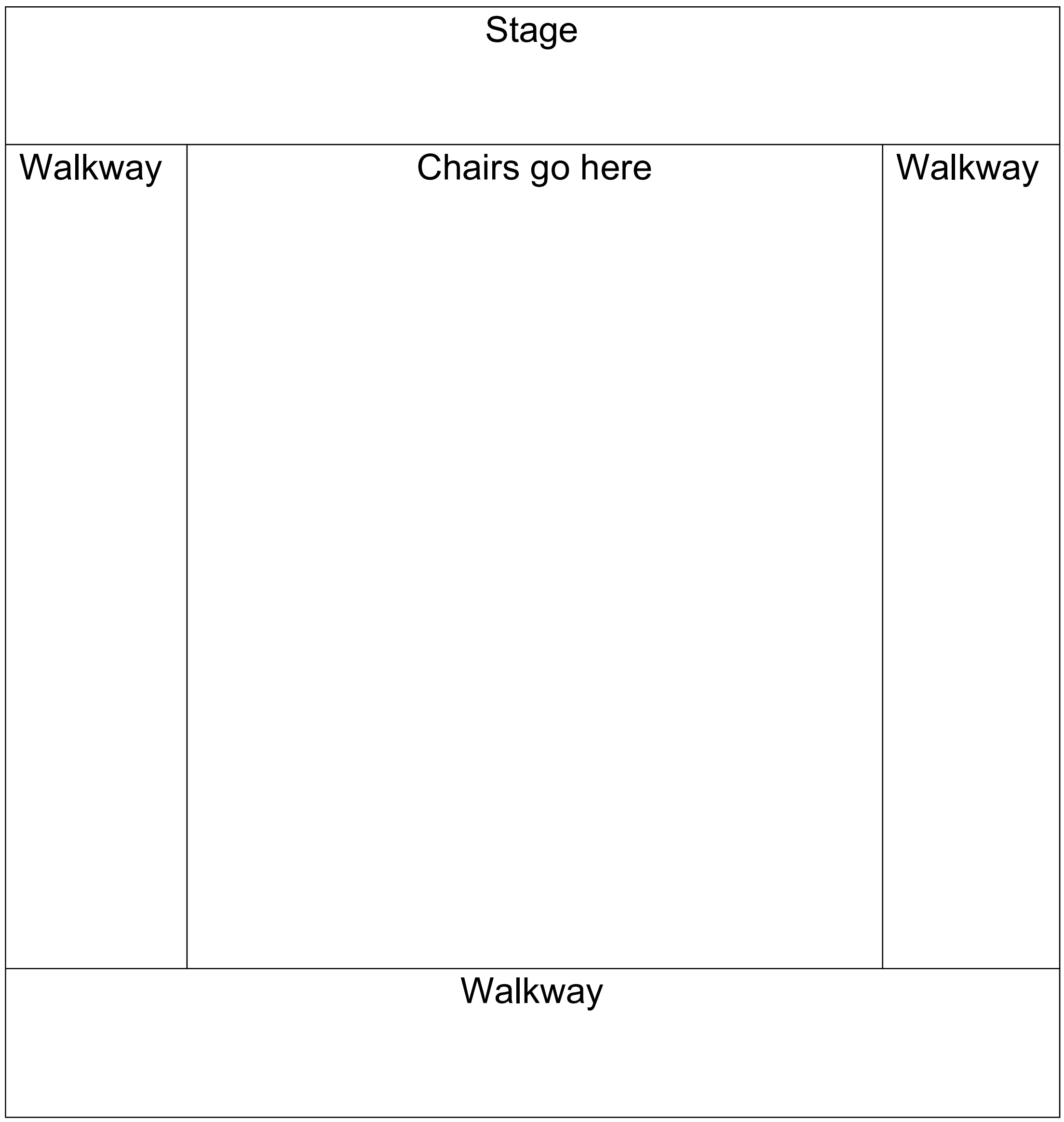


SIX-PACKS OF SODA



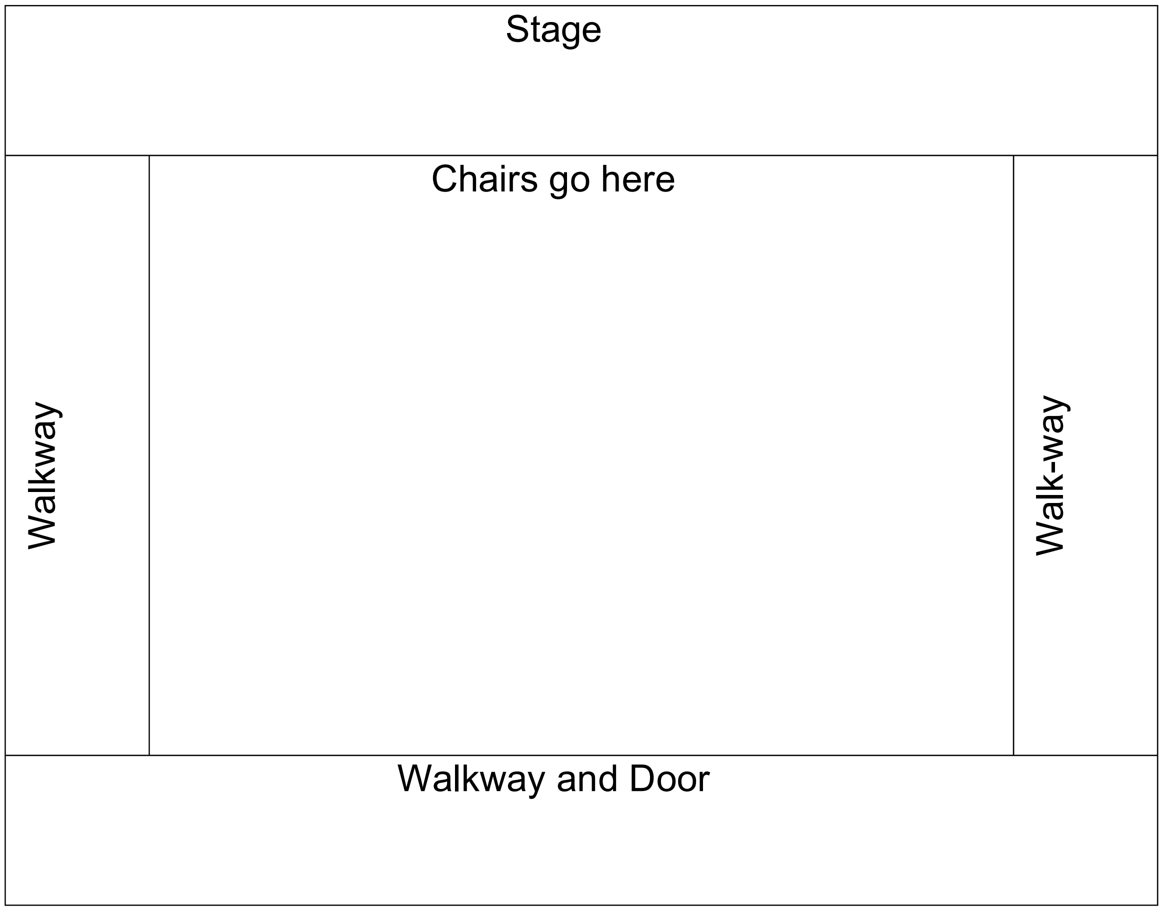
Acting Planet Nine Aliens

Arrangement A

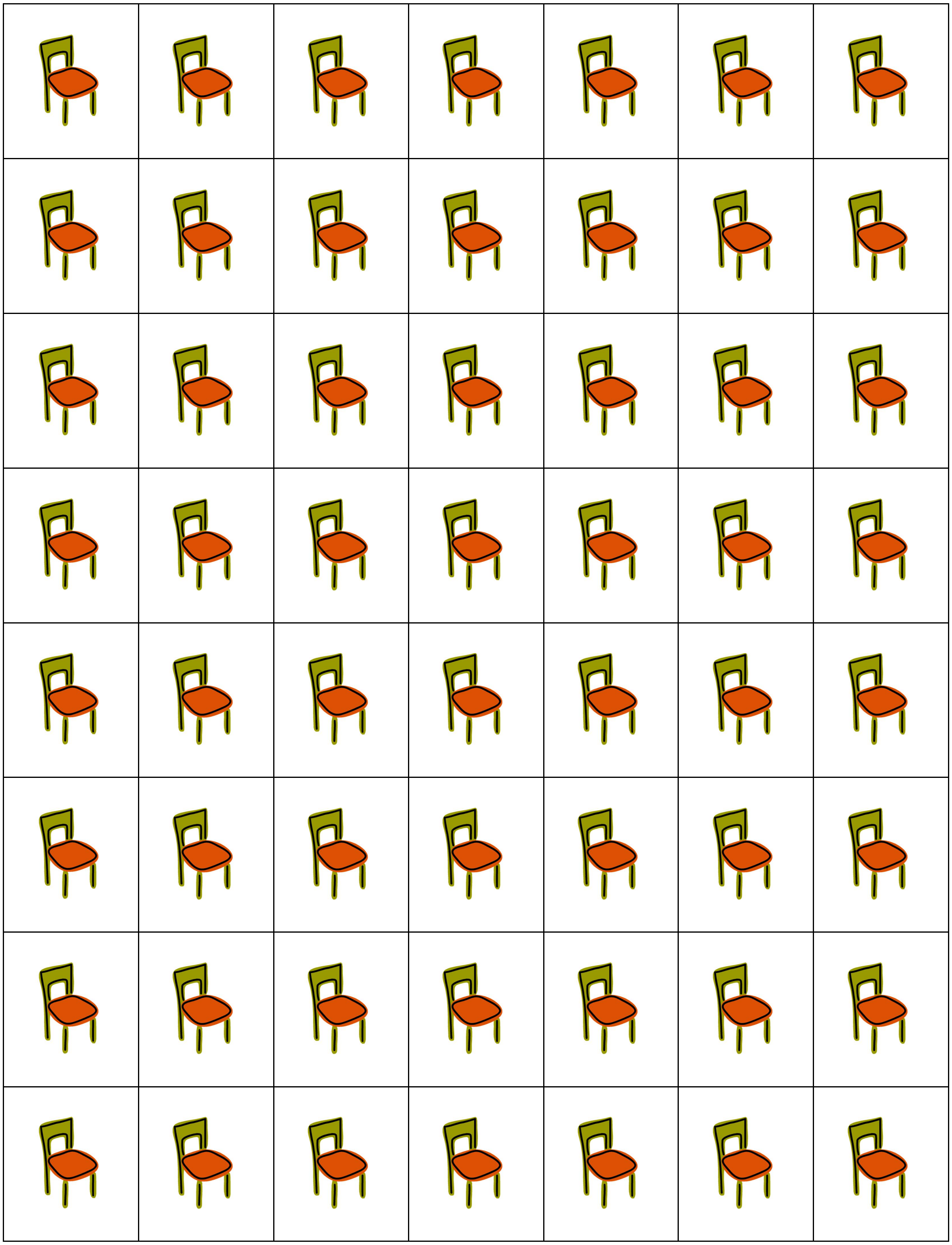


Acting Planet Nine Aliens

Arrangement B



Chair Manipulatives



Marching Band Planet Nine Aliens—Fibonacci
(Answers will vary)

The Planet Nine aliens really like to perform. One group of Planet Nine aliens brought instruments. They would like to perform in the "Welcome to Earth" Celebration. Unfortunately, the bandstand that the Planet Nine aliens will be performing on cannot hold all the Planet Nine aliens. If 5 Planet Nine aliens can fit on each row and there are 4 rows, how many Planet Nine aliens can perform on the bandstand? Explain your answer.

Possible response: *20 Planet Nine aliens. I know that there are 4 groups of 5, so if I add 5 + 5 + 5 + 5 I will get 20.*

The city is considering putting up 2 more bandstands that are the same size. How many Planet Nine aliens can perform now?

60 Planet Nine aliens

Marching Band Planet Nine Aliens—Diophantus

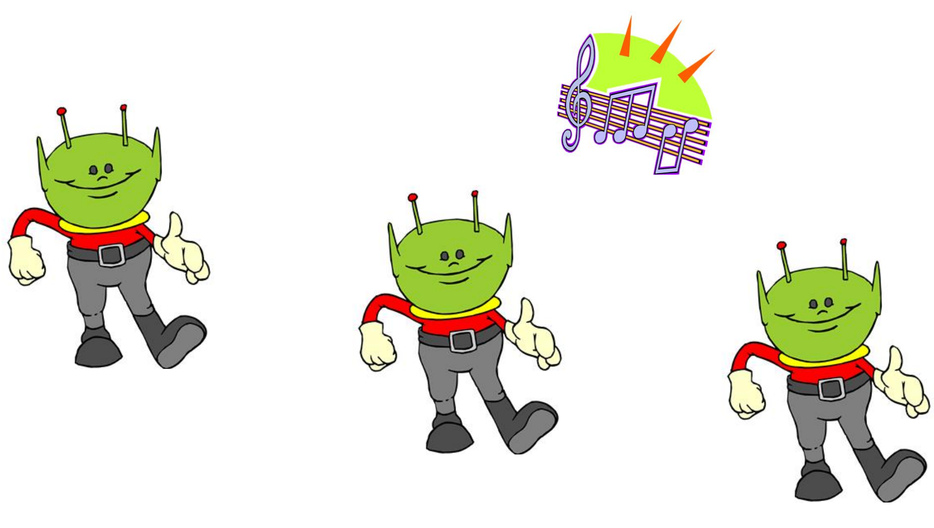
(Answers will vary)

The Planet Nine aliens really like to perform. One group of Planet Nine aliens brought instruments. They would like to perform in the "Welcome to Earth" Celebration. Unfortunately, the bandstand that the Planet Nine aliens will be performing on cannot hold all the Planet Nine aliens. If 7 Planet Nine aliens can fit on each row and there are 4 rows, how many Planet Nine aliens can perform on the bandstand? Explain your answer, and include the algorithm (number sentence).

Possible response: *There are 4 groups of 7 Planet Nine aliens, so that would be a total of 28 Planet Nine aliens. I could just add 7 four times or I could multiply 4 times 7. The number sentence would be 4 x 7 = 28.*

The city is considering putting up 2 more bandstands that are the same size. How many Planet Nine aliens can perform now?

84 Planet Nine aliens



Marching Band Planet Nine Aliens—Kovalevsky

(Answers will vary)

The Planet Nine aliens really like to perform. One group of Planet Nine aliens brought instruments. They would like to perform in the “Welcome to Earth” Celebration. Unfortunately, the bandstand that the Planet Nine aliens will be performing on cannot hold all the Planet Nine aliens. The city has decided to build a new bandstand. If 36 Planet Nine aliens want to perform, list the different ways the city could build one bandstand (remember each row needs to have an equal number of Planet Nine aliens).

Possible response: *The city could build a bandstand with 1 row and 36 columns, 36 rows and 1 column, 2 rows and 18 columns, 18 rows and 2 columns, 3 rows and 12 columns, 12 rows and 3 columns, 4 rows and 9 columns, 9 rows and 4 columns, or 6 rows and 6 columns.*

How do you know that you have listed all the ways to create the bandstand?

Possible response: *I know that I have all the ways because I listed all the factors that make 36.*

Possible response: *I know that I have all the ways because I made an organized list starting with 1 row, then 2 rows, and 3 rows and so on. I made sure that I switched the number of rows for the number of columns for each one, too.*

What if the city decided to build 2 bandstands for the 36 Planet Nine aliens—one for each side of the road. How many Planet Nine aliens could perform (remember each row needs to have an equal number of Planet Nine aliens)?

72 Planet Nine aliens



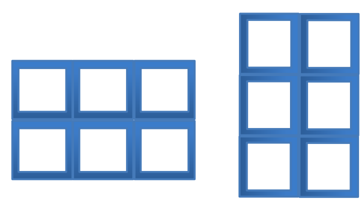

Collectibles—Fibonacci

Sloosa has lots of puppets for plays. She has decided to build a box with a space for each puppet. How many ways can she build her box if she wants the box to fit exactly 16 puppets? Fill in the chart below to show how many rows and columns there would be for each type of box.

Rows	Columns	Total Spaces for Puppets
1	*16*	*16*
16	*1*	*16*
2	*8*	*16*
8	*2*	*16*
4	*4*	*16*

How many ways can she build her box?

5 ways



Does it matter which way she places the box on the wall? Why or why not?

Possible response: *It doesn't matter which way the boxes are placed on the wall because she will still be able to fit all 16 puppets into the box.*

Sloosa’s friend Spudnie offers to build her 2 more boxes to hold an extra 16 puppets each. Choose one way that Sloosa could have had Spudnie build her the boxes. Draw a picture below of what the 3 boxes next to each other will look like.

Answers will vary. *Check student’s work.*

How many columns do you need? How many rows do you need?

Answers will vary. *Check student’s work.*

How many puppets can Sloosa fit on her wall now?

48 puppets

Write an algorithm or number sentence that explains how to find the total number of puppets.

Answers will vary. *Check student’s work.*
Possible response: *16 + 16 + 16 = 48*

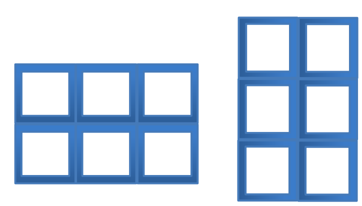
Collectibles—Diophantus

Sloosa has lots of puppets for plays. She has decided to build a box with a space for each puppet. How many ways can she build her box if she wants the box to fit exactly 24 puppets? Fill in the chart below to show how many rows and columns there would be for each type of box.

Rows	Columns	Total Spaces for Puppets
1	*24*	*24*
24	*1*	*24*
2	*12*	*24*
12	*2*	*24*
3	*8*	*24*
8	*3*	*24*
4	*6*	*24*
6	*4*	*24*

How many ways can she build her box?

8 ways



Does it matter which way she places the box on the wall? Why or why not?

Possible response: *It doesn't matter which way the boxes are placed on the wall because she will still be able to fit all 24 puppets into the box.*

Sloosa's friend Spudnie offers to build her 2 more boxes to hold an extra 16 puppets each. Choose one way that Sloosa could have Spudnie build her the boxes. Draw a picture below of what the 3 boxes next to each other will look like.

Answers will vary. *Check student's work.*

How many columns do you need? How many rows do you need?

Answers will vary. *Check student's work.*

How many puppets can Sloosa fit on her wall now?

72 puppets

Write an algorithm or number sentence that explains how to find the total number of puppets.

Answers will vary. *Check student's work.*
Possible response: *24 + 24 + 24 = 72 or 3 x 24 = 72*

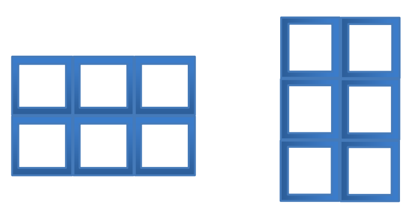
Collectibles—Kovalevsky

Sloosa has lots of puppets for plays. She has decided to build a box with a space for each puppet. How many ways can she build her box if she wants the box to fit exactly 36 puppets? Fill in the chart below to show how many rows and columns there would be for each type of box.

Rows	Columns	Total Spaces for Puppets
1	*36*	*36*
36	*1*	*36*
2	*18*	*36*
18	*2*	*36*
3	*12*	*36*
12	*3*	*36*
4	*9*	*36*
9	*4*	*36*
6	*6*	*36*

How many ways can she build her box?

9 ways



Does it matter which way she places the box on the wall? Why or why not?

Possible response: *It doesn't matter which way the boxes are placed on the wall because she will still be able to fit all 36 puppets into the box. For example, if she built her box with 4 rows of 9 spaces she could fit 36 puppets. If she built her box with 9 rows of 4 spaces, the commutative property says that the answer will be the same as 4 x 9.*

Is there a reason why it might matter which way she attaches the box to the wall?

Possible response: *It may matter if you have only a certain amount of space on your wall. If your wall is only 6 feet in length, a box that has 4 rows of 9 foot long spaces, it wouldn't fit.*

Sloosa's friend Spudnie offers to build her 2 more boxes to hold an extra 16 puppets each. Choose one way that Sloosa could have Spudnie build her the boxes. Draw a picture below of what the boxes next to each other will look like.

Answers will vary. *Check student's work.*

How many columns do you need? How many rows do you need?

Answers will vary. *Check student's work.*

How many puppets can Sloosa fit on her wall now?

108 puppets

Write an algorithm or number sentence that explains how to find the total number of puppets.

Answers will vary. *Check student's work.*
Possible response: *36 + 36 + 36 = 108 or 3 x 36 = 108.*

Exploring Factors of 12

The number I am factoring is: ____12___

1. Multiplication Representation

_*4*__ Piles x __*3*___ Shapes in Each Pile = ___*12*____ Total Shapes

Division Representation

$$\frac{\text{12 Total Shapes}}{\text{4 Piles}} = \text{3 Shapes in Each Pile}$$

2. Multiplication Representation

_*3*_ Piles x __*4*___ Shapes in Each Pile = ____*12*____ Total Shapes

Division Representation

$$\frac{\text{12 Total Shapes}}{\text{3 Piles}} = \text{4 Shapes in Each Pile}$$

The number I am factoring is: ____12___

3. Multiplication Representation

_*6*_ Piles x __*2*___ Shapes in Each Pile = ___*12*___ Total Shapes

Division Representation

$$\frac{\textit{12 Total Shapes}}{\textit{6 Piles}} = \textit{2 Shapes in Each Pile}$$

4. Multiplication Representation

_*2*__ Piles x __*6*__ Shapes in Each Pile = ____*12*____ Total Shapes

Division Representation

$$\frac{\textit{12 Total Shapes}}{\textit{2 Piles}} = \textit{6 Shapes in Each Pile}$$

The number I am factoring is: ____12___

5. Multiplication Representation

_*1*__ Pile x __*12*__ Shapes in Each Pile = ___*12*____ Total Shapes

Division Representation

$$\frac{\textit{12 Total Shapes}}{\textit{1 Pile}} = \textit{12 Shapes in Each Pile}$$

6. Multiplication Representation

_*12*_ Piles x __*1*__ Shape in Each Pile = ____*12*____ Total Shapes

Division Representation

$$\frac{\textit{12 Total Shapes}}{\textit{12 Piles}} = \textit{1 Shape in Each Pile}$$

The factors of 12 are: *1, 2, 3, 4, 6, 12*.

Finding Factors

(Answers will vary)

(Check student's work)

The number I am factoring is: __________

Multiplication Representation

____ Piles x ______ Shapes in Each Pile = _________ Total Shapes

Division Representation

Multiplication Representation

____ Piles x ______ Shapes in Each Pile = _________ Total Shapes

Division Representation

(Answers will vary)

(Check student's work)

The number I am factoring is: __________

Multiplication Representation

____ Piles x ______ Shapes in Each Pile = _________ Total Shapes

Division Representation

Multiplication Representation

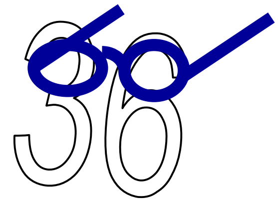
____ Piles x ______ Shapes in Each Pile = _________ Total Shapes

Division Representation

Number Sentences for 36

(Answers will vary)

Fill in each blank below with a number sentence that is equal to 36. Try to use number sentences that include addition, subtraction, multiplication, and division.



18 + 18 = *30 + 6* = *24 + 12* =

45 – 9 = *9 + 9 + 9 + 9* = *1 + 1 + 1 + 33* =

9 x 4 = *3 x 12* = *7 x 4 + 8* =

5 x 8 – 4 = *6 x 7 + 8 – 14* = *36 + 0* =

35 + 1 = *72 / 2* = *6 x 6 + 0* =

25 + 12 – 1 = *11 x 3 + 3* = *68 / 2 + 2* =

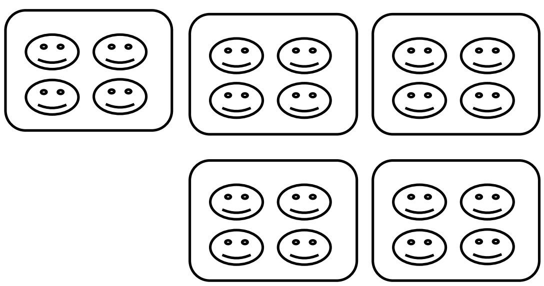
12 x 6 / 2 = *61 – 30 + 5* = *5 x 7 + 1* =

37 – 1 = *6 x 6* = 36

Assessment: Creative Mathematicians

(Answers will vary)

Directions: In each box, write your own multiplication problem. Show that you know what the commutative property means by rewriting the multiplication problem. Then pick 2 multiplication problems to create a picture that can be used to solve the problem.

Possible response: *5 x 4 = 20 4 x 5 = 20* *There were 5 cars filled with Planet Nine aliens who were on their way to the movies. Each car fit 4 Planet Nine aliens. How many Planet Nine aliens arrived at the movies in the five cars?* 	

Challenge: Create a story problem that goes along with your pictures.

Helping Nacci Unlock Her Bag—Fibonacci

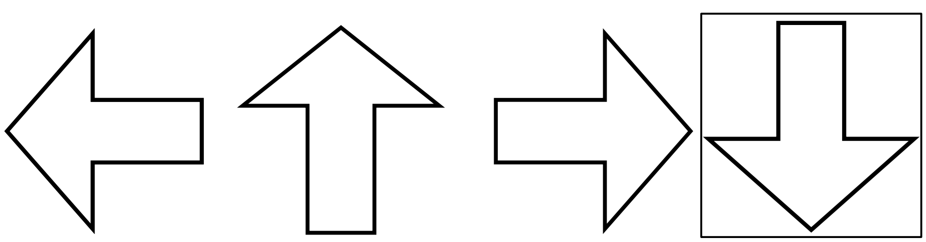
Nacci's mom sent her to Earth with a locked suitcase. Her mom was worried that she wouldn't remember the code so she set the first few parts of the lock for her.

Nacci,

I know you can be forgetful. The first parts of the lock are set. You just need to find a logical choice for the rest of the combination.

Love,
Mom

What should the last symbol in the lock combination look like? Draw it in the box.



Explain how you cracked the code that Nacci's mom gave her.

Possible response: *The arrow is rotated one-quarter turn for each lock.*

Helping Nacci Unlock Her Bag—Diophantus

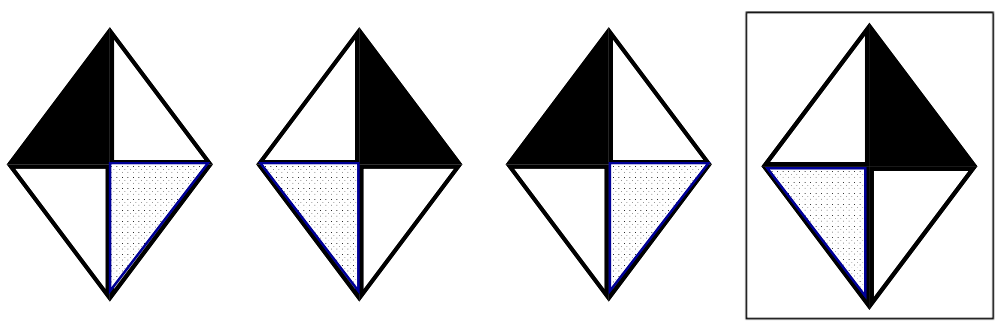
Nacci's mom sent her to Earth with a locked suitcase. Her mom was worried that she wouldn't remember the code so she set the first few parts of the lock for her.

Nacci,

I know you can be forgetful. The first parts of the lock are set. You just need to find a logical choice for the rest of the combination.

Love,
Mom

What should the last symbol in the lock combination look like?



Explain how you cracked the code that Nacci's mom gave her.

Possible response: *The shaded areas in the rhombus flips to the right or left for each part of the lock.*

Helping Nacci Unlock Her Bag—Kovalevsky

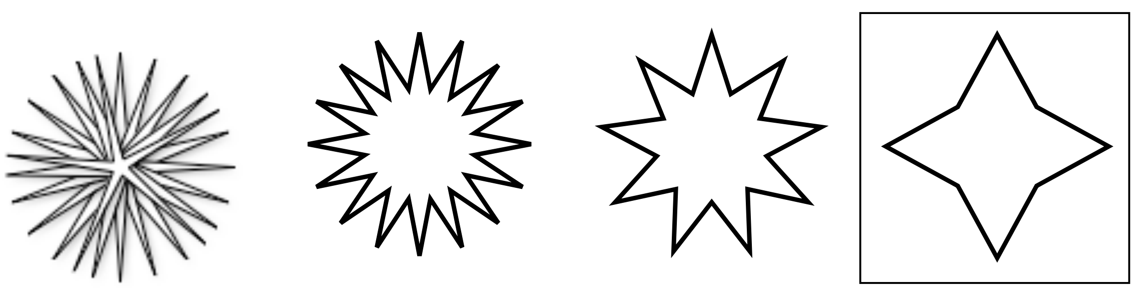
Nacci's mom sent her to Earth with a locked suitcase. Her mom was worried that she wouldn't remember the code so she set the first few parts of the lock for her.

Nacci,

I know you can be forgetful. The first parts of the lock are set. You just need to find a logical choice for the rest of the combination.

Love,
Mom

What should the last symbol in the lock combination look like?

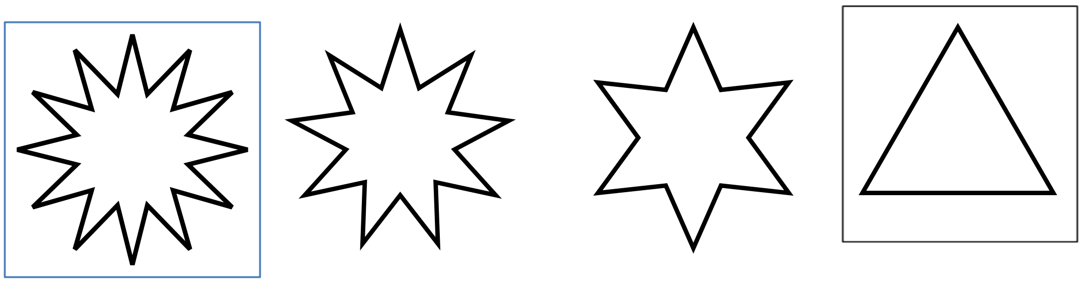


Explain how you cracked the code that Nacci's mom gave her.

Possible response: *The number of vertices is decreasing by perfect squares (i.e., 25, 16, 9, 4).*

Confounding Combinations—Fibonacci

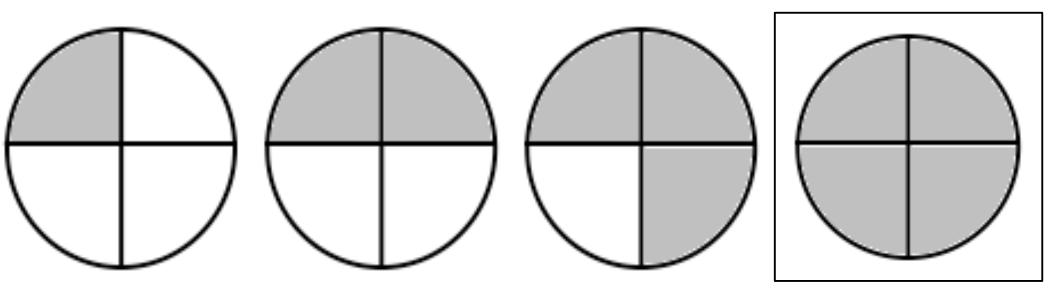
1. Unfortunately for Nacci that was not the only suitcase with a lock. Help her figure out the combinations for her other suitcases.



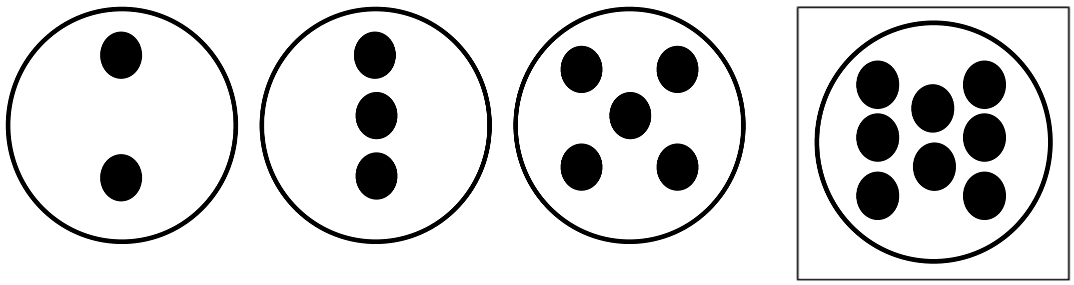
Explain how you cracked the code that Nacci's mom gave her.

Possible response: *The last shape has 3 vertices. The number of vertices is decreasing by multiples of 3.*

2. Practice opening more suitcases by filling in the last part of the lock. Explain how the pattern is growing or repeating.



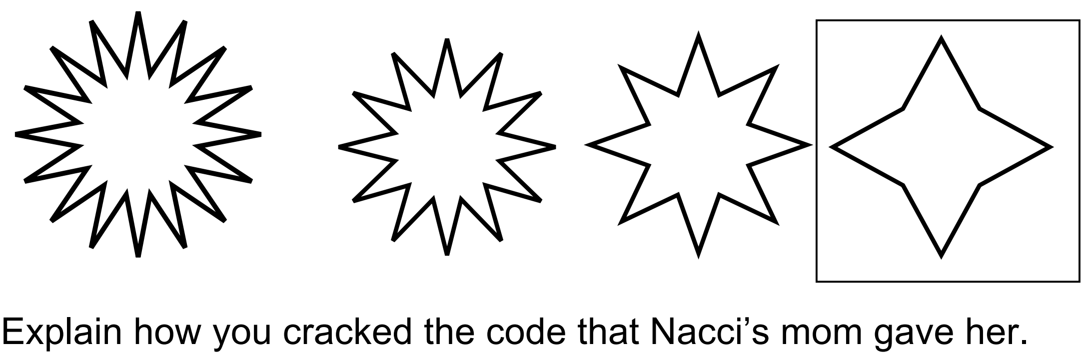
Possible response: *Increase the shading by one-fourth.*



Possible response: *This is the Fibonacci sequence or add an additional dot for each turn. +1, +2, +3 . . .*

Confounding Combinations—Diophantus

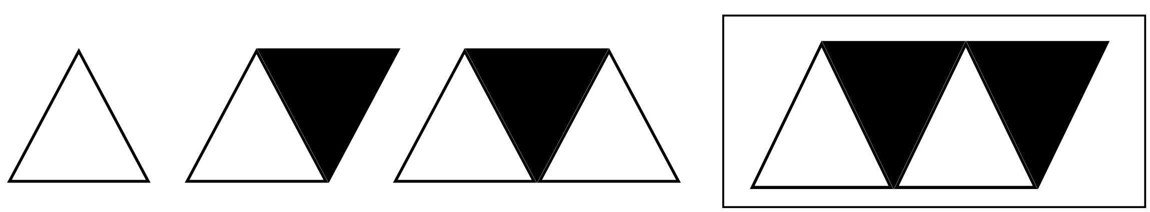
1. Unfortunately for Nacci that was not the only suitcase with a lock. Help her figure out the combinations for her other suitcases. Draw it in the box.



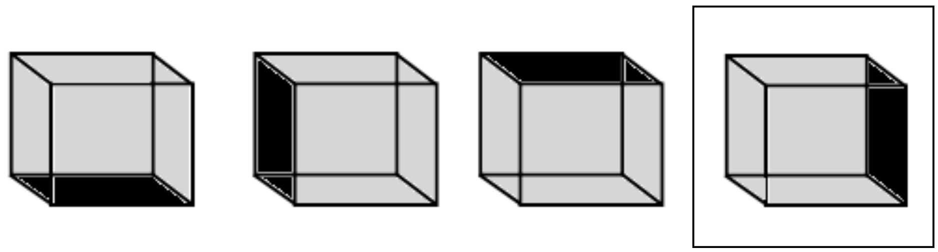
Explain how you cracked the code that Nacci’s mom gave her.

Possible response: *The last shape has 4 vertices. The number of vertices is decreasing by multiples of 4.*

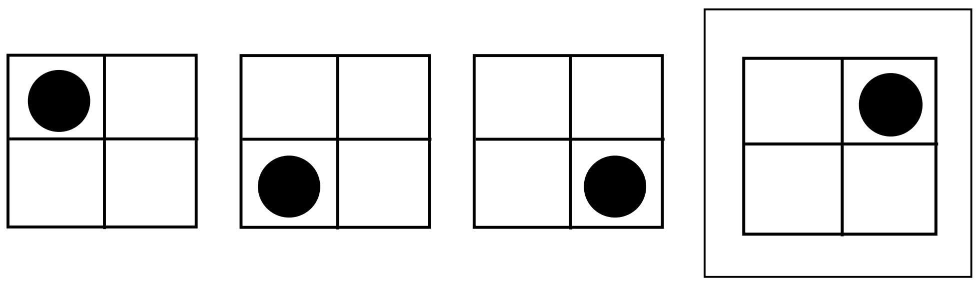
2. Practice opening more suitcases by filling in the last part of the lock. Explain how the pattern is growing or repeating.



Possible response: *Add another black triangle after the second white triangle.*



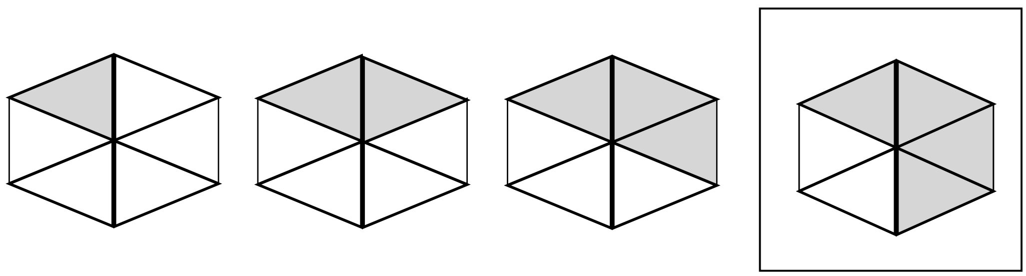
Possible response: *The box is **turned** once clockwise for each part of the lock.*



Possible response: *The dot moves clockwise three spaces each turn.*

Confounding Combinations—Kovalevsky

1. Unfortunately for Nacci that was not the only suitcase with a lock. Help her figure out the combinations for her other suitcases.

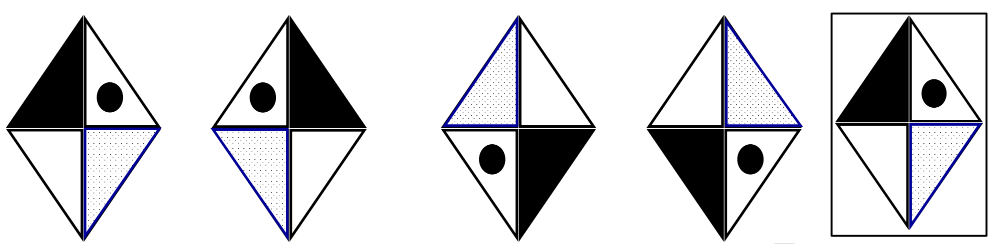


Possible response: *Increase the shading by 1/6 in clockwise.*

Explain how you cracked the code that Nacci's mom gave her.

Possible response: *Increase the shading by 1/6 in clockwise.*

2. Practice opening more suitcases by filling in the last part of the lock. Explain how the pattern is growing or repeating.

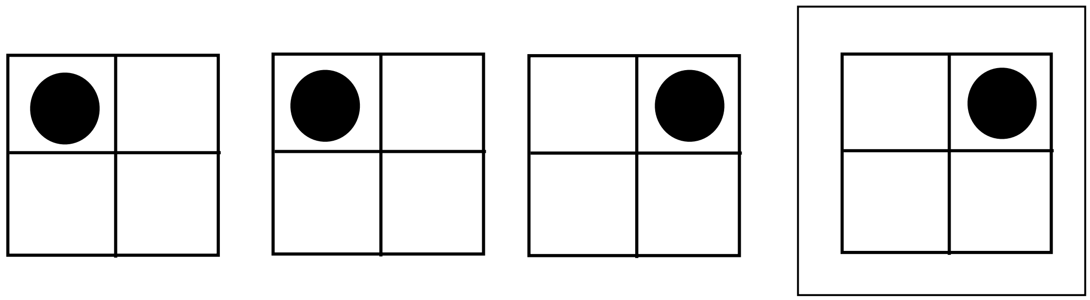


Possible response: *Flip over: Flip down*

3. Predict which quadrant (or triangle) the dot will be on the 20th move.

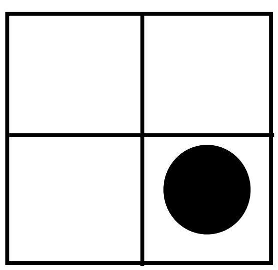
Possible response: *The dot will be on the top left quadrant.*

4. Fill in the last part of the lock. How many different ways can you explain the pattern?

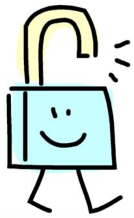


Possible response: *It is a repeating pattern that is in the top left quadrant twice and then the top right quadrant twice. OR The dot moves clockwise. First it moves 4 times, then it moves 9 times, then it moves 16 times. Note that these numbers are also perfect squares.*

Optional Challenge:
As students are completing the assignment, you may want to add the following visual to the above pattern, and have students re-examine the pattern, and write a new explanation.



Possible response: *It is still a repeating pattern. The dot is now in the bottom right quadrant. OR The dot moved clockwise 25 times which is also a perfect square. So starting with the first symbol, the dot moved clockwise 4, 9, 16, and then 25 times.*



Locking Into Creative Combinations—Homework

(Answers will vary)

(Check student's work)

Directions: Imagine that you are a locksmith for a Planet Nine alien luggage company! Put on your creative thinking cap and design your own repeating pattern combination using 5 symbols. Draw the symbols for your combination in the boxes below:

Explain why your combination is a repeating pattern.

Create another combination for the luggage. This time design your own growing pattern using 5 symbols. Draw the symbols for your combination in the boxes below:

Explain why your combination is a growing pattern.

Teaching Planet Nine Aliens Mathematical Terms

(Answers will vary)

A new species of Planet Nine aliens has arrived on Earth! It is very difficult to communicate with them because they don't speak English. Choose 3 words that you think the Planet Nine aliens should learn first. Write the 3 words on the lines below.

_________________ _________________ _________________

The Planet Nine aliens decide that they will learn the 3 words by repeating them over and over again in order.

1. What will be the 7th word that the Planet Nine aliens say? Explain your thinking.

Possible response: *The 7th is the same as the 1st word. I found this out by counting to 7 moving my finger over each word and going back to the 1st word after I counted the 3rd word.*

2. What will be the 30th word the Planet Nine aliens say? Explain your thinking.

Possible response: *The 30th word is the same as the 3rd word. I found this out by noticing any number that can be evenly divided by 3 will produce the 3rd word of 3.*

Improving Planet Nine Aliens' Vocabulary—Fibonacci

(Answers will vary)

The Planet Nine aliens have mastered the first 3 words you taught them. They are eager to learn more! Write the next 5 words you will teach the Planet Nine aliens on the lines below.

________________ ________________ ________________

________________ ________________

The Planet Nine aliens learn the 5 words by repeating them over and over again in order.

1. What will be the 20th word that the Planet Nine aliens say? Explain your thinking.

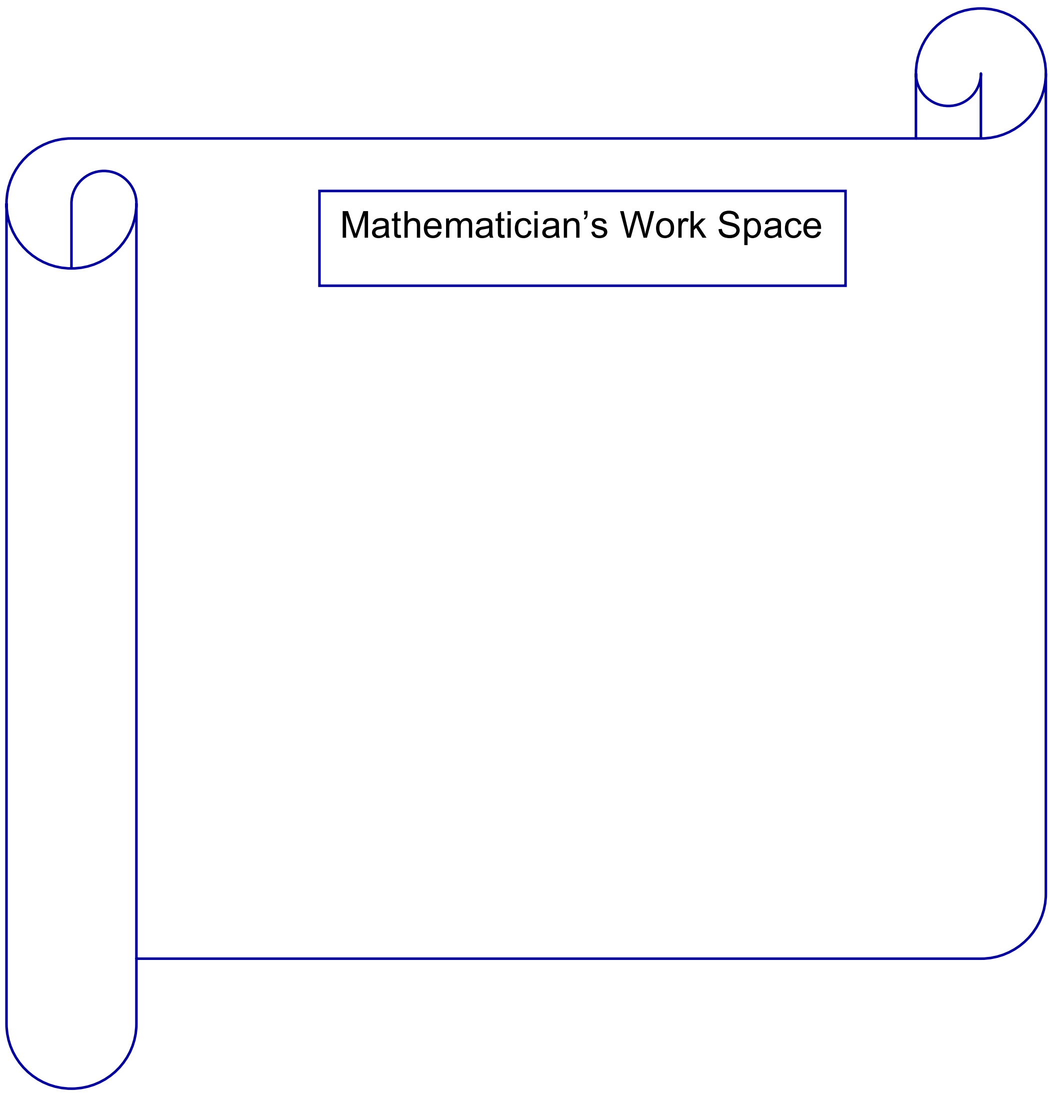
Possible response: *The 20th word is the same as the 5th. I know this because every number that can be divided evenly by 5 would result in the 5th word.*

2. What will be the 99th word the Planet Nine aliens say? Explain your thinking.

Possible response: *The 99th word is the same as the 4th. I know this because 99 is one less than a number evenly divided by 5, which makes the word said one less than the 5th, therefore the 4th.*

3. Can you think of a different way to figure out what the 99th word would be? Explain this method.

Possible response: *I could write out the words until I have reached 99 words and see what the 99th word is.*



Improving Planet Nine Aliens' Vocabulary—Diophantus

(Answers will vary)

The Planet Nine aliens have mastered the first 3 words you taught them. They are eager to learn more! Write the next 6 words you will teach the Planet Nine aliens on the lines below.

________________ ________________ ________________

________________ ________________

The Planet Nine aliens learn the 6 words by repeating them over and over again in order.

1. What will be the 20th word that the Planet Nine aliens say? Explain your thinking.

Possible response: *The 20th word would be the same as the 2nd word. I know this because every 6 words, the pattern repeats, so after three repeats, 18 words would be written, leaving 2 more words for the 20th word.*

2. What will be the 99th word the Planet Nine aliens say? Explain your thinking.

Possible response: *The 99th word would be the same as the 3rd word, because any number that divides evenly by 6 will result in the 6th word and 96 divides evenly by 6, leaving an additional 3 words for the 99th.*

3. Can you think of a different way to figure out what the 99th word would be? Explain this method.

Possible response: *I could also count upwards, touching each word and starting from the beginning after I go past the 6th word. When I reach 99, whatever number my finger is under is the 99th.*

Mathematician's Work Space

Teaching Students Planet Nine Alien Words

(Answers will vary)

The Planet Nine aliens have their own language called Alienese. They would like for you to learn how to say the 4 Alienese words listed below.

Jobuku   freligo   yuyu   bleebee

1. If you repeat the 4 words over and over again in order, what is the 20th word you will say? Explain your thinking.

Possible response: *I said the 4 words while keeping a tally, making the 20th bleebee.*

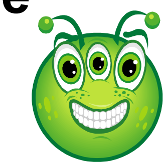
2. If you repeat the 4 words over and over again in order, what is the 54th word you will say? Explain your thinking.

Possible response: *The words repeat after the 4th word so the 52nd would be the same as the 4th that means the 54th is the same as the 2nd.*

3. Can you think of a different way to figure out what the 54th word would be? Explain this method.

Possible response: *I could write out the words until I have written out the 54th word, and see what word that is.*

Coasting at the Amusement Park! Table



Number of cars	Number of beings
3	*7*
4	*10*
5	*13*
6	*16*
7	*19*
8	*22*
9	*25*
10	*28*
11	*31*
12	*34*
13	*37*
14	*40*
15	*43*
16	*46*
17	*49*
18	*52*
19	*55*

Coasting at the Amusement Park!—Fibonacci

1. Toxo and his friends are going to ride the Rocket Rollercoaster at the Amusement Park. There are a total of 7 cars on the ride. How many of Toxo's friends can go for a ride?

19 friends (18 friends + Toxo)

2. Six of Toxo's friends arrived late at the rollercoaster, but all 10 cars were full! How many more middle cars would be needed so the 6 friends can ride the rollercoaster?

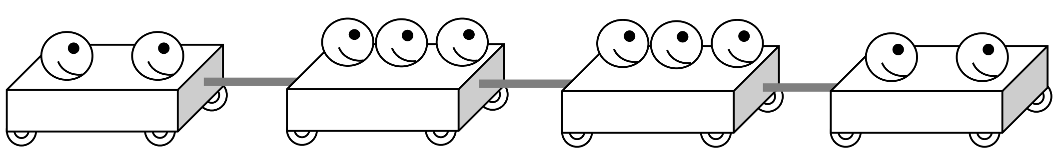
2 more middle cars are needed.

3. Imagine that there were 20 cars on the Rocket Rollercoaster! Now how many beings could ride altogether? Remember that the first and last car can only seat 2 beings.

58 "beings"

4. Explain how you found the answer.

Possible response: *Since there are 20 cars altogether, you would multiply 20 times 3 which is 60. Then you subtract 2 for the first and last car to get 58 beings altogether.*



Coasting at the Amusement Park!—Diophantus/Kovalevsky

1. Imagine that there were 19 cars on the Rocket Rollercoaster! Now how many beings could ride altogether? Remember that the first and last car can only seat 2 beings.

55 “beings”

2. Explain how you found the answer.

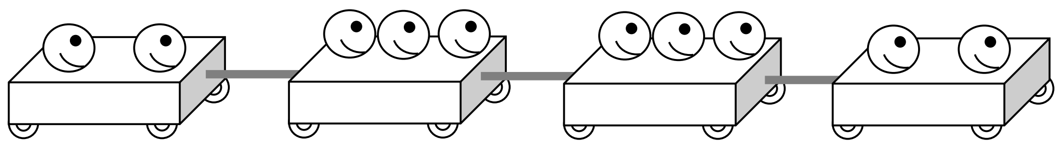
Possible response: *If you use the formula, 3n – 2, you would put 19 into the formula. To figure out 3 x 19, you could multiply 3 x 10 first, which is 30 and then add it to 3 x 9, which is 27. Add 30 + 27 to get 57. Next, subtract 2 from 57, which is 55. So 55 beings could fit in 19 rollercoaster cars.*

3. Toxo and his friends are going to ride the Rocket Rollercoaster at the Amusement Park. There are a total of 10 cars on the ride. Two of Toxo’s friends get into each car. How many of Toxo’s human friends can fill up the rest of the seats?

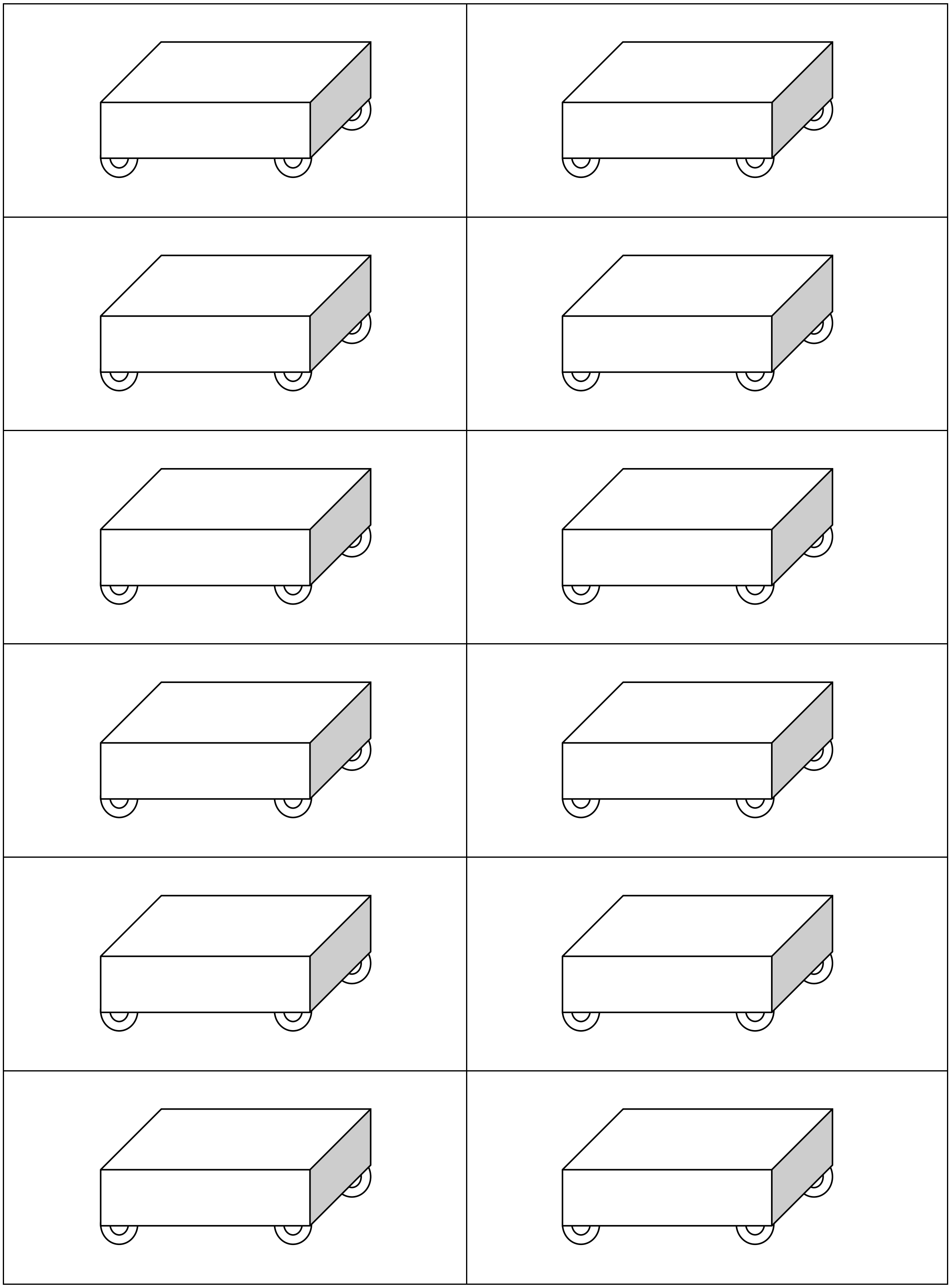
8 humans

4. Explain how you found the answer.

Possible response: *If 2 beings sit in each of the 10 cars, there would be 20 beings altogether so far. To figure out how many more can fit, you could find out how many beings could fit into 10 cars altogether by using the formula, 3n – 2. If you put in 10, the formula would read (3 x 10) – 2, which equals 28. Then subtract 28 – 20, which equals 8. So 8 beings can still fit into the 10 cars.*



Rollercoaster Cars Cutouts



Super Challenges

Super Challenge 1	Super Challenge 2
Imagine that the first and last rollercoaster cars have only 2 wheels and the middle cars have 4 wheels. How many wheels would there be altogether on 10 rollercoaster cars? *36 wheels* Create your own word problem about the rollercoaster cars. Trade problems with a partner and solve.	If one human sat in each of the rollercoaster cars, how many Planet Nine aliens could fill up the rest of the seats if there are 10 cars altogether? *18 Planet Nine aliens*
Super Challenge 3	**Super Challenge 4**
Toxo and 27 of his friends want to ride the Rocket Rollercoaster at the Amusement Park. How many rollercoaster cars are needed so that Toxo and all of his friends can ride at the same time? *10 cars*	Imagine that 3 “beings” could fit in the first and last rollercoaster cars and that 4 “beings” could fit in the middle cars! How many “beings” could fit in 10 cars now? *38 beings*

Coasting at the Amusement Park!—
Homework

1. Toxo's cousins from Pluto went for a ride on the Rocket Rollercoaster. There were 11 cars on the tracks of the rollercoaster. How many "beings" could go for a ride on the Rocket Rollercoaster? Explain your thinking.

Possible Response: *31 "beings" could ride the rollercoaster if there were 11 cars. Students' answers will vary for the explanation. Responses might include that students used repeated addition to find the number of "beings" for the first and last car* ($2 + 2 = 4$) *and the number of "beings" for the middle cars* ($3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 27$)*. The next step would be to add the two sums* ($4 + 27 = 31$ *"beings"). Students might solve the problem by calculating* $(9 \times 3) + (2 \times 2) = 31$*. Some students may be able to use the formula* $3n - 2$*, where* $n = 11$*. Therefore,* $3(11) - 2 = 31$*.*

2. How many "beings" can fit into 13 cars on a rollercoaster?

37 "beings"

3. How many cars are needed to fit 13 "beings?"

5 cars

4. If 3 Planet Nine aliens are already sitting in a car on the rollercoaster that has 5 cars, how many more Planet Nine aliens or humans can join them for a ride?

10 "beings"

5. How many rollercoaster cars are needed to fit 15 Planet Nine alien and human friends? Explain your thinking.

6 cars are needed to fit 15 Planet Nine alien and human friends. A possible explanation might include that 6 cars can fit 16 "beings" but 5 cars can only fit 13 "beings." So there must be enough cars to fit all 15 of the Planet Nine alien and human friends.

Toxo's Out of This World Ideas!—
Increasing and Decreasing Patterns

Term (t)	1	2	3	4	5	6	7
Number (n)	50	54	58	*62*	*66*	*70*	*74*

1. What is the rule for this pattern?

+4

2. What is the 7th term?

74

Term (t)	1	2	3	4	5	6	7
Number (n)	50	42	34	*26*	*18*	*10*	*2*

3. What is the rule for this pattern?

-8

4. What is the 7th term?

2

Toxo decided to frost and decorate the cookies he made. After an hour of frosting and decorating cookies he had 260 cookies left. After 2 hours he had 220 cookies left to frost and decorate. Fill in the table to figure out how many cookies Toxo had left to frost and decorate after 7 hours.

Hour	1	2	3	4	5	6	7
Number of cookies left	260	220	180	*140*	*100*	*60*	*20*

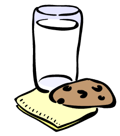
5. What is the rule for this pattern?

-40

6. What is the 7th term?

20

Toxo's Out of This World Cookie Recipe!—Fibonacci



Toxo would like to make 10 batches of cookies! Toxo needs your help to figure out how much flour he must buy at the store.

Directions: Identify the rule for each pattern. Fill in the following table to find the 10th term in the growing pattern. Answer each question below.

Number of batches	1	2	3	4	5	6	7	*8*	*9*	10
Cups of Zorg flour	4	8	12	*16*	*20*	*24*	*28*	*32*	*36*	*40*

1. What is the 10th term?

40

2. What is the rule for the growing pattern of cups of Zorg flour?

+4

3. Is there another way that you could figure out the 10th term for the cups of Zorg flour without having to fill out the table?

Possible response: *If one batch calls for 4 cups of flour, then 10 batches would need 10 groups of 4 cups of flour. I know that 4 x 10 = 40. So Toxo would need 40 cups of flour for 10 batches.*

Here is Toxo's table for his Zinky chips. He found a new recipe for 10 batches of cookies. But he only needs 4 batches. Look carefully at the pattern. Help Toxo finish filling in his table.

Number of batches	10	9	8	7	6	5	4
Small scoops of Zinky chips	30	27	24	*21*	*18*	*15*	*12*

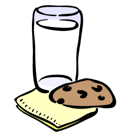
4. How many scoops of Zinky chips will Toxo need to buy to make 4 batches of cookies?

12 small scoops of Zinky chips

5. Is there another way that you could figure out how many scoops of Zinky chips are needed for 4 batches without having to fill out the whole table? Explain your thinking.

Possible response: *I know that the pattern is decreasing by 3s. To find out how many scoops of chips are needed in 4 batches of cookies, I would have to subtract 3 four times from 24. I know that 3 x 4 = 12 and that 24 – 12 = 12. So 4 batches need 12 scoops of chips.*

Toxo's Out of This World Cookie Recipe!—Diophantus



Toxo would like to make 12 batches of cookies! But now he is out of Zinky chips! How much Zorg flour does he need to buy at the store? Fill in the table below. Answer the questions.

Number of batches	1	2	3	4	5	6	7	*8*	*9*	*10*	*11*	12
Cups of Zorg flour	4	8	12	*16*	*20*	*24*	*28*	*32*	*36*	*40*	*44*	*48*

1. What is the rule for the growing pattern of cups of Zorg flour?

+4

2. How many cups of Zorg flour will Toxo need to buy to make 12 batches of cookies?

48

3. Is there another way that you could figure out the 12th term for the cups of Zorg flour without having to fill out the table?

Possible response: *If one batch calls for 4 cups of flour, then 12 batches would need 12 groups of 4 cups of flour. I know that 4 x 10 = 40. To get to 12 batches, I would add 8 more cups since 2 x 4 = 8. So Toxo would need 48 cups of flour for 12 batches.*

Here is Toxo's table for his Zinky chips. He found a new recipe for 10 batches of cookies. But he only needs 4 batches. Look carefully at the pattern. Help Toxo finish filling in his table.

Number of batches	10	9	8	7	6	5	4
Small scoops of Zinky chips	40	36	32	*28*	*24*	*20*	*16*

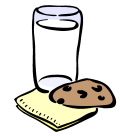
4. How many scoops of Zinky chips will Toxo need to buy to make 4 batches of cookies?

16 small scoops of Zinky chips

5. Is there another way that you could figure out how many scoops of Zinky chips are needed for 4 batches without having to fill out the whole table? Explain your thinking.

Possible response: *I know that the pattern is decreasing by 4s. To find out how many scoops of chips are needed in 4 batches of cookies, I would have to subtract 4 four times from 32. I know that 4 x 4 = 16 and that 32 – 16 = 16. So 4 batches need 16 scoops of chips.*

Toxo's Out of This World Cookie Recipe!—Kovalevsky



Toxo would like to make 12 batches of cookies! But now he is out of all his ingredients! How much of each ingredient does he need to buy at the store?

Directions: Answer each question below.

Number of batches	1	2	3	4	5	6	7	. . .	. . .	. . .	. . .	12
Cups of Zorg flour	7	*14*	*21*	*28*	*35*	*42*	*49*	*56*	*63*	*70*	*77*	*84*

1. What is the rule for the growing pattern cups of Zorg flour?

+7

2. How many cups of Zorg flour will Toxo need to buy to make 12 batches of cookies?

84

Toxo just got news that his cousins are joining him and his friends when they go to the Amusement Park! Now Toxo would like to bake 22 batches of cookies! Toxo decided to draw a table to figure out how much Zookie butter he needs to buy at the store, but he has run out of room! Help Toxo figure out a **different** way to find the 22nd terms in the growing patterns for each of the ingredients instead of extending the table.

Number of batches	1	2	3	4	. . .	. . .	22
Tablespoons of Zookie butter	8	*16*	*24*	*32*	*80*	*160*	*176*

3. How many tablespoons of Zookie butter will Toxo need?

176 tablespoons of Zookie butter

Explain how you solved this growing pattern problem.

Possible response: *If one batch needs 8 tablespoons of butter, then 22 batches would need 22 groups of 8 tablespoons of butter. I know that 20 x 8 = 160. To get 2 more groups, I would do 2 x 8, which is 16. Then I can add 160 + 16 to get 176 cups of butter.*

Here is Toxo's table for his Zinky chips. He found a new recipe for 10 batches of cookies. But he only needs 4 batches. Look carefully at the pattern. Help Toxo finish filling in his table.

Number of batches	10	9	8	7	6	5	4
Small scoops of Zinky chips	70	63	56	*49*	*42*	*35*	*28*

4. How many scoops of Zinky chips will Toxo need to buy to make 4 batches of cookies?

28 small scoops of Zinky chips

5. Is there another way that you could figure out how many scoops of Zinky chips are needed for 4 batches without having to fill out the whole table? Explain your thinking.

Possible response: *I know that the pattern is decreasing by 7s. To find out how many scoops of chips are needed in 4 batches of cookies, I would have to subtract 7 four times from 56. I know that 4 x 7 = 28 and that 56 – 28 = 28. So 4 batches need 28 scoops of chips.*

Toxo’s Out of This World Ideas!—Homework

Toxo took a ceramics class and painted his own mixing bowls for his cookies. On the first bowl he painted 6 stripes. On the second bowl he painted 13 stripes. Figure out the growing pattern and complete the table.

Mixing bowl number	1	2	3	4	5	6	*7*	*8*	*9*
Number of stripes	6	13	20	*27*	*34*	*41*	*48*	*55*	*62*

1. What is the rule for the growing pattern of number of stripes?

+7

2. How many stripes are on the 12th mixing bowl?

83 stripes

3. Explain how you found the 12th term in question #2.

Possible response: *It might include using repeated addition. Students could add 7 to the previous term, 20, 9 times to get 83. Some students may be able to figure out the 12th term using the formula: 6n + (n – 1), which is simplified to 7n – 1. In this case, 7(12) – 1 = 83.*

Toxo decided to make 43 mixing bowls for all of his friends so that they could bake Planet Nine alien cookies as well! Each day during ceramics class Toxo painted a certain number of bowls. Figure out the growing pattern and fill in the table.

Day	1	2	3	4	5	6	7	8	9
Number of bowls left to paint	43	38	33	*28*	*23*	*18*	*13*	*8*	*3*

4. What is the rule for the growing pattern of bowls to paint?

-5

5. How many bowls are left to paint after the 9th day?

3 bowls

6. Explain how you found the 9th term in question #5.

Possible response: *It might include using repeated subtraction. Students could subtract 5 from the third term, 33, 6 times to get 3.*

Super Challenges

Super Challenge #1

Create your own growing pattern about Toxo and his great ideas in the kitchen! Fill in a few of the numbers in the table and write a story problem. Exchange your story problem with a friend. Have your friend solve the problem.

	1	2	3	4	5	6	7

Write your story problem here:

Answers will vary. *Check student's work.*

Questions for your friend about your story problem:

1. What is the rule for my pattern?

   Answers will vary. *Check student's work.*

2. What is the 7th term?

   Answers will vary. *Check student's work.*

Super Challenge #2

Fill in the missing numbers in the table below. Think about the pattern first.

Number of batches	1	2	3	4	5	6	7	. . .	. . .	20
Teaspoons of Planet Nine alien vanilla	7	*14*	*21*	*28*	*35*	*42*	*49*			*140*
Small scoops of Zinky chips	8	*16*	*24*	*32*	*40*	*48*	*56*			*160*

Answers will vary. *Check student's work.*

Super Challenge #3

Fill in the missing numbers in the table below. Think about the patterns first.

Number of batches	1	2	3	4	5	6	7	. . .	. . .	10
Zoofy mini eggs	*6*	12	*18*	*24*	*30*	*36*	*42*			*60*
Teaspoons of Planet Nine alien vanilla	*5*	*10*	15	*20*	*25*	*30*	*35*			*50*

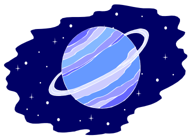
Answers will vary. *Check student's work.*

Super Challenge #4

Fill in the missing numbers in the table below. Think about the pattern first.

Term	1	2	3	4	5	6	7	8	9	10
Number	197	*195*	193	*191*	*189*	*187*	*185*	*183*	*181*	*179*

Answers will vary. *Check student's work.*



An Intergalactic Mission Play

Narrator: *The leader of the Planet Nine aliens that are visiting planet Earth, named Captain Chavir, just received an urgent intergalactic message commanding that he and his spaceship crew must return to a planet on the other side of the Milky Way as soon as possible for a top secret mission! Captain Chavir is waiting to beam his crew up to the spaceship, which is hovering in Earth's upper stratosphere.*

Captain Chavir: *Mozalk Navigator, come in, please! Do you read me?*

Mozalk Navigator*: Mozalk here! Captain, what can I do for you?*

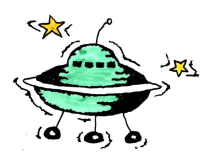
Captain Chavir: *Central command has ordered us to return home for a top secret mission! You must quickly help me get the crew back to the transporter station at the Earthly Alien Base. I can't beam up the crew because they are all spread out at the Amusement Park.*

Mozalk Navigator: *Yes, Captain! I will find the crew, but it may take a while.*

Captain Chavir: *No need to worry, Mozalk! I have calculated their estimated locations at the Amusement Park. I am going to send you secret encrypted clues to help you find the crew members along with a top secret map of the park.*

Mozalk Navigator: *Yes, Captain! I will stand by until I receive the top secret clues. Oh, and would it be ok if a few of my trusted human friends help me out?*

Captain Chavir: *Great idea, but make sure that they are mathematicians!*



Mozalk Navigator: *Yes, Captain!*

Find Captain Chavir's Crew!
Top Secret Clues!—Fibonacci

Find Diggy!
This Planet Nine alien can be found by finding the total number of toes and fingers of 3 of our Planet Nine alien friends. Each Planet Nine alien has 2 hands and 2 feet. Also, each Planet Nine alien has 5 fingers on each hand or 5 toes on each foot. **(Hint: Use mental math or draw a picture or a table.)**

Find Frazzy!
Here is a riddle to find Frazzy's location. Use the following information about Mr. Mailer's order of hay.

Four trucks were on their way.
Each bringing 3 bales of hay.

More bales were needed for Mr. Mailer,
so 2 extra bales were towed in a trailer.

How many bales of hay were delivered?

Find Quarg!

1. Find the 4th factor of the number 12 after all the factors have been placed in numerical order from least to greatest.
2. Now find the 4th multiple of 4.
3. To discover the number of this Ferris wheel car, find the difference between the two answers for the questions above.

Find Hilzo!

How many Planet Nine aliens can 5 ships hold?

# of ships	1	2	3 . . .
# of Planet Nine aliens	7	14	21

Find Larko!
Using only 2 addends, how many ways are possible to make a sum of 12?

Find Yile!
Find the missing numbers. Then find the sum of all the numbers in the boxes to discover the number for this Ferris wheel car.

$4 + 3 = \square + 5$ $12 - \square = 3 \times 2$

$\square + 7 = 20 - 6$ $\square \times 4 = 17 + 3$

Find Snork!
The Rocket Rollercoaster has a total of 6 cars. Four people can fit in the inside cars, but only 2 people can fit on the first and last cars. How many people can fit altogether in the 6 rollercoaster cars? **(Hint: Use mental math, make a table, draw a table, or use manipulatives.)**

Find Bleeb!
Solve for Y in the following equations:

$Y = 8 + 2 + 9 + 5$
$40 = 5 \times Y$
$24 / Y = 12$
$Y = 6 \times 3 + 7 + 3$

Find the sum of the four Y's. This will be the location of Bleeb.

“Captain Chavir, We Have Found Your Crew!”—Fibonacci

Diggy *60*	Frazzy *14*
Quarg *12*	Hilzo *35*
Larko *7*	Yile *20*
Snork *20*	Bleeb *62*

Find Captain Chavir's Crew!
Top Secret Clues!—Diophantus

Find Zilfred!
This Planet Nine alien can be found by finding the total number of toes and fingers of 5 of our Planet Nine alien friends. Each Planet Nine alien has 2 hands and 2 feet. Also, each Planet Nine alien has 4 fingers on each hand or 4 toes on each foot. **(Hint: Use mental math or draw a picture or a table.)**

Find Soland!
Here is a riddle to find Soland the Planet Nine alien's location. Use the following information about Mr. Mailer's order of hay.

Five trucks were on their way.
Each bringing 7 bales of hay.

More bales were needed for Mr. Mailer, so 2 extra bales were towed in a trailer.

How many bales of hay were delivered?

Find Nacci!
1. Find the 5th factor of the number 20 after all the factors have been placed in numerical order from least to greatest.
2. Now find the 5th multiple of 9.
3. To discover where Nacci is find the difference between the two answers for the questions above.

Find BeBop!

How many Planet Nine aliens can 5 ships hold?

# of ships	1	2	3 . . .
# of Planet Nine aliens	8	16	24

Find G4R4!
Using only 2 whole number addends, how many ways are possible to make the sum of 15?

Find Lindy!
Find the missing numbers. Then find the sum of all the numbers in the boxes to discover the number to locate this Planet Nine alien.

$14 + 3 = \square + 5$ $\quad$ $24 - \square = 3 \times 2$

$\square + 7 = 30 - 6$ $\quad$ $\square \times 4 = 17 + 3$

Find Heebie!
The Rocket Rollercoaster has a total of 8 cars. Seven people can fit in the inside cars, but only 4 people can fit on the first and last cars. How many people can fit altogether in the 8 rollercoaster cars? **(Hint: Use mental math, make a table, draw a table, or use manipulatives.)**

Find Im!
Solve for Y in the following equations:

$Y = 8 + 7 + 7 + 5$
$15 = 3 \times Y$
$36 / Y = 12$
$Y = (7 \times 7) + 7 + 5$

Find the sum of the four Y's. This will be the location of Im.

“Captain Chavir, We Have Found Your Crew!”—Diophantus

Find Zilfred! *80*	Find Soland! *37*
Find Nacci! *35*	Find BeBop! *40*
Find G4R4! *8*	Find Lindy! *52*
Find Heebie! *50*	Find Im! *96*

Find Captain Chavir's Crew!
Top Secret Clues!—Kovalevsky

Find Zroply!
This Planet Nine alien can be found by finding the total number of toes and fingers of 5 of our Planet Nine alien friends. Each Planet Nine alien has 3 hands and 2 feet. Also, each Planet Nine alien has 4 fingers on each hand or 4 toes on each foot.

Find Lilorlilan!
Here is a riddle to find Lilorlilan the Planet Nine alien's location. Use the following information about Mr. Mailer's order of hay.

Nine trucks were on their way.
Each bringing 9 bales of hay.

More bales were needed for Mr. Mailer,
so 7 extra bales were towed in a trailer.

How many bales of hay were delivered?

Find Vloop!
1. Find the 7th factor of the number 36 after all the factors have been placed in numerical order from least to greatest.
2. Now find the 5th multiple of 8.
3. To discover the number of this Ferris wheel car, find the difference between the two answers for the questions above.

Find Rukudig!

How many Planet Nine aliens can 6 ships hold?

# of ships	1	2	3 . . .
# of Planet Nine aliens	7	14	21

Find Zurp!
Using only 2 addends that are whole numbers, how many ways are possible to make the sum of 30?

Find Q'Lok!
Find the missing numbers. Then find the sum of all the numbers in the boxes to discover the number for this Ferris wheel car.

62 + 2 = $\square$ x 8   100 – $\square$ = 5 x 7

$\square$ + 7 = 76 – 50 $\square$ x 4 = 62 – 34

Find Hrtzzky-zzz
The Rocket Rollercoaster has a total of 10 cars. Five people can fit in the inside cars, but only 2 people can fit on the first and last cars. How many people can fit altogether in the 10 rollercoaster cars?

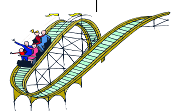
Find Paktaklak
Solve for Y in the following equations:
Y = 9 + 1 + 7 + 5 + 8
25 = 5 x Y
48 / Y = 12
Y = 81 / 27 – 18
Find the sum of the four Y's. This will be the location of Paktaklak.

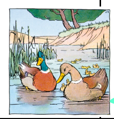
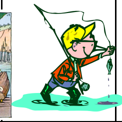
“Captain Chavir, We Have Found Your Crew!”—Kovalevsky

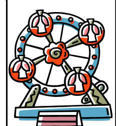
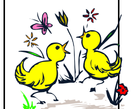
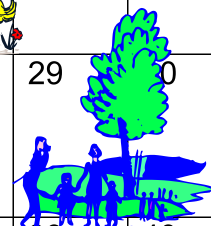
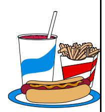
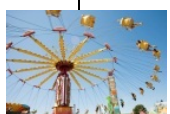
Find Zroply! *100*	Find Lilorlilan! *88*
Find Vloop! *28*	Find Rukudig! *42*
Find Zurp! *16*	Find Q’Lok! *99*
Find Hrtzzky-zzz! *44*	Find Paktaklak! *48*

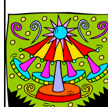
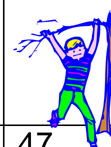
The Planet Nine Alien Amusement Park

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

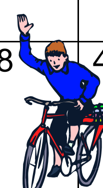


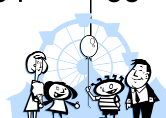



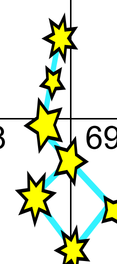







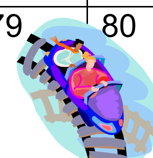




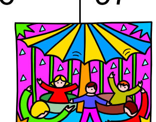
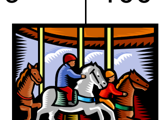




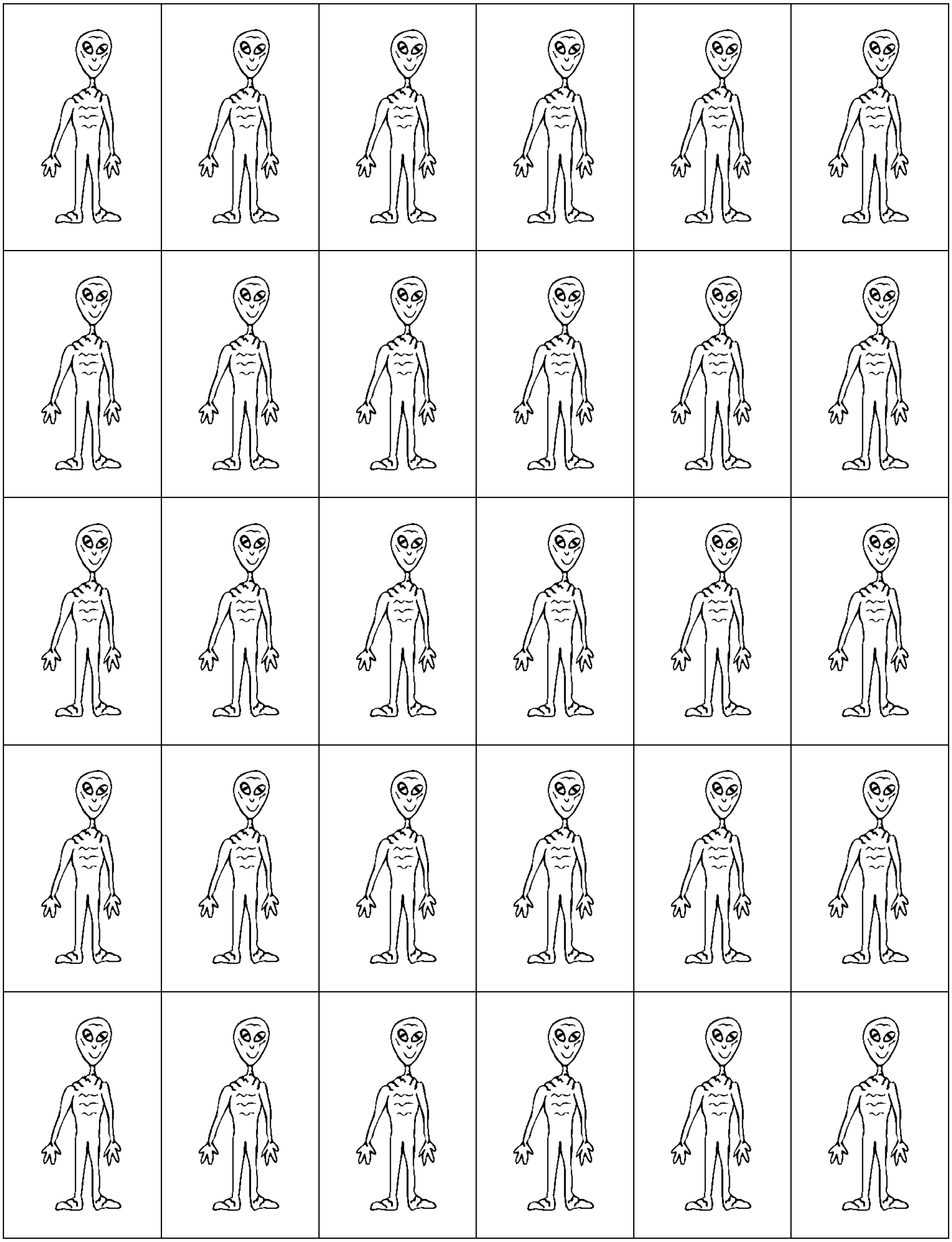




Planet Nine Alien Pictures (OPTIONAL)



If Aliens Taught Algebra Mathematicians' Glossary

Addend: A number that is being added to another number. Example: In $4 + 5 = 9$, numbers 4 and 5 are addends.

Algebra: A topic studied as part of mathematics; the study of mathematical symbols (e.g., numbers; equal, addition, or multiplication signs) and the rules for working with these symbols.

Array: A way to organize information in rows and columns.

Associative Property of Addition: The grouping of addends in a number sentence does not change the sum.

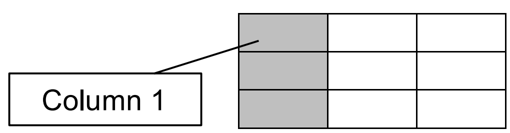
Astronomy: The scientific study of space, stars, planets, and other celestial bodies.

Astronomers: People who study astronomy.

Benchmark: A point of reference.

Column: A vertical arrangement of items or numbers in a list or table.

3
4
5

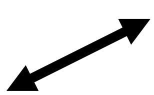


Commutative Property of Addition: The order that addends are added does not change the sum.

Commutative Property of Multiplication: The order that factors are multiplied does not change the product.

Decomposition: Breaking down a number to make a problem mentally easier to calculate.

Diagonal: A line that is on a slant compared to the top and sides of a page. Example:



Divisible: A whole number is *divisible* by another whole number if the remainder after dividing the two numbers is zero.

Division: A mathematical operation in which a number is subtracted from itself a certain number of times.

Equation: A mathematical sentence that contains an equal sign.

Estimate: An educated guess for the answer to an algorithm.

Estimating: Rounding numbers to calculate an answer such as a sum or difference.

Factors: The numbers in a multiplication problem that are multiplied together to arrive at the product.

Flip (Reflection): A term for describing the movement of a shape that is flipped to the left, right, up, or down.

Formula: A rule or function for a pattern of numbers to make a prediction about a specific term.

Function: A rule for calculating sums or differences when using input and output boxes.

Generalization: Stating a conclusion based on a small amount of information, instances, or items.

Growing Patterns: Patterns that increase or decrease in a linear manner. Patterns may “grow” in ascending order such as “2, 4, 6, 8, . . .” or they may “grow” in descending order such as “12, 9, 6, 3, . . .”

Horizontal: Parallel to the horizon. Example:

Inequality Sign: A sign used to represent a number sentence that is not equal (≠).

Mathematics: The study and use of numbers, patterns, and shapes.

Mathematicians: People who study or use mathematics in their work.

Multiple: The product of a whole number and any other whole number. Example: The multiples of 3 are 0, 3, 6, 9, 12, 15, . . .

Multiplication: A mathematical operation in which a number is added to itself a certain number of times.

Number Sentence: A mathematical sentence that contains any sign (equality or inequality)**;** an open number sentence that contains a variable or missing number.

Perfect Squares: Products that have the same two factors. For example, 81 is a perfect square because 9 x 9 = 81.

Prime Number: A whole number greater than one with two factors: 0 and itself.

Product: The answer to a multiplication problem.

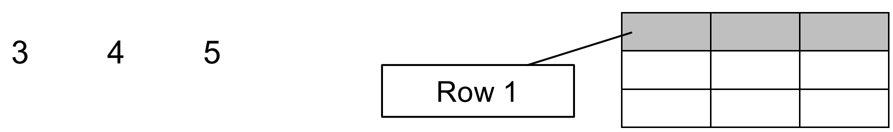
Property: A math rule.

Recompose: Putting numbers back together after decomposing them to make a problem mentally easier to calculate.

Repeating Pattern: A repeating arrangement of numbers or objects.

Rounding: Altering a number so that it is easier to use in calculations.

Row: A horizontal arrangement of items or numbers in a list or table.



Sets: Another term for a group.

Terms: Numbers in a pattern or sequence.

Turn (Rotation): A term for describing the movement of a shape in either a clockwise or counterclockwise direction.

Variable: A symbol or letter that represents a number or amount. Example: *a* is the variable in 4 x *a* = 12 or 4*a* = 12.

Vertical: At a right angle to the horizon (up and down). Example:

Thinking Like Mathematicians: Challenging All Grade 3 Students

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